

Extended MQCD
and
SUSY/non-SUSY duality

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- Introduction
- Review: IIA/M-theory brane picture
- SUSY/non-SUSY duality
- Partial SUSY breaking
- Conclusion

- Setup: Type IIB string on CY 3-fold
- Introducing varying bg B-field
- B-field through many shrinking 2-cycles
- Broken SUSY due to relatively different Kahler moduli (orientation)

$$\Delta t \sim B_{NS} = \sum_{k=0}^{n-1} t_k v^k$$

Gauge theory & geometry

- Engineer 4d gauge theory:

Wrapping D5-branes on 2-cycles

UV gauge coupling \sim 2-cycle Kahler moduli

- IR confining phase (effective superpotential)

\rightarrow geometric transition of CY

- IR scalar potential is non-zero for non-SUSY case

- In order to study non-SUSY case ~~is~~ more than one 2-cycle
- Focus on CY w/ A1-type singularity, say,

$$X : uz + w^2 - W'(v)^2 = 0$$

- Positions of 2-cycles encoded in tree-level superpotential $W(v)$
- Gauge coupling at each shrinking 2-cycle:

$$\alpha = \frac{\theta}{2\pi} + \frac{4\pi i}{g_{YM}^2} = \int_{\mathbb{P}^1} B_0(v), \quad B_0 = B_{RR} + \frac{i}{g_s} B_{NS}$$

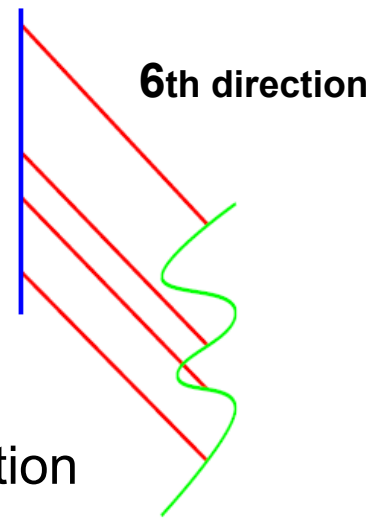
Review: IIA/M-theory brane picture

- T-dualizing along $(u, z, w, v) \rightarrow (\lambda u, \lambda^{-1}z, w, v)$, $\lambda \in \mathbb{C}^*$.
one goes to Type IIA picture
- For a conifold w/ D5-branes wrapped on the only 2-cycle
~~↗~~ two perpendicular NS5-branes and D4-branes in between

	0123	4	5	6	7	8	9
NS5	○	○	○				
NS5'	○				○	○	
D4	○			○			

- For more general CYs w/ more 2-cycles

of NS5: type of singularity
 # of stacks of D4: way of fibration



- Size of 2-cycle \rightarrow length of D4 along T-dual direction

$$\frac{1}{g_{YM}^2} = \frac{l}{8\pi^2 g_s \ell_s} = \frac{1}{4\pi g_s} \int_{\mathbb{P}^1} B_{NS}$$

- Orientation of 2-cycle \rightarrow sign of RR-charge of D4
- Non-SUSY vacua due to simultaneous D4/anti-D4

\rightarrow switch on bg NS 3-flux

\rightarrow SUSY/non-SUSY duality !

M-theory lift Witten

- Open up M-cycle: large g_s limit for finite $\frac{1}{g_{YM}^2} = \frac{l}{8\pi^2 g_s \ell_s}$
- IIA branes are unified by one smooth M5-brane
- Except for 0123, a holomorphic curve appears which contains IR information of gauge theory, i.e. (degenerated) SW curve or planar loop eq.

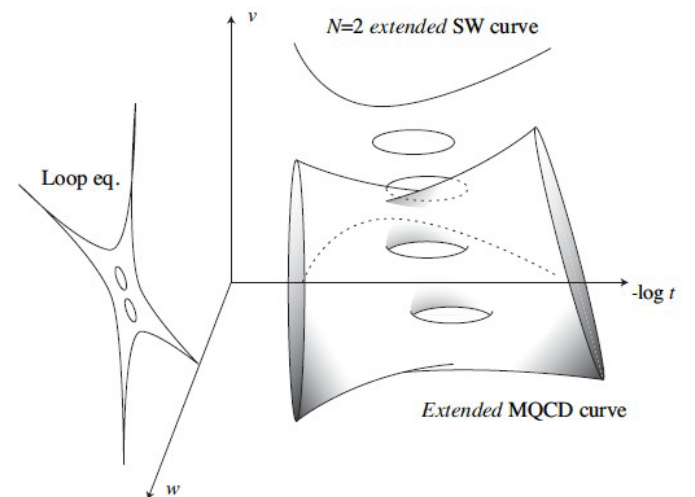
- In fact, coefficients t and a are related to each other, once we extremize M-theory curve

$$W(v) = \sum_{k=1}^{n+1} a_k v^k$$

shape on (v,w) -plane

$$B_0(v) = \sum_{k=0}^{n-1} t_k v^k$$

shape on (t,w) -plane



- There is a parallel in $N=1$ gauge theory, where all glueball vevs are determined by degenerated SW curve once M-theory curve is extremized

SUSY/non-SUSY duality

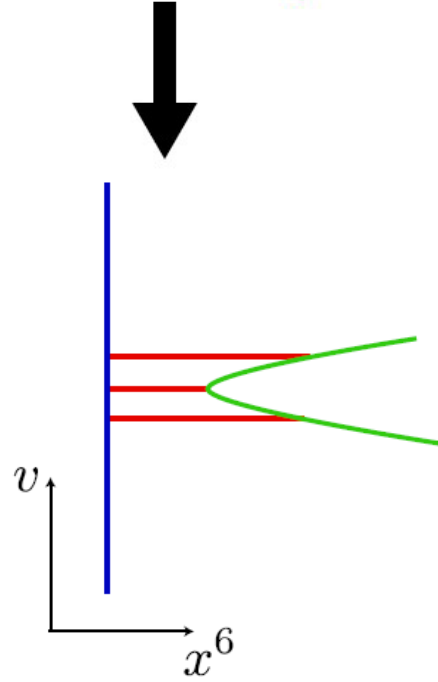
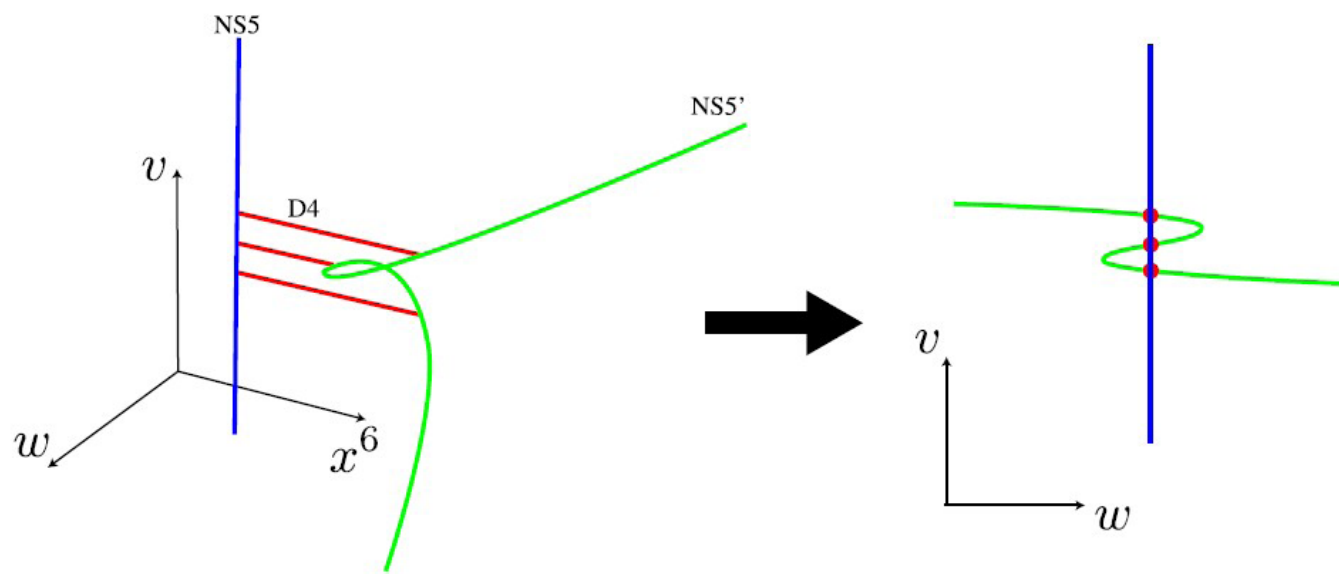
- Example:

$$\mathcal{F}_{UV}(\Phi) = \text{Tr} \left(\frac{t_2}{12} \Phi^4 + \frac{t_1}{6} \Phi^3 + \frac{t_0}{2} \Phi^2 \right),$$

$$\mathcal{F}_{UV}''(v) = B_0(v)$$

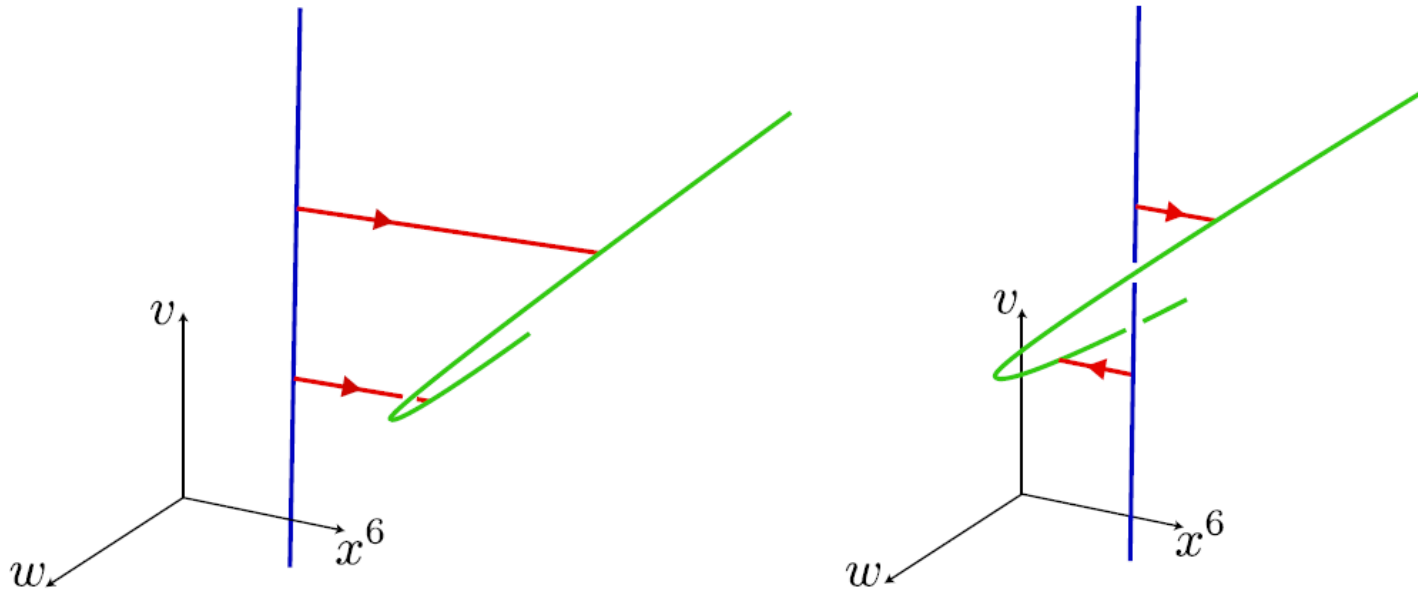
$$W(\Phi) = \text{Tr} (a_4 \Phi^4 + a_3 \Phi^3 + a_2 \Phi^2 + a_1 \Phi)$$

- A *new extended* brane configuration appears due to the UV prepotential polynomial



- On (v,w) -plane: singular CY geometry encoded
- On (v,t) -plane: *extended* SW theory, different gauge coupling for each stack of D4s

Non-SUSY phase

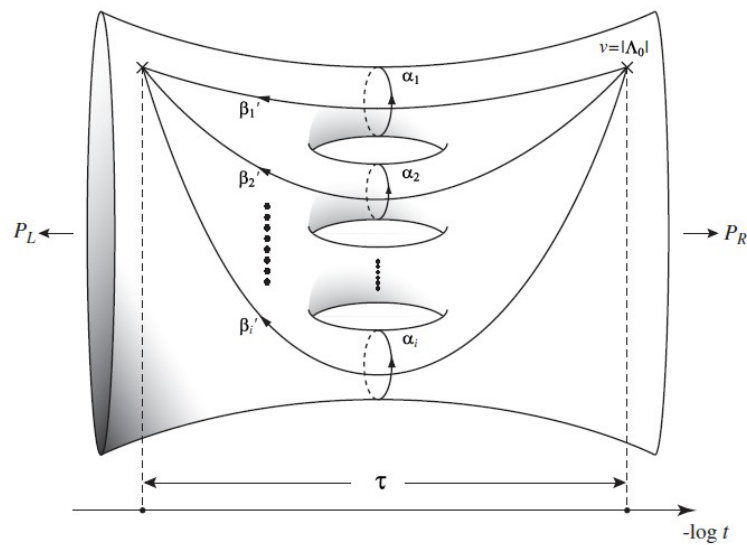


- Anti-D4s appear if we tune the coefficients of prepotential due to

$$l(v_i) = \mathcal{F}_{UV}''(v_i)$$

The terminology "SUSY/non-SUSY duality" bears similarity to [Seiberg duality](#) because they amount to exchanging NS5s and thereby changing the coupling constant

$N = 1$ effective superpotential



$$W_{eff} = - \int_{\Sigma} w dv \wedge \frac{dt}{t} = \sum_i \left(\oint_{\alpha_i} \frac{dt}{t} \int_{\beta_i} w dv - \int_{\beta_i} \frac{dt}{t} \oint_{\alpha_i} w dv \right) \quad (\text{Riemann Identity})$$

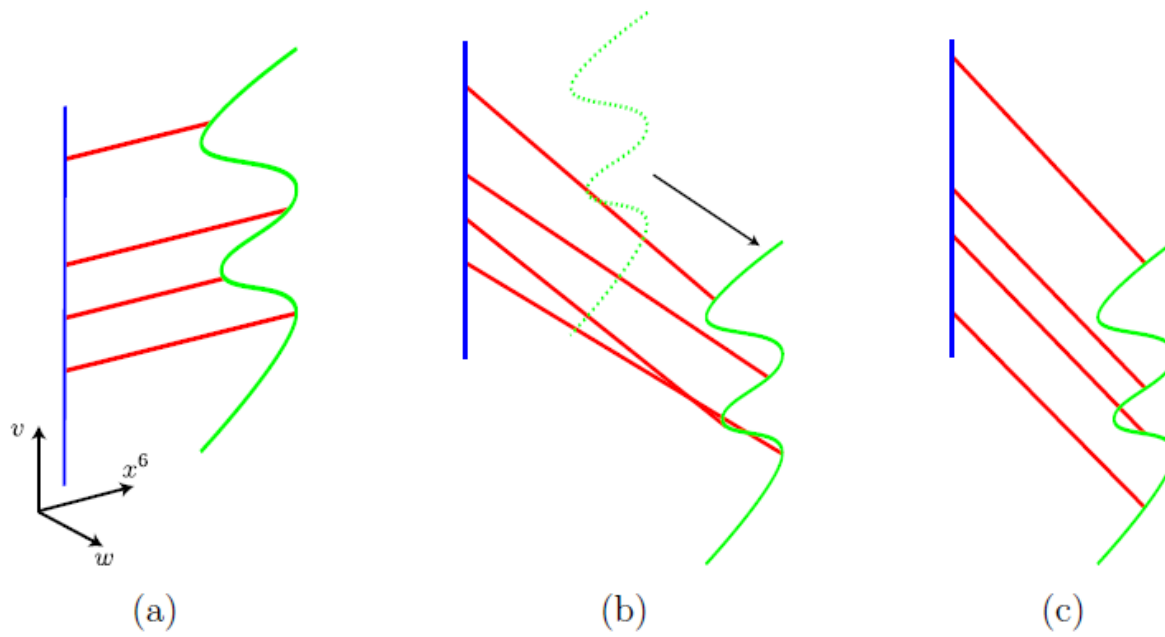
$$W_{eff} = \sum_i \left(N_i \Pi_i + 2\pi i \oint_{\alpha_i} \alpha_i(v) w(v) dv \right)$$

$$S_i \equiv \oint_{\alpha_i} w dv \quad \frac{\partial \mathcal{F}_0}{\partial S_i} = \Pi_i \equiv \int_{\beta'_i} w dv \quad (\text{dual periods on DV Riemann surface})$$

Partial SUSY breaking

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- In terms of Type IIA *extended* brane picture, partial SUSY breaking from $N = 2$ configuration to $N = 1$ one is clear visualized
- That is, $N=2$ *extended* Seiberg-Witten theory goes to a Dijkgraaf-Vafa model at tree-level



(1) Turning on FI parameters (789 directions), SUSY of the *extended* $N = 2$ theory gets completely broken (off-shell) temporarily

(2) SUSY is recovered (on-shell) again at critical loci

$$\mathcal{F}''(\Phi) = W'(\Phi) = 0$$

but only $N = 1$ is now preserved

Conclusion

1. SUSY/non-SUSY duality can have a corresponding Type IIA Brane picture
2. Changing the orientation of local 2-cycles is achieved by introducing a varying bg NS-flux
3. Anti-D4s appear naturally between crossing NS5s
4. A way to realize various non-SUSY vacua
5. From the *extended* brane setup, partial SUSY breaking is easily understood