# Non-perturbative determination of running coupling with twisted Polyakov line calculation

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# Outline

1. Introduction
2. Methods
3. Simulation details
4. Results
5. Summary

Related presentation : 伊藤悦子さん, Poster
"Wilson loopによる格子ゲージ理論の結合定数の測定"

# Introduction

Long-term goal

To study physics of (approximate) conformal gauge theories

Theoretical interest

Walking Technicolor

etc.....

A candidate of conformal gauge theories is

large flavor QCD (SU(3) gauge theory).

In $8 < N_f \leq 16$, 2-loop $\beta(\alpha)$ function has zero at $\alpha \neq 0$.

IR fixed point beyond perturbative calculation?

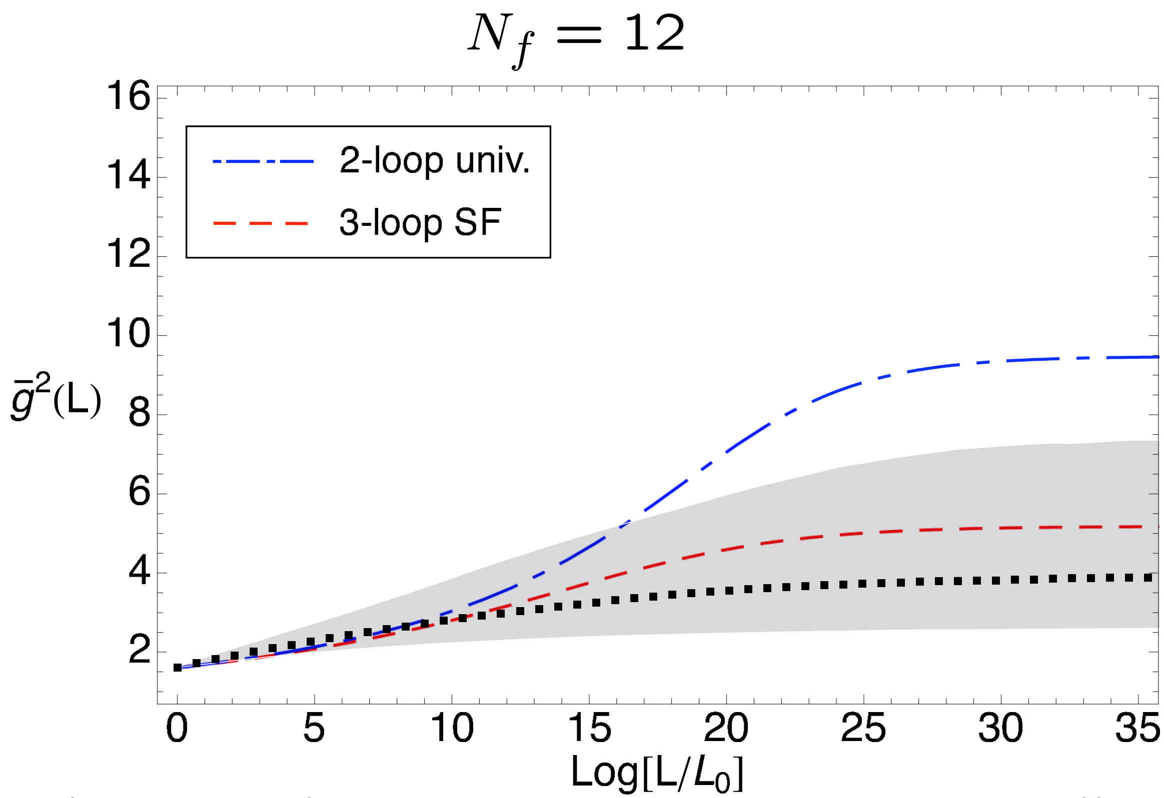
## Previous works of lattice QCD

- Iwasaki et al. PRD69:014507
  various $N_f$ and search phase transition in $\beta$-$m$ plane
  $7 \leq N_f \leq 16$ IR fixed point

- Appelquist et al. PRL100:171607
  Wilson gauge and massless staggered fermion
  Running coupling in $N_f = 8$ and 12
  Step scaling procedure with Schrödinger functional scheme
  $N_f = 8$ no evidence for IR fixed point
  $N_f = 12$ IR fixed point

- etc.

# Appelquist et al. PRL100:171607




large scale $\Longleftrightarrow$ small scale

Flat $g^2(L)$ in small scale region with small statistical error

Large systematic error (presented by shaded band) due to different continuum extrapolations

# New schemes without large systematic error

Systematic error of Schrödinger functional scheme

$\Rightarrow O(a)$ discretization error of boundary counter term

## Purpose of this work

Before large flavor calculation

study several schemes without $O(a)$ discretization error

find schemes which can control statistical and systematical errors in quenched QCD

Step scaling procedure with

· Wilson loop scheme with periodic(twisted) b.c.

$\rightarrow$ 伊藤悦子さん(Poster)

· Twisted Polyakov line scheme

$\rightarrow$ this talk

# Methods

## Step scaling (Lüscher et al. NPB359:221)

Renormalized coupling on finite volume $L^4$

$$g^2(\mu) \;=\; \frac{A(\mu)}{A_0(\mu)} \cdot g_0^2 = \frac{A(\mu)}{k}, \qquad A_0(\mu) = k\, g_0^2 \text{ (Tree amplitude)}$$

Usually $\mu = p$, while $\mu = 1/L$ on $L^4$ through $A_L(1/L)$

On lattice $A_L(1/L) \to A_L^{NP}(a, L/a) = A_L^{NP}(a/L, 1/L) = k\, g^2(a/L, 1/L)$

$$g^2(\mu) \;=\; g^2(1/L) = \lim_{a\to 0} g^2(a/L, 1/L)\Big|_L$$

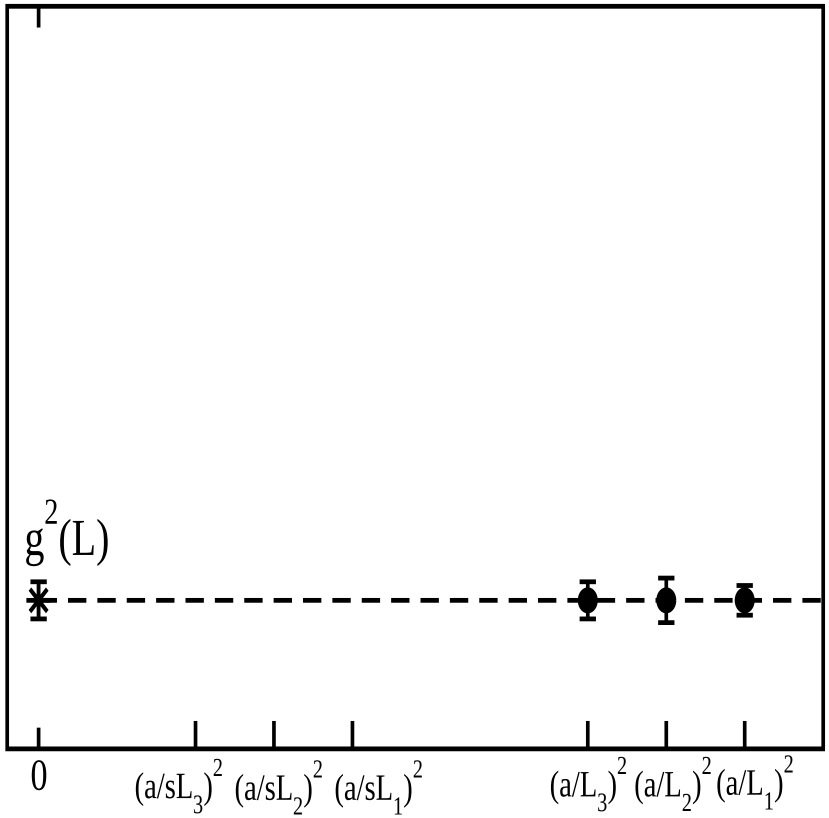
Taking continuum limit $a \to 0$ on a constant physics (fixed $L$)

e.g., Sommer scale, $f_\pi$, $m_N$, etc

## Step scaling $\mu \to \mu/s \iff L \to sL$

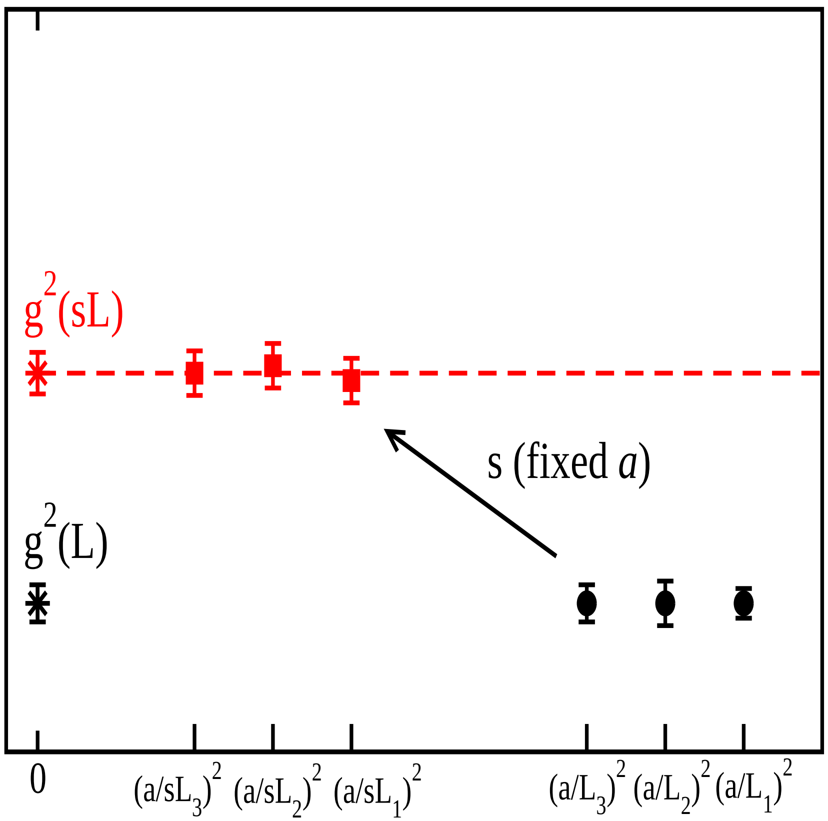
$$g^2(\mu/s) \;=\; g^2(1/sL) = \lim_{a\to 0} g^2(a/sL, 1/sL)\Big|_L$$

## Step scaling



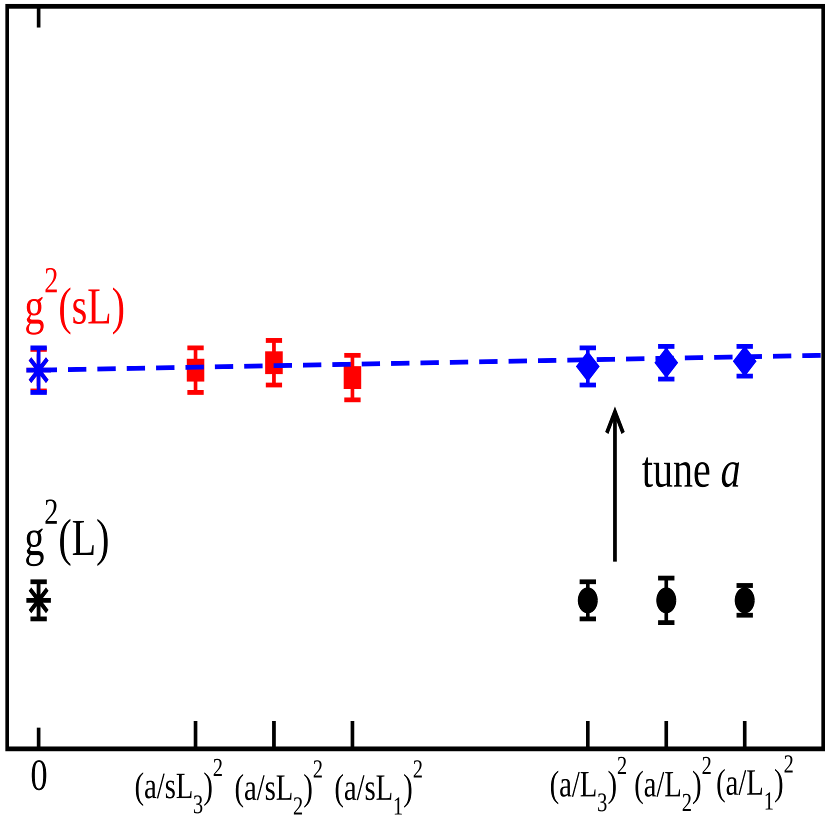

Calculate $g^2(a/L, L)$ with an input on each $(a, L/a)$, then $a \to 0$

## Step scaling



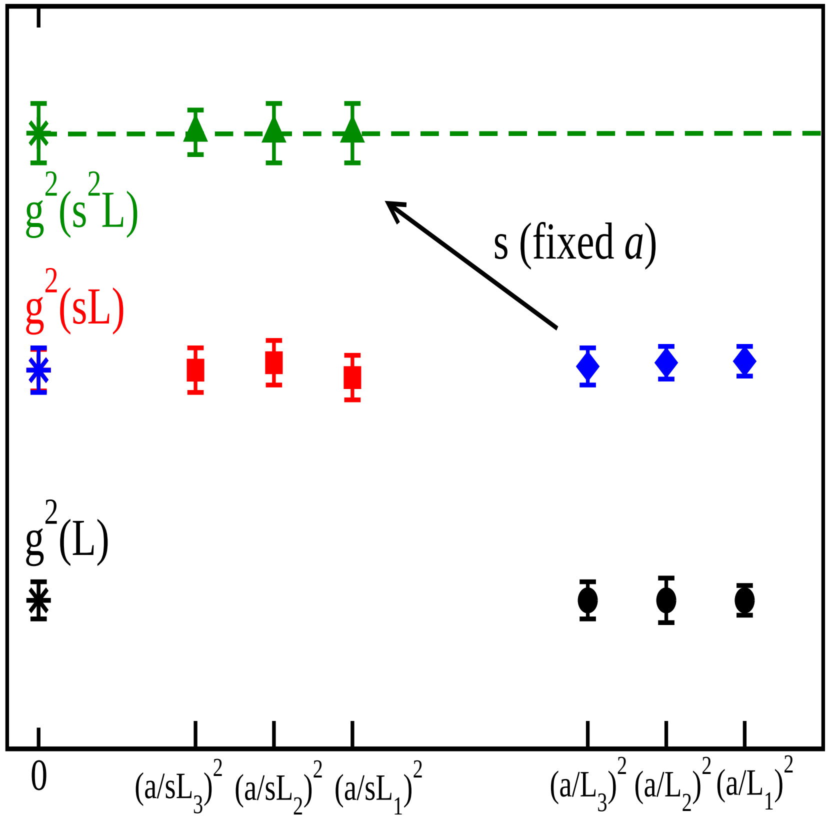

Calculate $g^2(a/sL, sL)$ at same $a$, then $a \to 0$

## Step scaling




Tune $a$ on $L/a$ to get same $g^2(sL)$ in $a \to 0$

## Step scaling



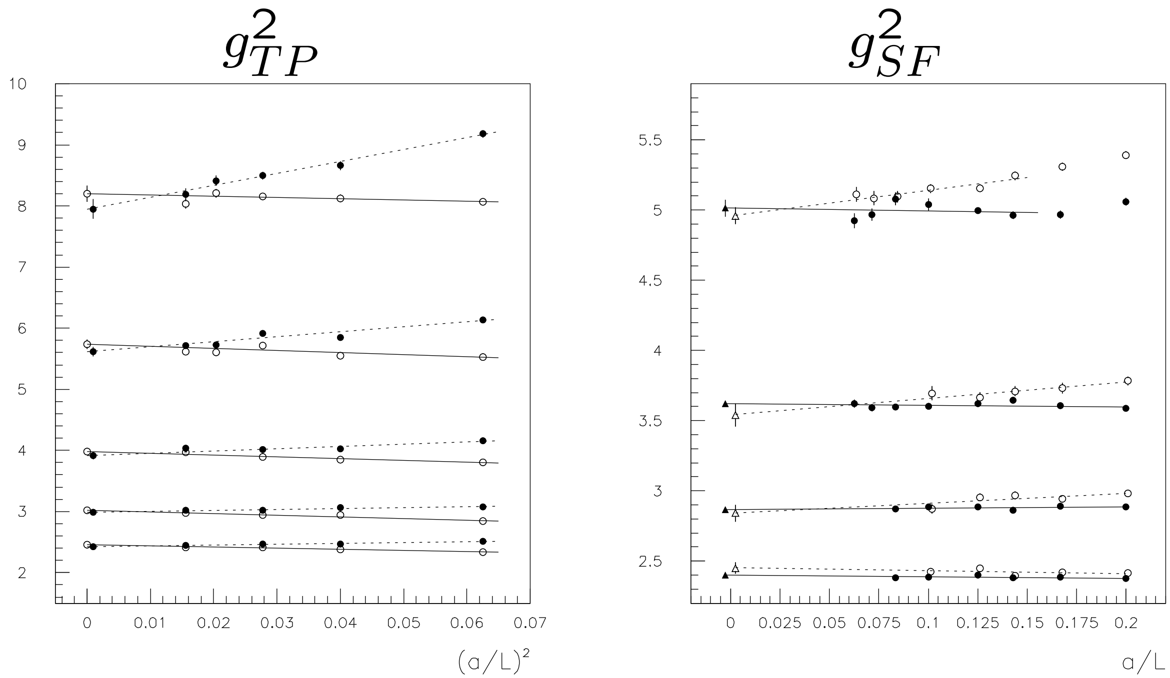

Calculate $g^2(s^2L)$ at same $a$, then $a \to 0$

On each step we need $a \to 0$.

# Twisted Polyakov line scheme

Previous works of SU(2) gauge theory (NPB433:390, NPB437:447)

more than 10 years ago




Nice $O(a^2)$ scaling even at small $L/a$

Large statistical fluctuation $\rightarrow$ method to reduce fluctuation

Calculation cost $\sim$ 10 $\times$ SF at fixed $L/a$ with the method

which is effective only in quenched QCD

$\Rightarrow$ SF became major method, but TP did not.

# Twisted Polyakov line scheme

Previous works of SU(2) gauge theory (NPB433:390, NPB437:447)

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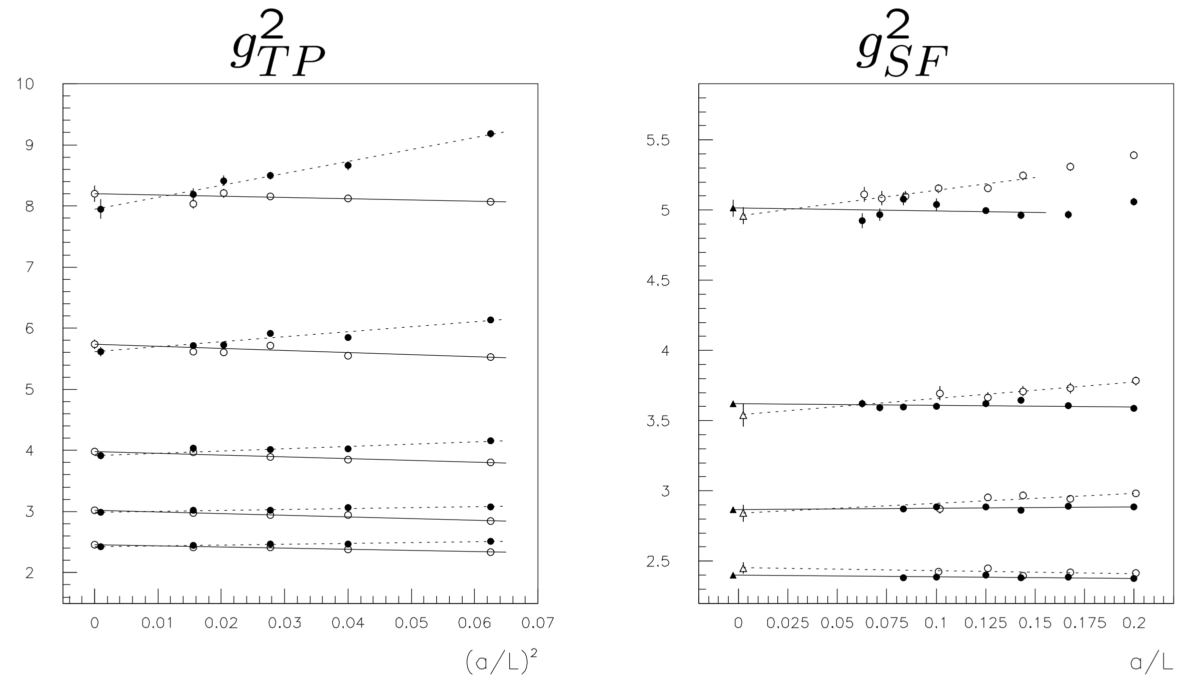



Nice $O(a^2)$ scaling even at small $L/a$

Large statistical fluctuation $\rightarrow$ method to reduce fluctuation

Calculation cost $\sim$ 10 $\times$ SF at fixed $L/a$ with the method

Suitable for IR fixed point search without $O(a)$ error

Require method to reduce statistical error in full QCD

## Twisted boundary condition ('t Hooft NPB153:131)

$$U_\mu(x+\hat{\nu}L/a) = \Omega_\nu U_\mu(x)\Omega_\nu^\dagger \quad (\nu = 1,2)$$

Kill the zero mode of gauge fields on finite volume

$$\begin{aligned}\Omega_\mu\Omega_\nu &= e^{i2\pi/3}\Omega_\nu\Omega_\mu \quad (\mu,\nu = 1,2,\mu\neq\nu)\\ \Omega_\mu\Omega_\mu^\dagger &= 1, \quad (\Omega_\mu)^3 = 1, \quad \mathrm{Tr}[\Omega_\mu] = 0\end{aligned}$$

First property guarantees consistency of different order of twist.

$$\begin{aligned}U_\mu(x+\hat{\nu}L/a+\hat{\rho}L/a) &= \Omega_\rho\Omega_\nu U_\mu(x)\Omega_\nu^\dagger\Omega_\rho^\dagger\\ &= \Omega_\nu\Omega_\rho U_\mu(x)\Omega_\rho^\dagger\Omega_\nu^\dagger\end{aligned}$$

Typical twist matrix (PRD65:094502)

$$\Omega_1 = \begin{pmatrix} 0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0\end{pmatrix}, \quad \Omega_2 = \begin{pmatrix} e^{-i2\pi/3} & 0 & 0\\ 0 & e^{i2\pi/3} & 0\\ 0 & 0 & 1\end{pmatrix}$$

## Twisted Polyakov line (NPB433:390, NPB437:447)

Polyakov line

$$P_3(x,y,t) = \mathrm{Tr}\left[\prod_{z=1}^{L/a} U_3(x,y,z,t)\right] \quad \text{(periodic b.c.)}$$

$$P_1(y,z,t) = \mathrm{Tr}\left[\left\{\prod_{x=1}^{L/a} U_1(x,y,z,t)\right\}\Omega_1\right] e^{-i\frac{2\pi y}{3L}} \quad \text{(twisted b.c.)}$$

$\Omega_1$ and $e^{-i2\pi y/3L}$ guarantee translational invariance and periodicity of $P_1(y,z,t)$ in $x$ and $y$ directions, respectively.

### Running coupling of twisted Polyakov line scheme on $L^4$

$$g^2_{TP} = \frac{1}{k}\cdot\frac{\langle 0|\sum_{y,z} P_1(y,z,L/2a)P_1(0,0,0)^*|0\rangle}{\langle 0|\sum_{x,y} P_3(x,y,L/2a)P_3(0,0,0)^*|0\rangle}$$

$$g^2_{TP}\Big|_{\text{tree}} = k\, g_0^2$$

$$k = \frac{1}{12\pi^2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{n^2+(1/3)^2} = 0.0636942294\ldots \quad \text{(Preliminary)}$$

# Simulation details

## Strategy

Measure Polyakov line at every Monte Carlo sweeps

(comparable computatinal cost with Wilson loop scheme)

Compensate autocorrelation by jackknife analysis with large bin size

## Parameters

- Quenched QCD with Wilson gauge action on $(L/a)^4$
- Input for constant physics : $g^2_{SF}$ (Alpha collaboration NPB544:669)
- Scaling step $s = 2$

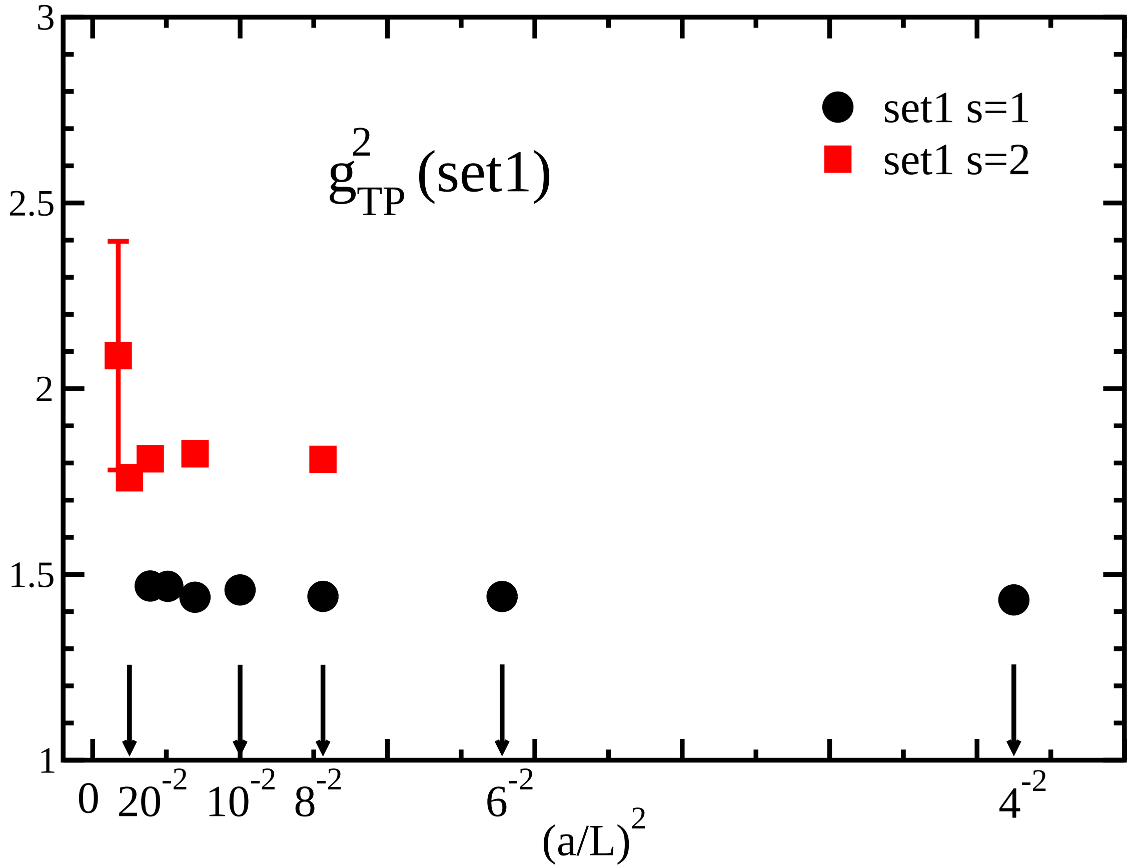
| set1 | | | set2 | | | set3 | | | set4 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\beta$ | $L/a$ | $2L/a$ | $\beta$ | $L/a$ | $2L/a$ | $\beta$ | $L/a$ | $2L/a$ | $\beta$ | $L/a$ | $2L/a$ |
| 7.6631 | 4 | 8 | 7.0644 | 4 | 8 | 6.4346 | 4 | 8 | 5.8932 | 4 | 8 |
| 7.9993 | 6 | 12 | 7.4082 | 6 | 12 | 6.7807 | 6 | 12 | 6.2204 | 6 | 12 |
| 8.2500 | 8 | 16 | 7.6547 | 8 | 16 | 7.0197 | 8 | 16 | 6.4527 | 8 | 16 |
| 8.4677 | 10 | 20 | 7.8500 | 10 | 20 | 7.2098 | 10 | 20 | 6.6629 | 10 | 20 |
| 8.5985 | 12 | 24 | 7.9993 | 12 | 24 | 7.3551 | 12 | 24 | 6.7750 | 12 | 24 |
| 8.7289 | 14 | – | 8.1352 | 14 | – | 7.4986 | 14 | – | 6.9169 | 14 | – |
| 8.8323 | 16 | – | 8.2415 | 16 | – | 7.6101 | 16 | – | 7.0203 | 16 | – |

large scale $\Longleftrightarrow$ small scale

Calculations are carried out on SX-8 at YITP.

# Results

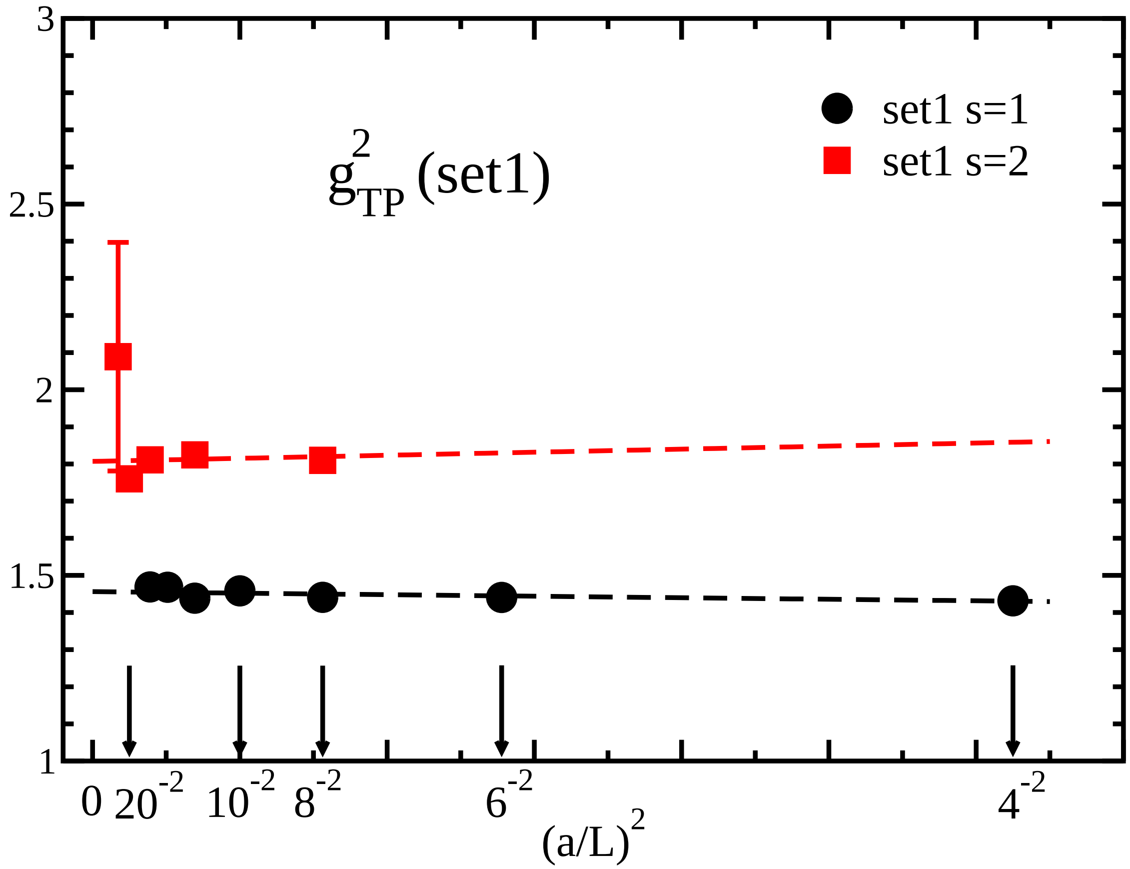
## Scaling of $g^2_{TP}$ at large scale (set1)




Small statistical error except $L/a = 24$ in $s = 2$

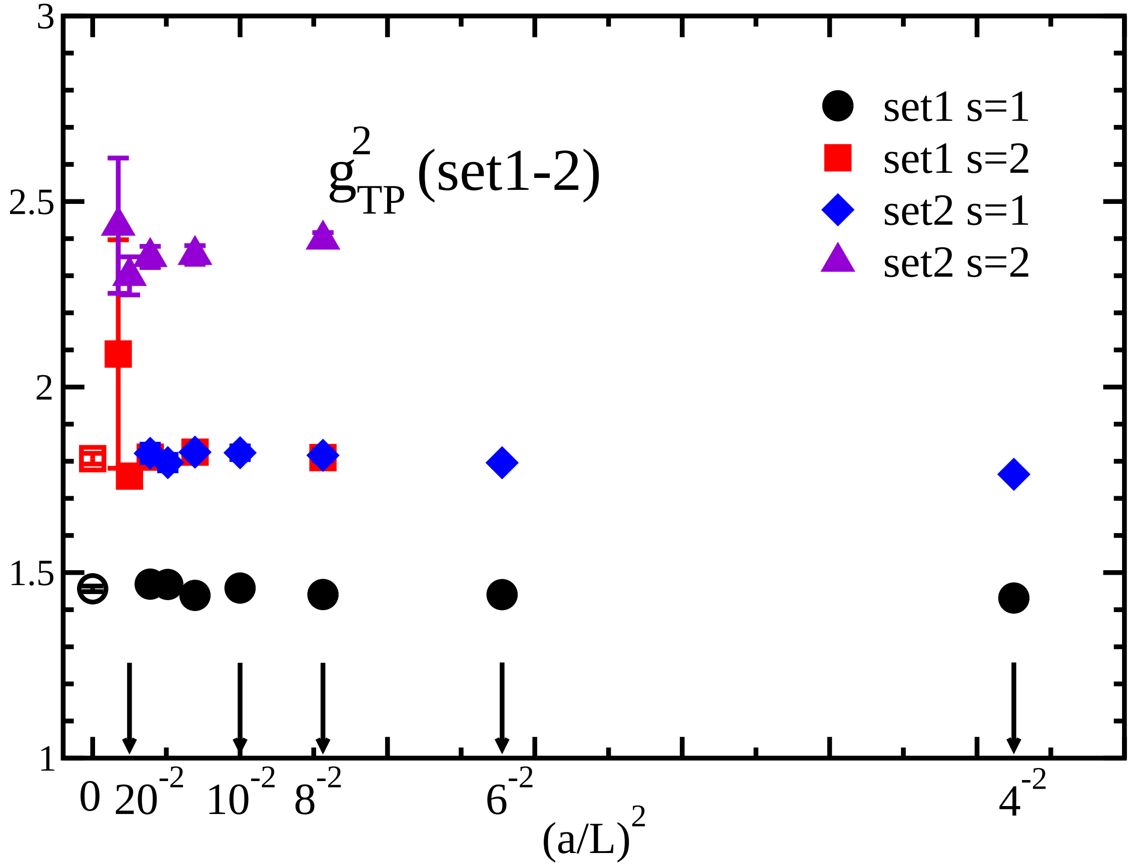
Reasonably flat from $L/a = 4$ to 16 in $s = 1$

# Scaling of $g^2_{TP}$ at large scale (set1)




$$g^2_{TP} = \begin{cases} 1.4562(76) & s = 1 \\ 1.807(15) & s = 2 \end{cases}$$

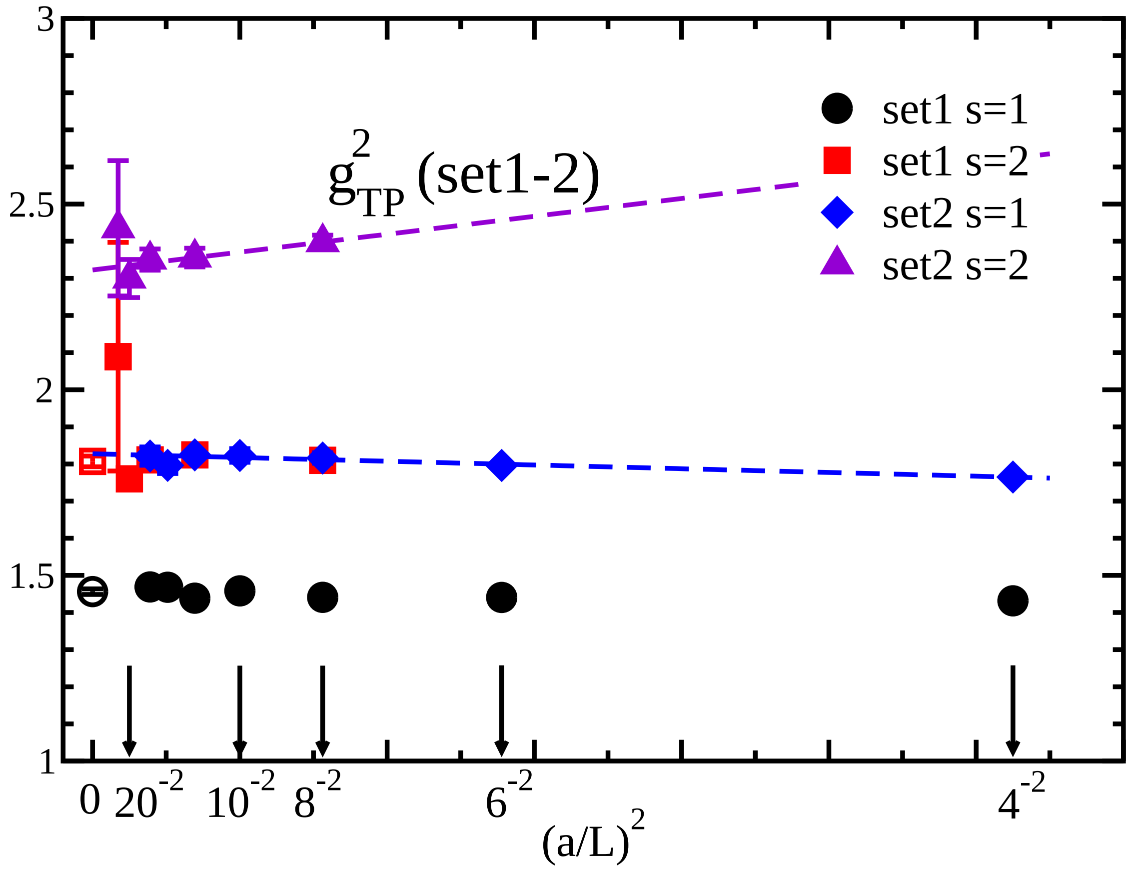
# Scaling of $g^2_{TP}$ (set1–2)




set2 $s = 1$ is well consistent with set1 $s = 2$.

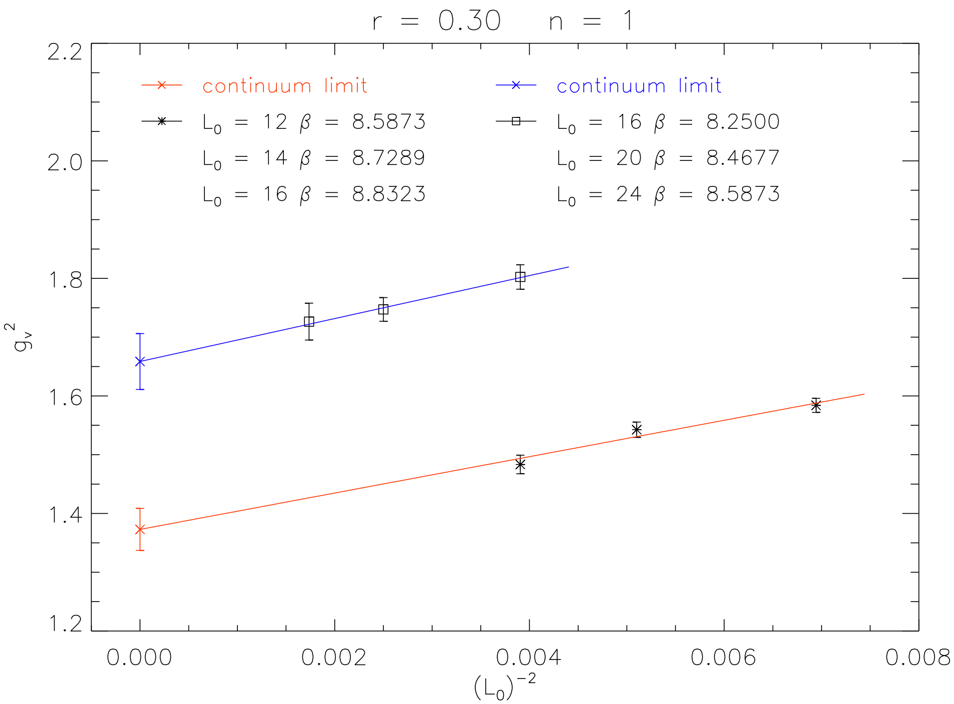
set2 $s = 1$ is reasonably flat from $L/a = 4$.

## Scaling of $g^2_{TP}$ (set1–2)

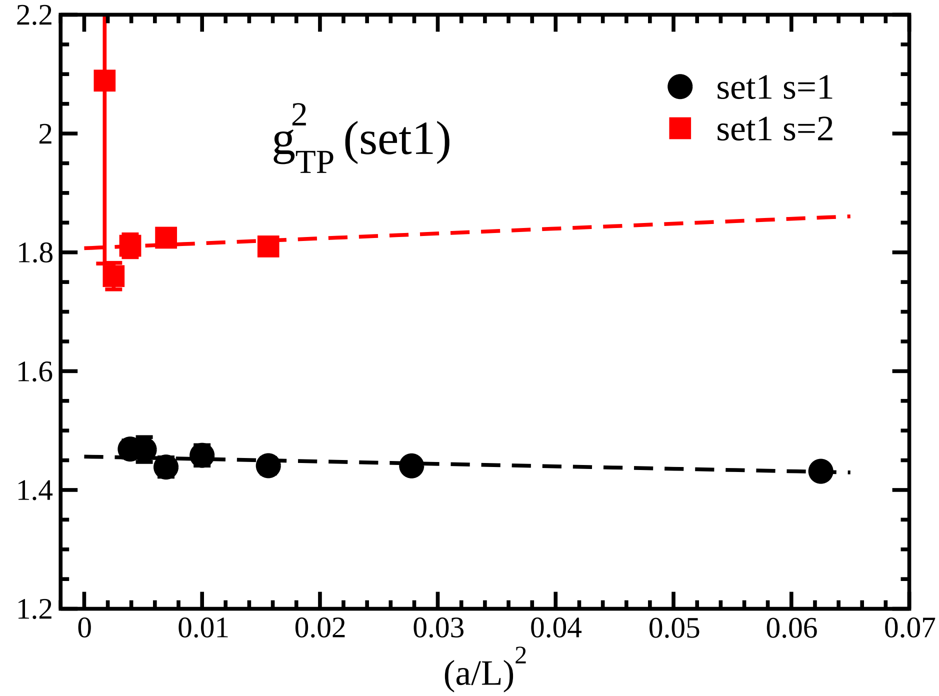



$$g^2_{TP} = \begin{cases} 1.8277(72) & s = 1 \qquad 1.807(15)\ (\text{set1 } s = 2) \\ 2.323(27) & s = 2 \end{cases}$$

# Comparison of scheme




Wilson loop $g_W^2$

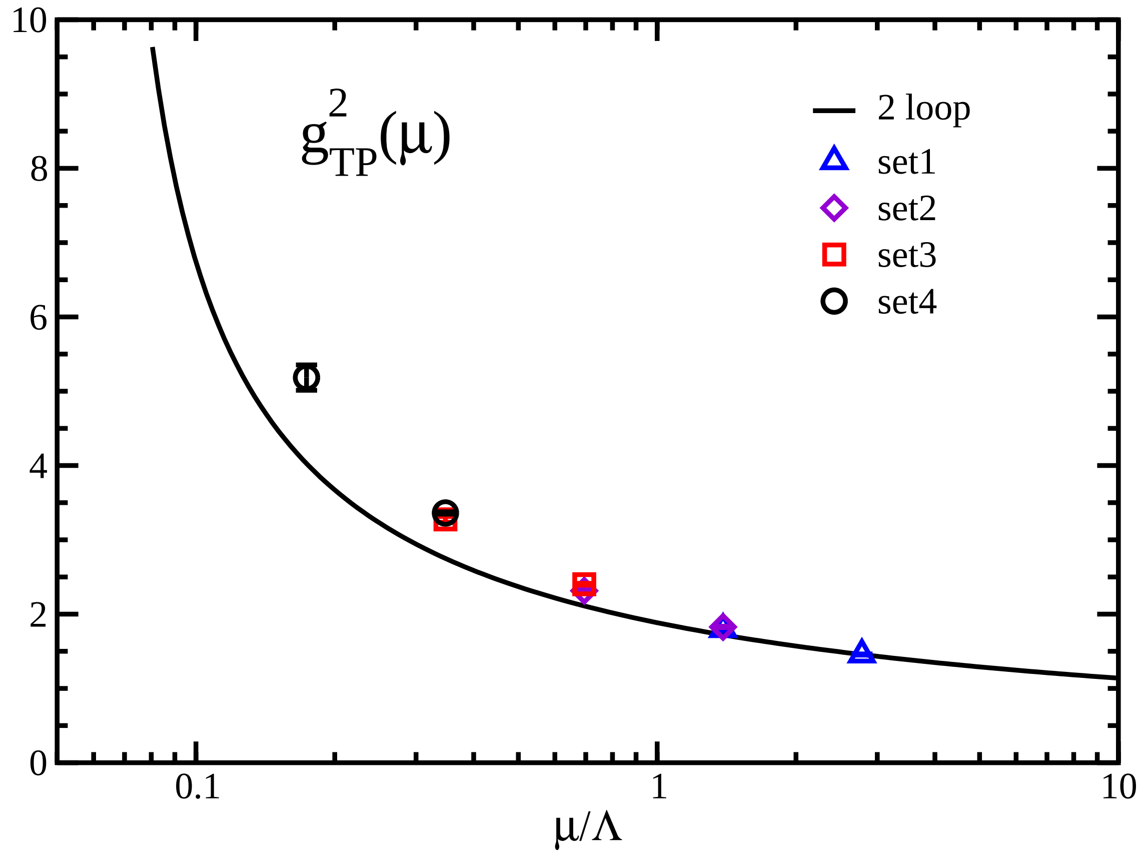



Twisted Polyakov $g_{TP}^2$

$g_W^2$ : Larger $L/a$ possible, but hard smaller $L/a$
Small statistical error with smearing method

$g_{TP}^2$ : Smaller $L/a$ possible, but hard larger $L/a$

## Running of $g^2_{TP}$ (Preliminary)




Reasonably connected on each step

Comparable to 2-loop coupling, while rough scale setting in $g^2_{TP}$

# Summary

We have investigated twisted Polyakov line scheme.

1. Twisted Polyakov line scheme works in quenched QCD.
2. Problem of large statistical fluctuation is resolved by every sweep measurements. (easy to utilize in full QCD calculation)
3. Scaling is well even from smaller volume.

Twisted Polyakov line scheme is promissing method, as well as Wilson loop scheme method, to control both statistical and systematic errors.

We will try both methods for IR fixed point search in large flavor QCD.

# Backup Slides

# Twisted Polyakov line

$$g_{TP}^2 = \frac{1}{k} \cdot \frac{\langle 0|\sum_{y,z} P_1(y,z,L/2) P_1(0,0,0)^* |0\rangle}{\langle 0|\sum_{x,y} P_3(x,y,L/2) P_3(0,0,0)^* |0\rangle} = \frac{1}{k} \cdot \frac{C_t}{C_p}$$

Tree level

$$\begin{aligned}
C_p|_{\text{tree}} &\propto \text{Tr}[1] \times \text{Tr}[1] = O(1) \\
C_t|_{\text{tree}} &\propto \text{Tr}[\Omega_1] \times \text{Tr}[\Omega_1] \ (= 0) \\
&\quad + g_0 \text{Tr}[\Omega_1 T^a] \times \text{Tr}[\Omega_1] A_1^a \ (= 0) \\
&\quad + g_0^2 \text{Tr}[\Omega_1 T^a] \times \text{Tr}[\Omega_1 T^b] A_1^a A_1^b \\
&= O(g_0^2) \\
C_t/C_p|_{\text{tree}} &= k\, g_0^2
\end{aligned}$$

$$k = \frac{1}{12\pi^2} \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{n^2 + (1/3)^2} = 0.0636942294...$$

(Preliminary)