Heterotic--F Theory Duality Revisited

Aug. 01, '08 at YITP workshop

Taizan Watari (Tokyo)

- based on 0805.1057 in collaboration with

  林博貴, R. Tatar, 戸田幸伸 and 山崎雅人

  (東大理) (Liverpool) (東大IPMU) (東大理)

- Closely related papers:
  - Donagi Wijnholt 0802.2969
  - Beaseley Heckman Vafa 0802.3391
- References on $R^1\pi_{Z*}\wedge^2 V$:
  - Donagi He Ovrut Reinbacher 0405014
  - Blumenhagen Moster Reinbacher Weigand 0612039

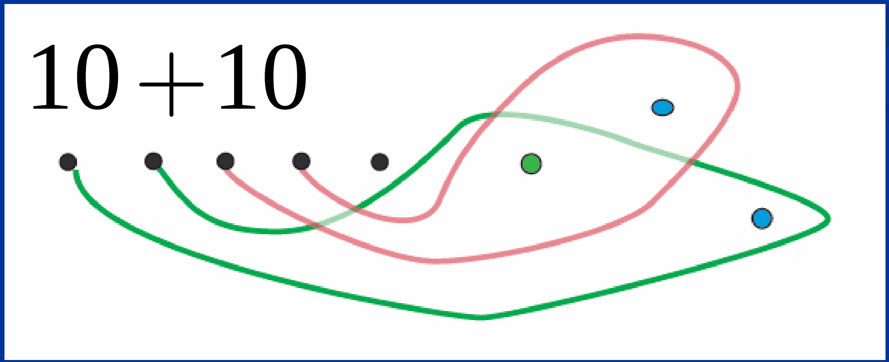
Yukawa couplings in F/M-Theory

- Need a framework that has all of

$$10^{ab}\,10^{cd}\,H(5)^{e}\;\varepsilon_{abcde}+\bar{5}_a\,10^{ab}\,\overline{H}(\bar{5})_b+N\,\bar{5}_a\,H(5)^{a}.$$

  - important in q/l masses and SUSY flavor problem

- How to get such Yukawa couplings?



eg. with (p,q) 7-branes (i.e. in F-theory), the u-type Yukawa is OK.

[Gaberdiel Zwiebach '97]

- Either E_7 or E_8 to get them all in F/M. [Tatar TW '06]

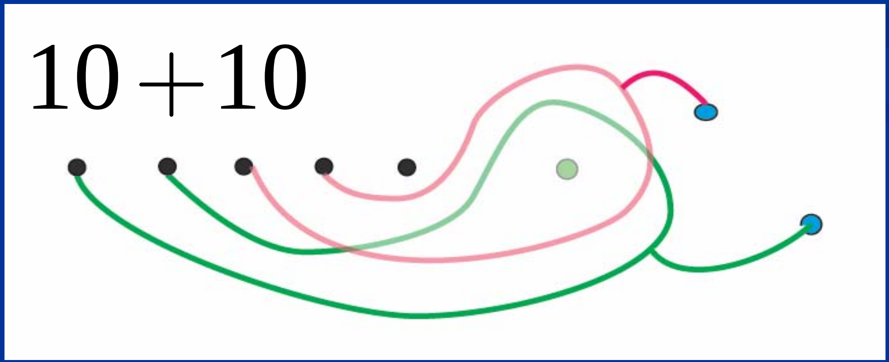
Yukawa couplings in F/M-Theory

- Need a framework that has all of

  $$10^{ab}\,10^{cd}\,H(5)^e\,\varepsilon_{abcde} + \bar{5}_a\,10^{ab}\,\overline{H}(\bar{5})_b + N\,\bar{5}_a\,H(5)^a.$$

  - important in q/l masses and SUSY flavor problem

- How to get such Yukawa couplings?



eg. with (p,q) 7-branes (i.e. in F-theory), the u-type Yukawa is OK.

[Gaberdiel Zwiebach '97]

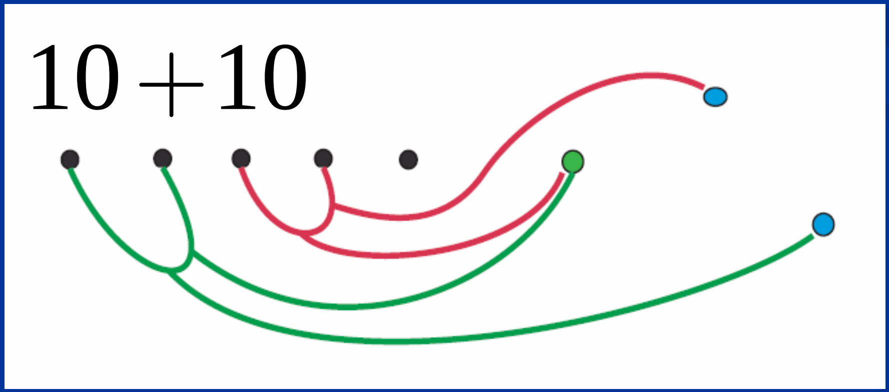
- Either E_7 or E_8 to get them all in F/M. [Tatar TW '06]

Yukawa couplings in F/M-Theory

- Need a framework that has all of

  $$10^{ab}\,10^{cd}\,H(5)^{e}\ \varepsilon_{abcde} + \bar{5}_a\,10^{ab}\,\overline{H}(\bar{5})_b + N\,\bar{5}_a\,H(5)^a .$$

  - important in q/l masses and SUSY flavor problem
- How to get such Yukawa couplings?



eg. with (p,q) 7-branes (i.e. in F-theory), the u-type Yukawa is OK.

[Gaberdiel Zwiebach '97]

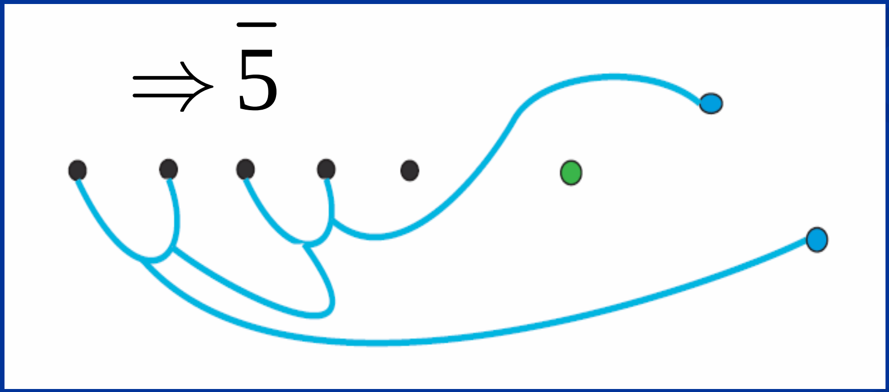
- Either E_7 or E_8 to get them all in F/M. [Tatar TW '06]

Yukawa couplings in F/M-Theory

- Need a framework that has all of

  $$10^{ab}\,10^{cd}\,H(5)^e\;\varepsilon_{abcde}+\bar{5}_a\,10^{ab}\,\overline{H}(\bar{5})_b+N\,\bar{5}_a\,H(5)^a.$$

  - important in q/l masses and SUSY flavor problem
- How to get such Yukawa couplings?



eg. with (p,q) 7-branes (i.e. in F-theory), the u-type Yukawa is OK.

[Gaberdiel Zwiebach '97]

- Either E_7 or E_8 to get them all in F/M. [Tatar TW '06]

low-energy matter multiplets

Heterotic theory on CY 3-fold Z w/ bundle V

for SU(5) bdle V within E_8,

$$10\text{'s} = H^1(Z;V),$$

$$\bar{5}\text{'s} = H^1(Z;\wedge^2 V).$$

Type IIB on CY orientifold X w/ D7-branes and O7-planes

five D7-branes on Σ_5 and an O7 on S_0

$$10\text{'s} = H^0(c_{10}; Tc_{10}{}^{-1/2} \otimes E_5{}^{\otimes 2}),$$

$$c_{10} = \Sigma_5 \cdot S_0.$$

an extra D7-brane wrapped on Σ_1,

$$\bar{5}\text{'s} = H^0(c_5; Tc_5{}^{-1/2} \otimes E_5^* \otimes E_1)$$

$$c_5 = \Sigma_5 \cdot \Sigma_1$$

Type IIA on CY orientifold w/ D6-branes and O6-planes

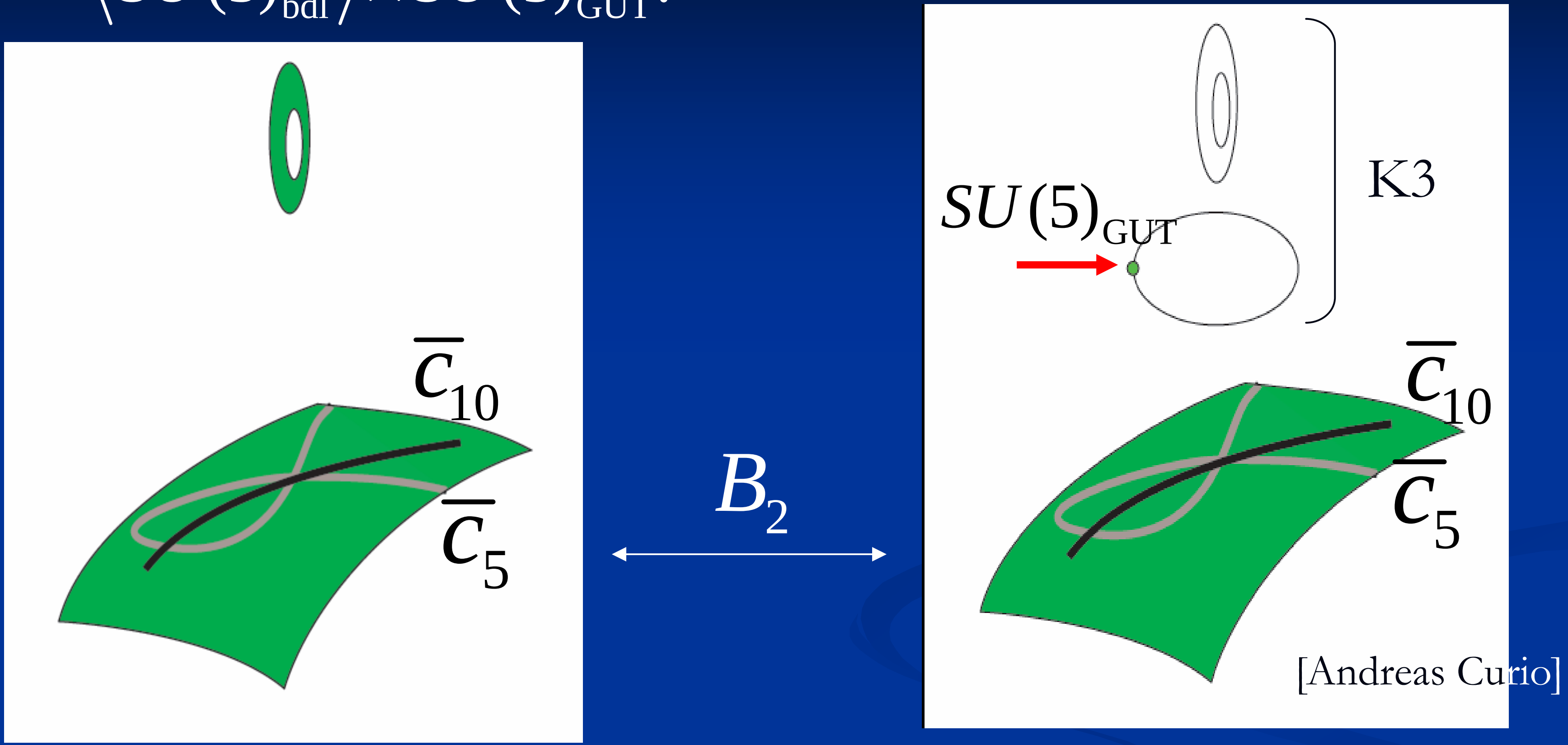
one chiral mutiplet at each D6-D6 intersection point

F-theory [Type IIB with (p,q) 7-branes]

→ ?

$$\mathrm{Het}/(T^2 \to Z \to B_2) \Longleftrightarrow F/(K3 \to X \to B_2)$$

e.g. $E_8 \to \langle SU(5)_{\mathrm{bdl}} \rangle \times SU(5)_{\mathrm{GUT}}$.



$$10's = H^1(Z;V) \cong H^0(\bar{c}_{10}; K_{B_2} \otimes \mathcal{N}_V)$$

$$\bar{5}'s = H^1(Z;\wedge^2 V) \cong H^0(\bar{c}_5; K_{B_2} \otimes \mathcal{N}_{\wedge^2 V})$$

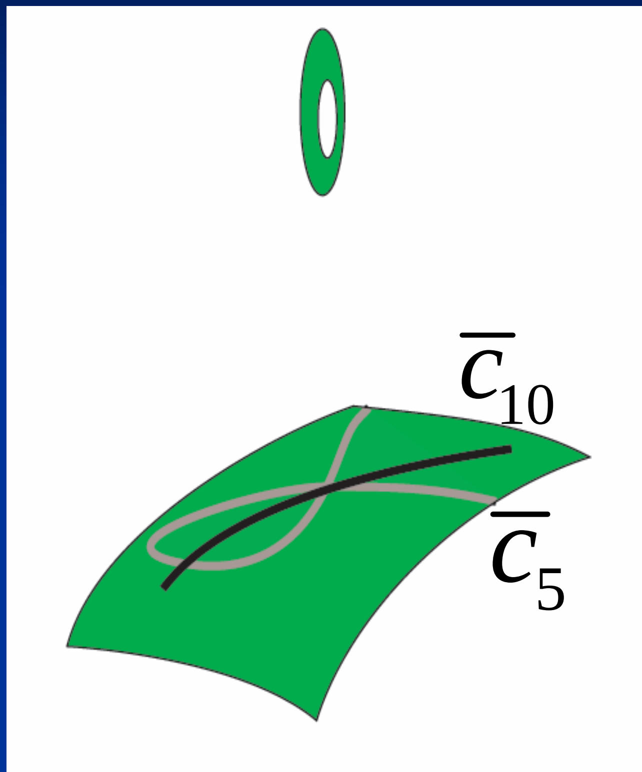
? $\Leftrightarrow$

Curio, Diaconescu Ionesei ’98

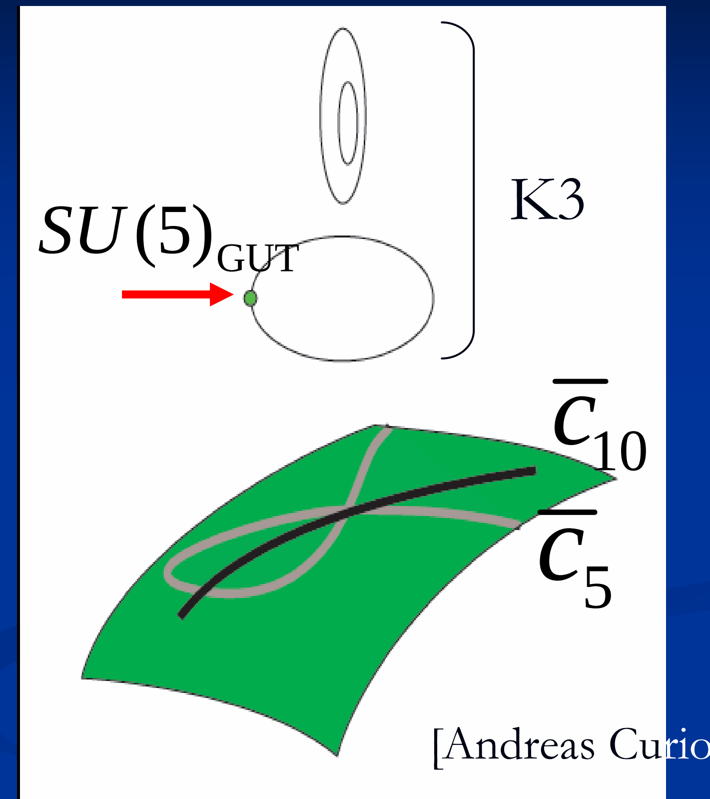
Donagi et.al. ’04, Blumenhagen et.al. ‘06

$$\text{Het}/(T^2 \to Z \to B_2) \Longleftrightarrow F/(K3 \to X \to B_2)$$

e.g. $E_8 \to \langle SU(5)_{\text{bdl}} \rangle \times SU(5)_{\text{GUT}}$.



B_2



$$10's = H^1(Z;V) \cong H^0(\overline{c}_{10}; K_{B_2} \otimes \mathcal{N}_V) \quad ? \quad H^0(\overline{c}_{10}; K_{\overline{c}_{10}}^{1/2} \otimes \mathcal{L}_G)$$

$$\overline{5}'s = H^1(Z;\wedge^2 V) \cong H^0(\overline{c}_5; K_{B_2} \otimes \mathcal{N}_{\wedge^2 V}) \Longleftrightarrow H^0(\overline{c}_5; K_{\overline{c}_5}^{1/2} \otimes \mathcal{L}_G).$$

Curio, Diaconescu Ionesei ’98
Donagi et.al. ’04, Blumenhagen et.al. ‘06

Donagi Wijnholt ’08
Beaseley Heckman Vafa ‘08

何が issue なのか？

- IIB with D7 and O7 では手が出ない、(p,q) 7-brane が交差する codim.-3 特異点では何が起きているのか？
- matter curve (7-brane intersection) の上の sheaf は、どのような構造を持っているのか？
  - 単純にベクトルバンドルと思っていいのか、どのようなバンドルか？
- "$K_{\bar{c}}^{1/2}$" だとか "$\mathcal{L}_G$" といった代物の正確な定義は何なのか？

- やはり、Heterotic-F duality をつかってちゃんと調べてみよう。
- 詳細は、0805.1057 [hep-th] (Hayashi et.al.) をご覧ください。

Results I

- e.g. in F-theory with unbroken SU(5) symmetry

$\deg K_{\tilde{\bar{c}}_5}^{1/2}$

$$\bar{5}'s = H^0(\tilde{\bar{c}}_5; \mathcal{O}(K_s + (\tilde{b}^{(c1)} + \tilde{b}^{(c2)})/2 + \gamma))$$

Double points resolved.

$\Delta \sim z^7.$

codim-3 singularity in B_3

$\gamma = \int_C G^{(4)}.$

integrated over the vanishing 2-cycle

It is $\tilde{\bar{C}}$, not $\bar{C}$, that parametrizes vanishing 2-cycles.

Results II

- e.g. in F-theory with unbroken SO(10) symmetry

$$\deg K_{\tilde{\bar{c}}_{vec}}^{1/2}$$

$$vec's = H^0(\tilde{\bar{c}}_{vec}; \mathcal{O}(K_s + \tilde{b}^{(c)} + \gamma))$$

degree-2 cover on $\bar{c}_{vec}$.

$\Delta \sim z^9$.

codim-3 singularity in B_3

$\gamma = \int_C G^{(4)}$.

integrated over the vanishing 2-cycle

$$= H^0(\bar{c}_{vec}; E)$$

E: rank-2 bundle.

back-up slides

Fourier—Mukai Transform of $\wedge^2 V$

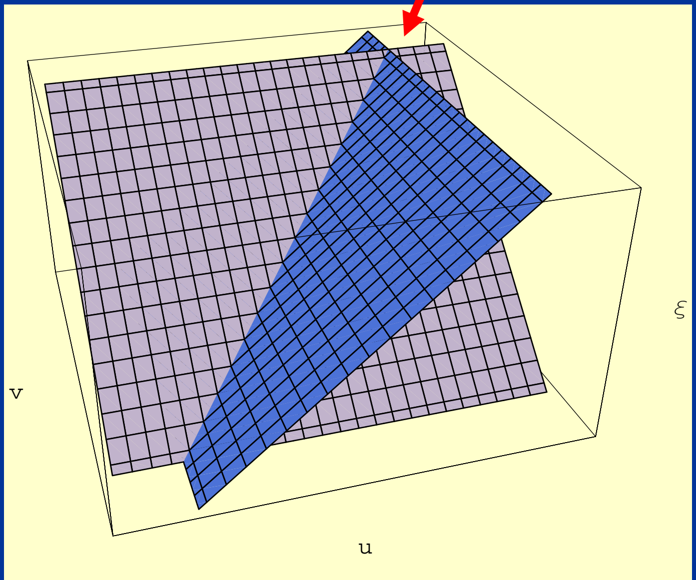
- $V \Leftrightarrow (C_V, \mathcal{N}_V)$
- $\wedge^2 V \Rightarrow (C_{\wedge^2 V}, \mathcal{N}_{\wedge^2 V})$

C_V : smooth

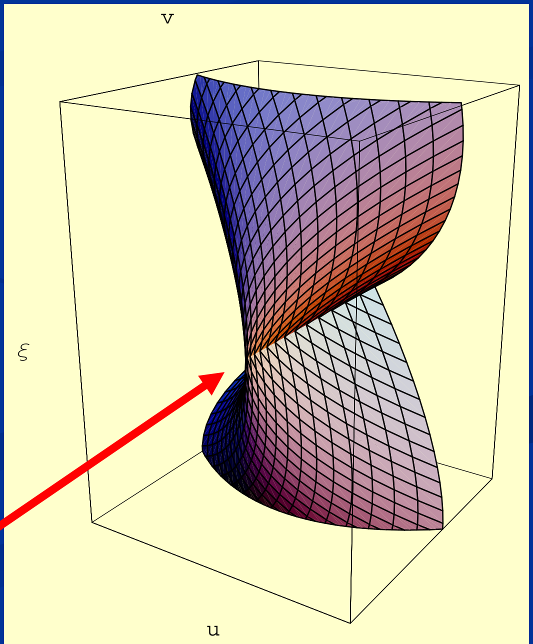
$\mathcal{N}_V$: a line bdl on C_V

$$i_{C_{\wedge^2 V}*}\mathcal{N}_{\wedge^2 V} = R^1 p_{1*}[P_B^{-1} \otimes K_B^{-1} \otimes p_2^*(\wedge^2 V)].$$

(double curve)



spectral surface $C_{\wedge^2 V}$



(pinch point)

Fourier—Mukai Transform of $\wedge^2 V$

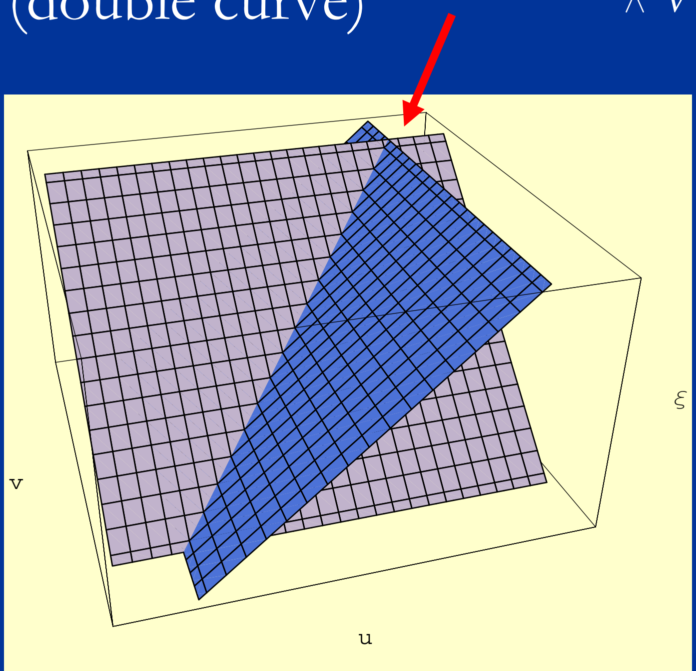
C_V : smooth

$\mathcal{N}_V$: a line bdl on C_V

- $V \Leftrightarrow (C_V, \mathcal{N}_V)$
- $\wedge^2 V \Rightarrow (C_{\wedge^2 V}, \mathcal{N}_{\wedge^2 V})$

$$i_{C_{\wedge^2 V}*}\mathcal{N}_{\wedge^2 V} = R^1 p_{1*}[P_B^{-1} \otimes K_B^{-1} \otimes p_2^*(\wedge^2 V)].$$

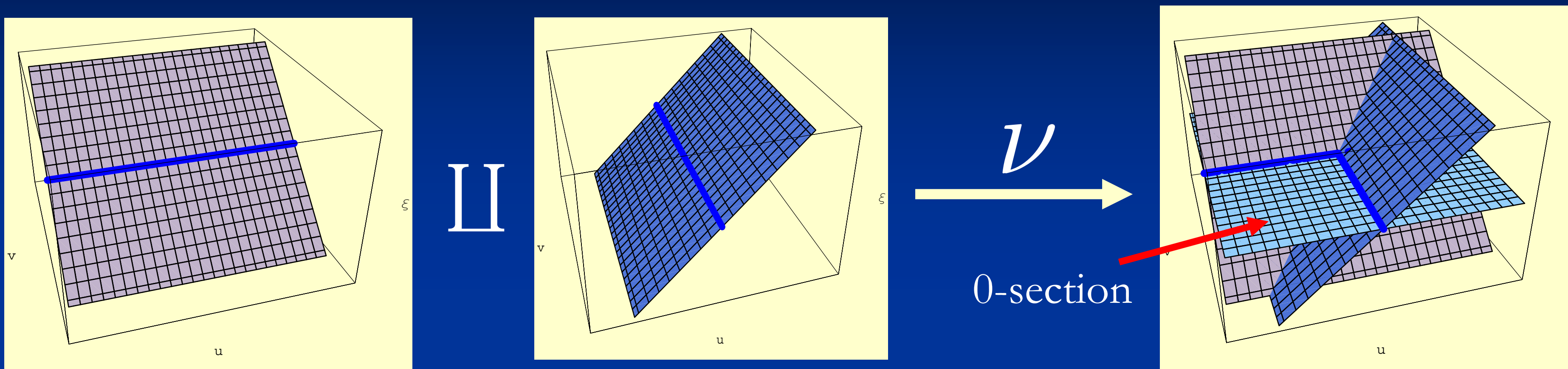
(double curve)



$$V \simeq \oplus_i \mathcal{O}(p_i - e_0)$$

$$\wedge^2 V = \mathcal{O}(p_i - e_0) \otimes \mathcal{O}(p_j - e_0)$$
$$\oplus \mathcal{O}(p_k - e_0) \otimes \mathcal{O}(p_l - e_0) \oplus \cdots .$$

a line bdle $\tilde{\mathcal{N}}_{\wedge^2 V}$ on $\tilde{C}_{\wedge^2 V}$ $\qquad \mathcal{N}_{\wedge^2 V} = \nu_* \tilde{\mathcal{N}}_{\wedge^2 V}$



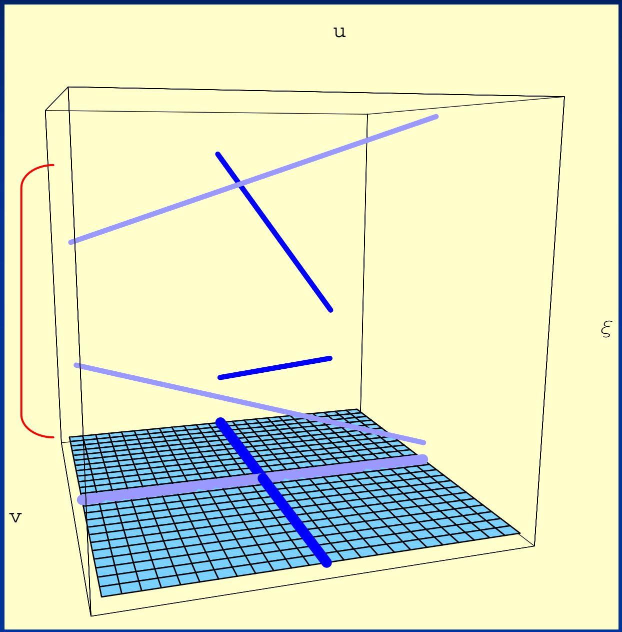
$$\tilde{\mathcal{F}}_{\wedge^2 V} = K_{B_2} \otimes \tilde{\mathcal{N}}_{\wedge^2 V} \xrightarrow{\ \nu_*\ } \mathcal{F}_{\wedge^2 V} = K_{B_2} \otimes \mathcal{N}_{\wedge^2 V}$$

on $\bar{\tilde{C}}_{\wedge^2 V}$ $\qquad$ on $\bar{C}_{\wedge^2 V}$

To Determine $\tilde{\mathcal{N}}_{\wedge^2 V}$ on $\tilde{C}_{\wedge^2 V}$,

cf. Donagi et.al. '04
Blumenhagen et.al. '06

D



T^2 fiber

$$\tilde{\mathcal{N}}_{\wedge^2 V} = \tilde{\pi}_{D*}(\mathcal{N}_V \otimes \mathcal{O}(-R/2))$$
$$= \mathcal{O}(\tilde{\pi}_{D*}((r|_D - R)/2 + \gamma)),$$

where $c_1(\mathcal{N}_V) = r/2 + \gamma.$

r: ramification divisor of

$$\pi_C : C_V \longrightarrow B_2.$$

R: ramification divisor of

$$\tilde{\pi}_D : D \longrightarrow \tilde{\tilde{c}}_{\wedge^2 V}$$

$$p_i \oplus p_j = e_0,$$
$$p_k \oplus p_l = e_0,$$

$$p_{i,j,k,l} \in E_b.$$

$$\deg\left[i^* K_{B_2} + \frac{1}{2}\tilde{\pi}_{D*}(r|_D - R)\right] = \deg K_{\tilde{\tilde{c}}_{\wedge^2 V}}^{1/2}$$

Heterotic—F Translation

- 2-form $\gamma_{\text{on } C_V} = \int_l G^{(4)} = \int_C G^{(4)}.$
  - determines $G^{(4)} \mod H^4(B_2;\mathbb{Q})$
  - $\chi(\wedge^2 V) = \int_{C \times \tilde{\tilde{c}}_{\wedge^2 V}} G^{(4)}.$
  - $\tilde{\tilde{c}}_{\wedge^2 V}$ parametrizes a family of vanishing 2-cycle.
- divisor $\tilde{\pi}_{D*}(r|_D - R)$

  (most of its components are) identified with codim.-3 singularities in base 3-fold of F-theory

$$\Delta \sim z^c \to z^{c+2}.$$

around a pinch point (rank V=4)