# More on $\beta$-deformed matrix model of M-theory


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based on arxiv:0804.3236

- $D=11$ M-theory plays an essential role in non-perturbative prop. of String theory.

However,

Formulation of M-theory is not yet established.

Degrees of freedom (DOF) ?

Action ?

- Best candidate = <u>Matrix Model</u> (de Wit, Hoppe, Nicolai; Banks, Fischler, Shenker, Susskind)

DOF : $X^\alpha$ $N \times N$ matrices ($\alpha = 1, \cdots, 9$)

$$S = \int \mathrm{Tr}\left( (D_t X^\alpha)^2 + \frac{1}{2}\left([X^\alpha, X^\beta]\right)^2 \right) dt + (\text{Fermi})$$

$\Uparrow$ matrix regularisation (mathematically analogous to canonical quantisation)

Membrane theory (in lightcone gauge)

DOF : $X^\alpha(\sigma^1, \sigma^2)$

$$S = \iint (D_\tau X^\alpha)^2 - \frac{1}{2}\left(\{X^\alpha, X^\beta\}\right)^2 \, d^2\sigma \, d\tau + (\text{Fermi})$$

$$\left( \{f, g\} \stackrel{\text{def}}{=} \frac{\partial f}{\partial \sigma^1}\frac{\partial g}{\partial \sigma^2} - \frac{\partial f}{\partial \sigma^2}\frac{\partial g}{\partial \sigma^1} \right)$$

- <u>Problems</u>

$N \to \infty$ limit? ~RNG flow

$D = 11$ Lorentz inv? Proven for membranes (de Wit, Marquard, Nicolai; Ezawa, Matsuo, Murakami)

- Still far from establishing matrix model formulation

$\downarrow$

Needs more insight into physics of (membranes, matrix model)

## $\beta$-deform. for matrix model (HS)

$$[Z,W] \rightsquigarrow [Z,W]_* = ZWe^{i\pi\beta} - WZe^{-i\pi\beta}$$

$$\left(Z = X^1 + iX^2,\ W = X^3 + iX^4,\quad \beta \sim O\left(\tfrac{1}{N}\right)\right)$$

$$\begin{array}{ccc}
\boxed{\begin{array}{c}\text{Matrix Model}\\ -\mathrm{Tr}[X^\alpha, X^\beta]^2\end{array}} & \xrightarrow[\quad]{[Z,W]\rightsquigarrow[Z,W]_*} & \boxed{\begin{array}{c}\beta\text{-dfm. Matrix Model}\\ -\mathrm{Tr}[X^\alpha, X^\beta]_*^2\end{array}} \\
\Big\Uparrow \text{matrix reg.} & & \text{matrix reg.} \Big\Uparrow \\
\boxed{\begin{array}{c}\text{membrane on}\\ \text{flat space}\end{array}} & \xrightarrow[G^{--},\ F_{+\alpha\beta\gamma}]{\text{change BKG}} & \boxed{\begin{array}{c}\text{membrane on a}\\ \text{certain curved}\\ \text{BKG}\end{array}}
\end{array}$$

- <u>Background</u> $\alpha = -\frac{\beta N}{P_-}$

$$G^{--} = 64\pi^4\alpha^2|z|^2|w|^2,\quad F_{+zw\bar{w}} = 8\pi^2\alpha\bar{z}, \cdots$$

★ solves $D=11$ SUGRA eq. of motion

$$\begin{cases} \frac{1}{2}\partial_\alpha\partial^\alpha G^{--} = \frac{1}{12}F_{+\alpha\beta\gamma}F_+{}^{\alpha\beta\gamma} \\ \partial^\alpha F_{+\alpha\beta\gamma} = 0 \end{cases}$$

★ pp-wave, non-const. flux.

(cf. const. flux Berenstein, Maldacena, Nastase)

# Outline

1. Introduction

membrane $\xrightarrow[\text{matrix reg.}]{}$ matrix model.

$$[Z, W] \overset{\beta\text{-dfm.}}{\rightsquigarrow} [Z, W]_* = e^{i\pi\beta} ZW - e^{-i\pi\beta} WZ$$

2. Stable membrane configurations which are inequivalent become indistinguishable in matrix model.

3. mass deformation (BMN-) (const. flux) + $\beta$- deformation (linear flux)

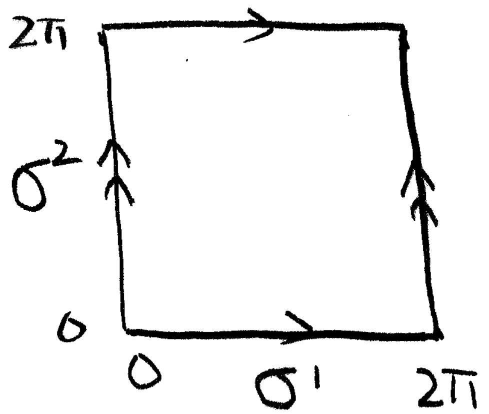
$\longrightarrow$ Stable solution

Sphere $\leftrightarrow$ torus

2.

Inequivalent membranes (with different winding numbers) are indistinguishable in matrix model.

# Matrix reg. for torus (Fairlie, Fletcher, Zachos, Floratos)

- 


periodic func

$$q = e^{i\frac{2\pi}{N}}$$

- $$e^{i\sigma^1} \longleftrightarrow h_1 = \begin{pmatrix} 1 & & & 0 \\ & q & & \\ & & \ddots & \\ 0 & & & q^{N-1} \end{pmatrix}$$

$$e^{i\sigma^2} \longleftrightarrow h_2 = \begin{pmatrix} 0 & 1 & & 0 \\ & 0 & \ddots & \\ & & \ddots & 1 \\ 1 & & & 0 \end{pmatrix}$$

$$e^{i(m_1\sigma^1 + m_2\sigma^2)} \longleftrightarrow h_m = e^{i\frac{m_1 m_2}{N}\pi} h_1^{m_1} h_2^{m_2}$$

- $$h_1 h_2 = e^{-i\frac{2\pi}{N}} h_2 h_1$$

- $$[h_m, h_n] = -2i \sin\left(\frac{\pi}{N} m \times n\right) h_{m+n}$$

$$\{e^{im\cdot\sigma}, e^{in\cdot\sigma}\} = -m \times n \, e^{i(m+n)\cdot\sigma}$$

# Stable solution of β-dfm. model

potential energy $= - \mathrm{Tr}\left([X^\alpha, X^\beta]_*\right)^2 \geq 0$

$0 = [Z, W]_* \Leftrightarrow ZW = WZe^{-i2\pi\beta}$

gives stable solutions, automatically.

Can be solved using $h_1, h_2$

when $\beta N = \pm 1, \pm 2, \cdots$

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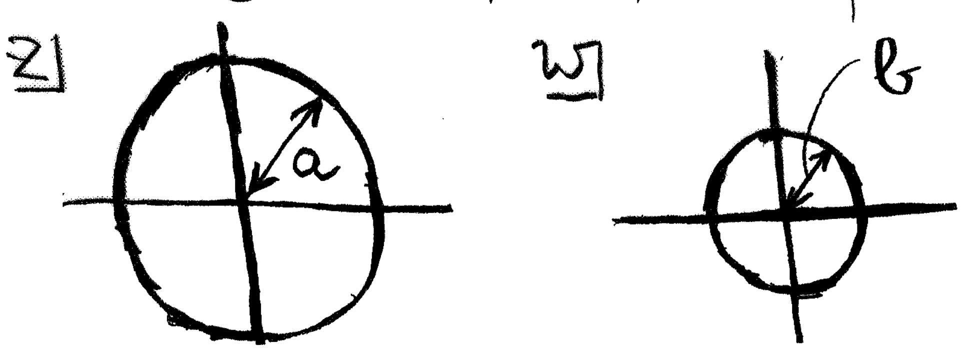
$\underline{\beta N = 1}$ $\quad ZW = WZe^{-i\frac{2\pi}{N}}$

$Z = a h_1, \; W = b h_2$

$\updownarrow$

$Z = a e^{i\sigma^1}, \; w = b e^{i\sigma^2}$

a membrane with $S^1 \times S^1$ shape



$\beta N = 2 \rightarrow ZW = WZ e^{-i2\frac{2\pi}{N}}$, $N$ : odd **9.**

- fluct. spectrum (Low energy ; $m, n \ll N$)

$$\omega \fallingdotseq \sqrt{\left(\frac{m}{\underline{2}a}\right)^2 + \left(\frac{n}{\underline{2}b}\right)^2}$$

- $(Z_{(1)}, W_{(1)})$ & $(Z_{(2)}, W_{(2)})$ are unitary equiv

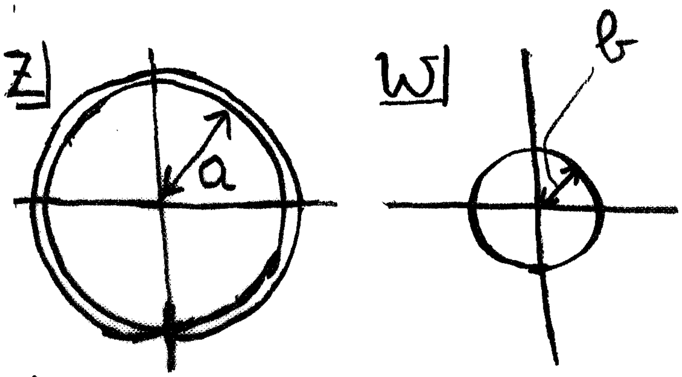
$\exists U : UU^\dagger = 1$, $UZ_{(1)}U^{-1} = Z_{(2)}$, $UW_{(1)}U^{-1} = W_{(2)}$

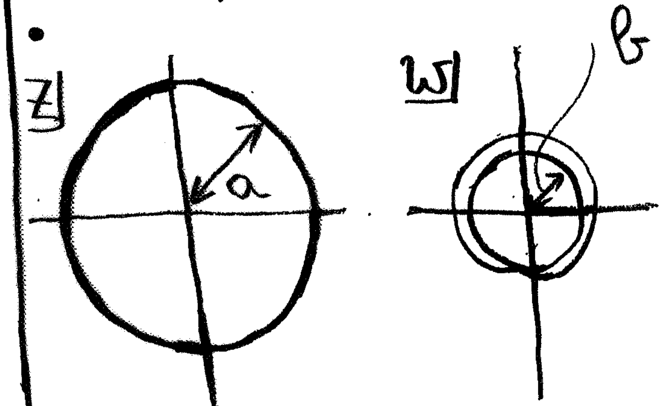
$$\begin{cases} Z_{(1)} = a h_1^2 \\ W_{(1)} = b h_2 \end{cases} \qquad \begin{cases} Z_{(2)} = a h_1 \\ W_{(2)} = b h_2^2 \end{cases}$$

↑ matrix

↓ membrane

- $\begin{cases} Z_{(1)} = a e^{i2\sigma^1} \\ w_{(1)} = b e^{i\sigma^2} \end{cases}$ - $\begin{cases} Z_{(2)} = a e^{i\sigma^1} \\ w_{(2)} = b e^{i2\sigma^2} \end{cases}$




fluct. spectrum

$$\omega_{(1)} = \sqrt{\left(\frac{m}{\underline{2}a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

- fluct. spectrum

$$\omega_{(2)} = \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{\underline{2}b}\right)^2}$$

- matrix ver.

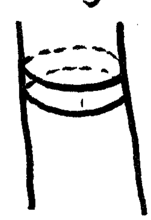
$$Z_{(1)} = a h_1^2 , \; W_{(1)} = b h_2 \; ; \; Z_{(2)} = a h_1 , \; W_{(2)} = b h_2^2$$

of inequivalent membrane config

$$Z_{(1)} = a e^{i 2\sigma^1} , \; w_{(1)} = b e^{i\sigma^2} ; \; Z_{(2)} = a e^{i\sigma^1} , \; w_{(2)} = b e^{i2\sigma^2}$$

are unitary equiv., therefore indistinguishable.

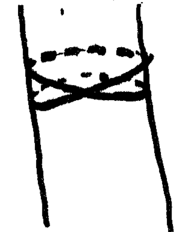
- <u>Possible Interpretation</u>
- Membrane might be non-Abelian obj. like D-branes
- extra DOF → exotic boundary condition
  → winding numbers become "vague"
- 2 coincident D-branes singly-wrapped



$$X^\alpha(2\pi R) = X^\alpha(0)$$ 2x2 matrices

- $X^\alpha(2\pi R) = U X^\alpha(0) U^{-1}$ should be allowed.

when $U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ → 1 double-wrapped D-brane



- cf. matrix string

3. Simultaneously introduce mass-deformation & $\beta$-deformation

$\longrightarrow$ Stable solution interpolates between sphere & torus

- simultaneous introduction of BKG for $\beta$- and mass-deformation

$$\begin{cases} F_{+\alpha\beta\gamma} = f_{\alpha\beta\gamma} + F^{(1)}_{+\alpha\beta\gamma} \\ G^{--} = \mu_{\alpha\beta} x^\alpha x^\beta + \underset{\substack{\uparrow \\ \text{cross-term.}}}{K_{\alpha\beta\gamma}} x^\alpha x^\beta x^\gamma + G^{--}_{(4)} \end{cases}$$

mass-def. $\qquad$ cross-term. $\qquad$ $\beta$-def.

- Should satisfy Einstein eq.

$$\frac{1}{2}\partial^\alpha \partial_\alpha G^{--} = \frac{1}{12} F_{+\alpha\beta\gamma} F_+{}^{\alpha\beta\gamma}$$

- one can tune $K_{\alpha\beta\gamma}$ such that

$$[X^\alpha, X^\beta]_* = i a \epsilon^{\alpha\beta}{}_\gamma X^\gamma$$

$\epsilon_{123} = +1$, $\epsilon_{124} = 0$, $\cdots$

gives stable solutions

- $\beta \to 0$ : pure mass dfm. $\to$ sphere

  $a \to 0$ : pure $\beta$-dfm. $\to$ torus

  $\beta N = \pm 1, \pm 2, \cdots$

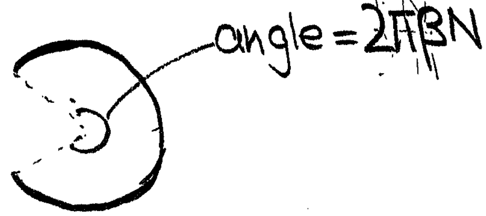
- Continuum version can be solved by the ansatz

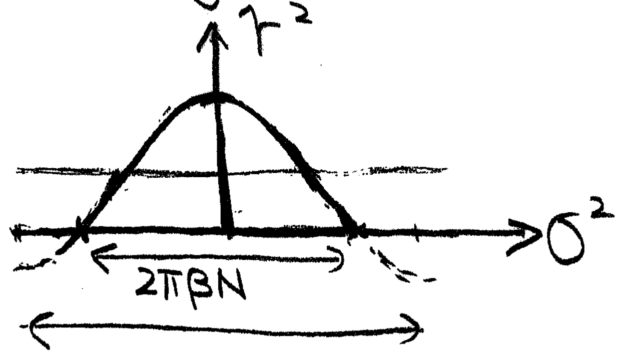
$$\begin{cases} Z = r(\sigma^2)\, e^{i\sigma^1} \\ w = w(\sigma^2) \end{cases}$$

$$\left( \begin{array}{l} r = \sqrt{R^2 - (\sigma^2)^2},\ w(\sigma^2) = 0;\ \sigma^2 \in [-R, +R] \rightarrow \text{sphere} \\ r = a,\ w = b e^{i\sigma^2} \rightarrow \text{torus} \end{array} \right)$$

- Solution (1) $\beta N$ : not integer

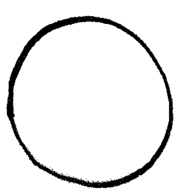
$w$

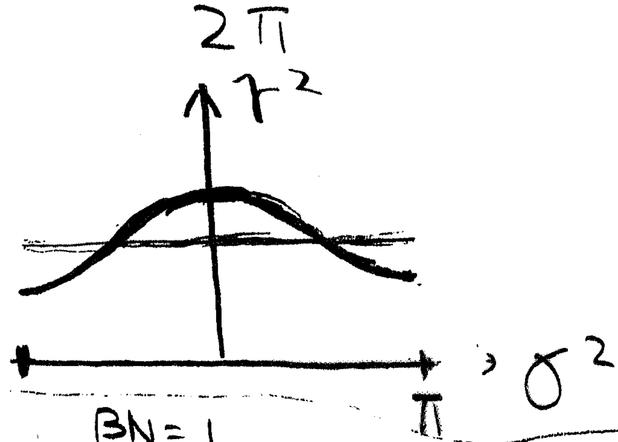



(2) $\beta N$ : integer

$w$



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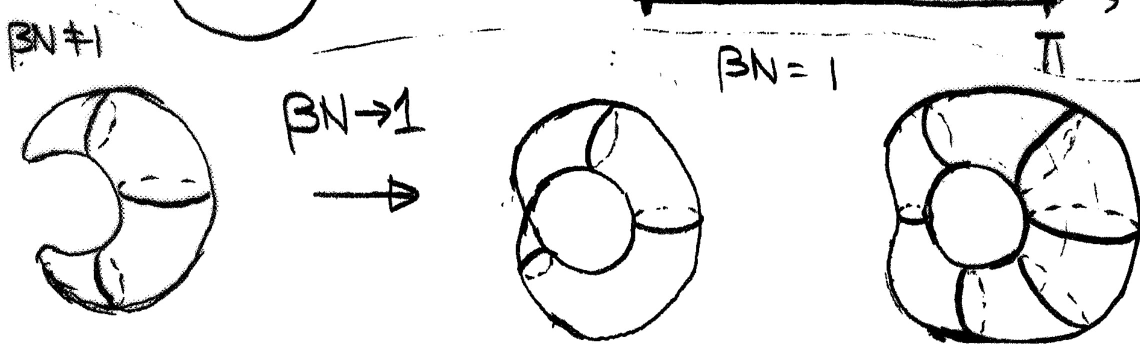


## Summary

1. $[Z,W] \to e^{i\pi\beta} ZW - e^{-i\pi\beta} WZ$

Corresponds to curved BKG

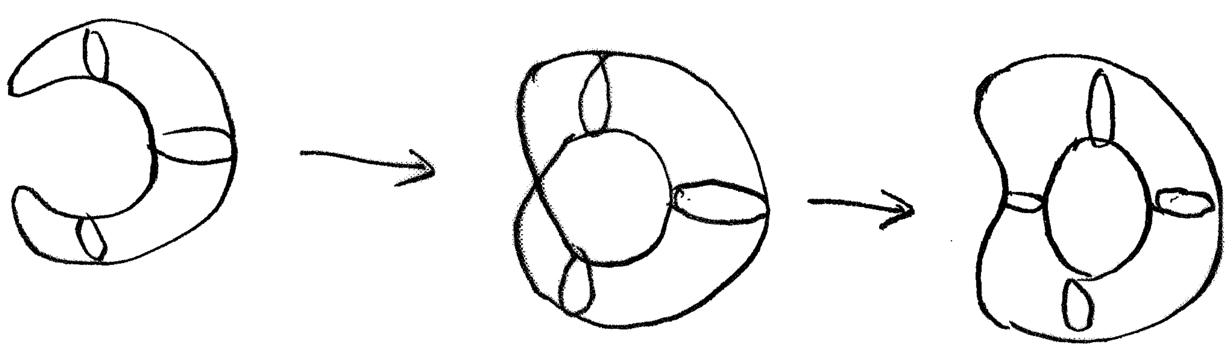
$G^{--} \sim \alpha |z|^2 |w|^2$, $F_{+z\bar{z}\bar{w}} \sim \alpha w$

$\left( \alpha = \frac{\beta N}{P_-} \right)$

2. • inequiv. membrane configs become indistinguishable

• winding number become "vague"

• might be the sign of non-Abelian nature of membranes.

3. 

## Directions

1. non-Abelian membrane from matrix model?
2. quantum corrections?
3. more general pp-wave BKG?
   (Kim, Kim, Park, Pletka)

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Many more...