Super Schrödinger in Super Conformal

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“Super Schrödinger in Super Conformal,”
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“More super Schrödinger algebras from psu(2,2|4),”
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1. Introduction

AdS/CFT correspondence as weak/strong coupling duality [Maldacena'97]

IIB string in $AdS_5 \times S^5 \Longleftrightarrow \mathcal{N}=4$ super Yang-Mills in 4-dim.

psu(2,2|4) $\supset$ so(2,4)$\times$su(4)

non-relativistic CFT (NRCFT) : QFT with Galilei enlarged to Schrödinger

cf) CFT : QFT with Poincare enlarged to conformal

NRCFT: accessible experimentally

fermions at unitarity (strongly coupled)

[Hagen'72, Mehen-Stewart-Wise'99, Nishida-Son'07,...]

A method to derive a NR system in d-dim from a relativistic system in $(d+1)$-dim

A scalar in $(d+1)$-dimensions $\partial^2\phi = 0$.

Let $t = \frac{1}{\sqrt{2}}(x^0 - x^d)$ and $\xi = \frac{1}{\sqrt{2}}(x^0 + x^d)$, and consider a mode with definite momentum along ξ, $\phi = \mathrm{e}^{-im\xi}\psi(t, x^i)$, and then $\left(i\partial_t + \frac{\nabla^2}{2m}\right)\psi = 0$.

This is invariant under the group of the Schrödinger algebra, Sch$(d-1)$:

- Galilean symmetry $\{H,\ P_i,\ K_i,\ J_{ij}\}$,
- dilatation D,
- Galilean special conformal C,
- central charge N (the momentum along ξ).

Schrödinger algebra Sch$(d-1)$ is simply related to conformal algebra so$(2, d+1)$.

Super Schrödinger algebras from psu(2,2|4), osp(8|4) and osp(8*|4)

[Yoshida-MS'0805,'0806]

What's the gravity dual ?

- AdS$_5$ on x^+ [Goldberger'0806, Barbon-Fuertes'0806]

  Schrödinger symmetry via boundary condition

  24 supersymmetric (see Appendix E in [Maldacena-Martelli-Tachikawa'0807])

- deformed AdS$_5$ [Son'0804,Balasubramanian-McGreevy'0804]

  string background with B, finite temperature and finite density [Herzog-Rangamani-Ross'0807, Maldacena-Martelli-Tachikawa'0807, Adams-Balasubramanian-McGreevy'0807]

  Non supersymmetric (even in zero temperature)

Contents

2. $\mathcal{N}=4$ superconformal algebra psu(2,2|4)

The super isometry of AdS$_5\times$S^5, psu(2,2|4) $\qquad$ $a=0,...,4$ and $a'=5,...,9$

$$\mathrm{so}(2,4)=\{P_a, J_{ab}\} \quad ; \quad \mathrm{su}(4)\cong\mathrm{so}(6)=\{P_{a'}, J_{a'b'}\} \quad ; \quad 32 \ \text{supercharges}\ \mathcal{Q}=\begin{pmatrix}\mathcal{Q}^1\\ \mathcal{Q}^2\end{pmatrix}$$

Let $\mu=0,1,2,3$ and $\qquad$ $p_\pm=\dfrac{1}{2}(1\pm\Gamma^{0123}i\sigma_2)$

$$\begin{array}{ccccc}
-1 & -\frac{1}{2} & 0 & \frac{1}{2} & 1\\
\tilde{K}_\mu=\frac{1}{2}(P_\mu+J_{\mu 4}) & \tilde{S}=\mathcal{Q}p_+ & \tilde{J}_{\mu\nu}=J_{\mu\nu} & \tilde{Q}=\mathcal{Q}p_- & \tilde{P}_\mu=\frac{1}{2}(P_\mu-J_{\mu 4})\\
 & & \tilde{D}=P_4 & & \\
 & & P_{a'},\ J_{a'b'} & &
\end{array}$$

and then these generate $\mathcal{N}=4$ superconformal algebra with su(4) R-symmetry in $d=1+3$.

3. Schrödinger algebra

Let $\mu = (0, i, 3)$ with $i = 1, 2.$

$$P_{\pm} = \frac{1}{\sqrt{2}}(\tilde{P}_0 \pm \tilde{P}_3) \ , \quad K_{\pm} = \frac{1}{\sqrt{2}}(\tilde{K}_0 \pm \tilde{K}_3) \ , \quad J_{i\pm} = \frac{1}{\sqrt{2}}(\tilde{J}_{i0} \pm \tilde{J}_{i3}) \ ,$$

$$D = \frac{1}{2}(\tilde{D} - \tilde{J}_{03}) \ , \quad D' = \frac{1}{2}(\tilde{D} + \tilde{J}_{03}) \ , \quad P_i = \tilde{P}_i \ , \quad K_i = \tilde{K}_i \ , \quad J_{ij} = \tilde{J}_{ij}$$

and then a set of generators

$$\boxed{\begin{array}{l} \text{Sch(2)} : \{J_{ij},\ \underset{(K_i)}{J_{i+}},\ D,\ \underset{(N)}{P_+},\ \underset{(H)}{P_-},\ P_i,\ \underset{(C)}{K_+}\} \end{array}}$$

forms a subalgebra, the Schrödinger algebra

$$[P_i, K_+] = \frac{1}{2}J_{i+} \ , \quad [P_i, J_{j+}] = \eta_{ij}P_+ \ , \quad [J_{i+}, P_-] = -P_i \ , \quad [P_-, K_+] = -D \ ,$$

$$[D, J_{i+}] = -\frac{1}{2}J_{i+} \ , \quad [D, P_-] = P_- \ , \quad [D, P_i] = \frac{1}{2}P_i \ , \quad [D, K_+] = -K_+ \ ,$$

$$[J_{ij}, J_{k+}] = \eta_{jk}J_{i+} - \eta_{ik}J_{j+} \ , \quad [J_{ij}, P_k] = \eta_{jk}P_i - \eta_{ik}P_j$$

where P_+ is center, and $\{H,\ D,\ C\} = \text{sl}(2;\mathbb{R})$.

4. Super Schrödinger algebra with 24 susy

Let [Yoshida-MS'0805]

$$S = \tilde{S}\ell_- \ , \quad S' = \tilde{S}\ell_+ \ , \quad Q = \tilde{Q}\ell_- \ , \quad Q' = \tilde{Q}\ell_+ \ , \qquad \ell_\pm = \frac{1}{2}(1 \pm \Gamma^{03}) = \frac{1}{2}\Gamma^\pm\Gamma^\mp$$

and we find

$$\{Q, Q\} \sim P_+ \ , \quad \{Q, Q'\} \sim P_i \ , \quad \{Q', Q'\} \sim P_- \ , \quad \{S, S\} \sim K_+ \ ,$$

$$\{Q, S\} \sim J_{i+} \ , \quad \{Q', S\} \sim J_{ij} + D + P_{a'} + J_{a'b'} \ ,$$

$$[K_+, Q'] \sim S \ , \quad [P_-, S] \sim Q' \ , \quad [P_i, S] \sim Q \ , \quad [J_{i+}, Q'] \sim Q \ ,$$

$$[D, Q'] \sim -Q' \ , \quad [D, S] \sim S \ . \qquad \text{(in addition to Lorents and R-symmetry)}$$

Thus the set of generators

$$\boxed{\{\underbrace{J_{ij},\ J_{i+},\ D,\ P_+,\ P_-,\ P_i,\ K_+}_{\text{Sch}(2)},\ \underbrace{P_{a'},\ J_{a'b'}}_{\text{su}(4)},\ \underbrace{Q,\ Q',\ S}_{8\times 3=24\text{ susy}}\ \} \subset \text{psu}(2,2|4)}$$

forms the super Schrödinger algebra with 24 susy and su(4) R-symmetry.

- $\mathrm{AdS}_5\times\mathrm{S}^5$ on x^+ [Goldberger'0806,Barbon-Fuertes'0806]

  is 24-supersymmetric : $\delta\psi = 0$ [Maldacena-Martelli-Tachikawa'0807]

- $\{$Sch(2), su(4), $Q,\ Q',\ S\} \supset \{$Sch(2), su(4), $Q\} \supset \{$Sch(2), $Q\}$

5. Super Schrödinger with less susy

[Yoshida-MS'0806]

(1) $\mathcal{N}=2$ superconformal with su(2)$^2\times$u(1) $\subset$ psu(2,2|4)

Further introduce 1/2 projector $q_+ = \frac{1}{2}(1+\Gamma^{5678})$ and let $\mathcal{Q} = \mathcal{Q}q_+$.
su(4) $\supset$ so(4)$\times$u(1) ; $J_{\bar{a}'\bar{b}'}$ $(\bar{a}' = 5,6,7,8)$ and P_9

$\Longrightarrow$ super Schrödinger with 12 susy and su(2)$^2\times$u(1)
generated by $\{$Sch(2), $J_{\bar{a}'\bar{b}'}$, P_9, Q, Q', $S\}$

(2) $\mathcal{N}=1$ superconformal with u(1)^3 $\subset$ psu(2,2|4)

Further introduce 1/4 projector $q_+ = \frac{1}{2}(1+i\Gamma^{56})\frac{1}{2}(1+i\Gamma^{78})$ and let $\mathcal{Q} = \mathcal{Q}q_+$.
su(4) $\supset$ u(1)^3 ; J_{56}, J_{78} and P_9

$\Longrightarrow$ super Schrödinger with 6 susy and u(1)^3
generated by $\{$Sch(2), J_{56}, J_{78}, P_9, Q, Q', $S\}$

(3) $\mathcal{N}=1$ superconformal with u(1), su(2,2|1)

let R be the diagonal u(1) $\subset$ u(1)^3 in (2),
and then su(2,2|1) is generated by $\{\text{conformal}, R, \tilde{Q}, \tilde{S}\}$

$\Longrightarrow$ super Schrödinger with 6 susy and u(1)

The super Schrödinger with u(1) is the symmetry of the non-relativistic Chern-Simons-matter system [Leblanc-Lozano-Min'92]

two-comp. real spinor to complex spinor

Convert a two-component real spinor into a complex spinor.
$Q = \mathcal{Q}p_- q_+ \ell_-$ is a real two-component spinor. Let $q = Qk_+$ and $q' = Qk_-$ with $k_\pm = \frac{1}{2}(1 \pm i\Gamma^{12})$, and then $q^* = q'$ as $k_\pm^* = k_\mp$. So q is a complex one-component spinor. Similarly for Q' and S.

No other example

6. Super Schrödinger in osp(8|4) and osp(8*|4)

(1) super $\mathrm{AdS}_4 \times \mathrm{S}^7$ algebra osp(8|4) [Yoshida-MS'0805]

$\Downarrow$ $\tilde{Q} = \mathcal{Q}p_-$ and $\tilde{S} = \mathcal{Q}p_+$ with $p_\pm = \frac{1}{2}(1 \pm \Gamma^{012})$

$\mathcal{N} = 8$ superconformal with so(8) in $d = 1 + 2$

$\Longrightarrow$ super Schrödinger with 24 susy and so(8) R-symmetry in $d = 1 + 1$

generated by $\{$Sch(1), so(8) generators, $Q,\ Q',\ S\}$

(2) super $\mathrm{AdS}_7 \times \mathrm{S}^4$ algebra osp(8*|4)

$\Downarrow$ $\tilde{Q} = \mathcal{Q}p_-$ and $\tilde{S} = \mathcal{Q}p_+$ with $p_\pm = \frac{1}{2}(1 \pm \Gamma^{6789\natural})$

$\mathcal{N} = 2$ superconformal with sp(4) in $d = 1 + 5$

$\Longrightarrow$ super Schrödinger with 24 susy and sp(4) R-symmetry in $d = 1 + 4$

generated by $\{$Sch(4), sp(4) generators, $Q,\ Q',\ S\}$

7. Summary and Discussions

Summary

- super Schrödinger algebras [Yoshida-MS'0805,'0806]

d	# susy	R-symmetry	subalgebra of	
$1+2$	24	su(4)	psu(2,2|4)	$N=4,\ d=1+3$
$1+2$	12	su(2)$^2\times$u(1)	psu(2,2|4)	$N=2^*,\ d=1+3$
$1+2$	6	u(1)^3	psu(2,2|4)	$N=1^*,\ d=1+3$
$1+2$	6	u(1)	su(2,2|1)	$N=1,\ d=1+3$
$1+1$	24	so(8)	osp(8|4)	$N=8,\ d=1+2$
$1+4$	24	sp(4)	osp(8*|4)	$N=2,\ d=1+5$

– half of superconformal charges is projected out

– super Schrödinger with su(4) is the symmetry of AdS$_5\times$S^5 on x^+

- super Schrödinger with u(1) is the symmetry of the NR Chern-Simons-matter system

Future directions

- non-relativistic systems with super Schrödinger symmetries found above
- $\mathcal{N} = 4$ SYM on x^+ and super Schrödinger with su(4)
- super Schrödinger with less susy in $d = 1 + 1$ and $1 + 4$
- index for NR CFT with super Schrödinger symmetry found above
  super Schrödinger of SU(2,2|1) [Nakayama'0807]
- and so on