Structures on Doubled Geometry

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Workshop on Field Theory and String Theory
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Plan

- The Problem: T-duality
- Doubled Geometry
- Generalised Geometry
- Comparison
- Summary

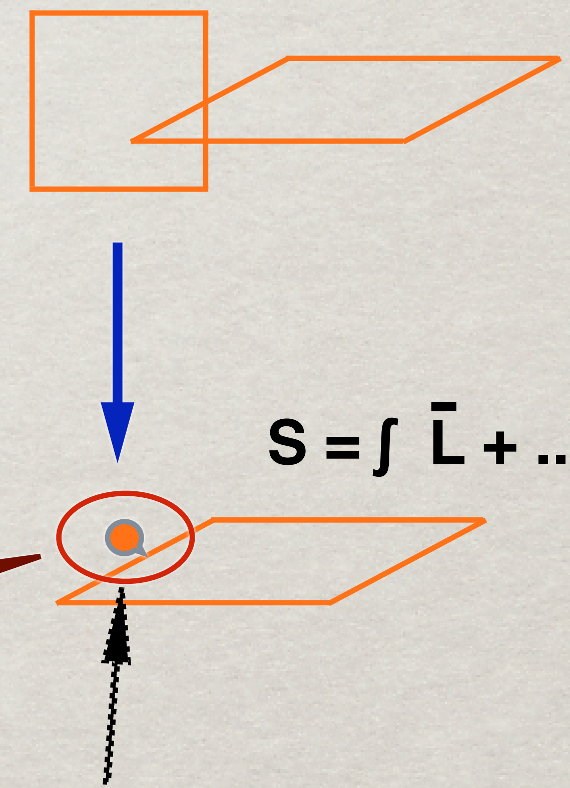
T-duality

String theory: 10 dim

$S = \int L$

Compactify
on 6 dim!

Real world: 4 dim



$S = \int \bar{L} + \dots$

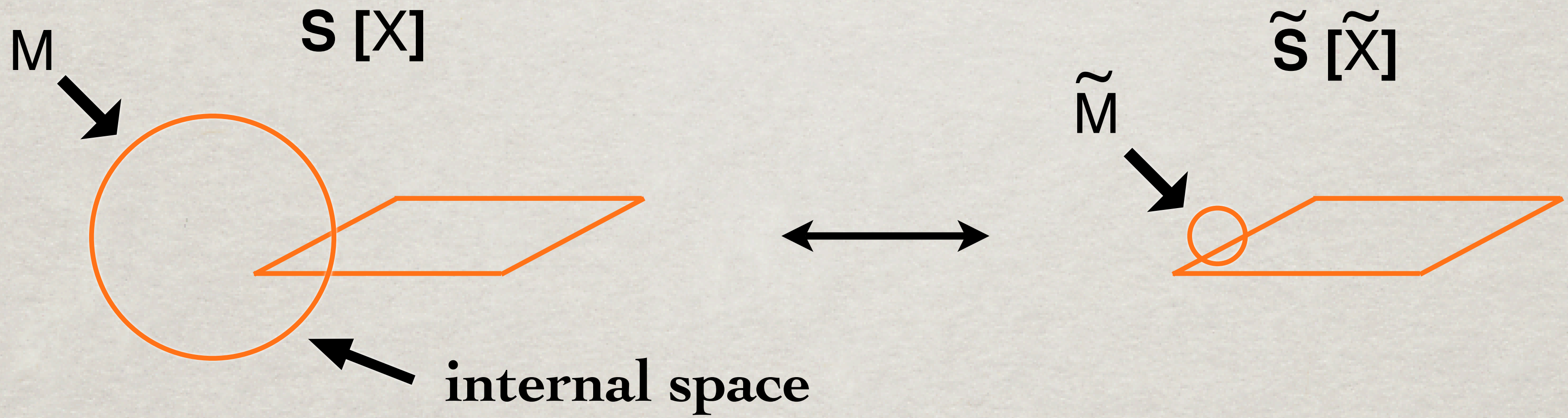
**Describe using
Generalised Geometry or
Doubled Geometry!**

**orbifolds, Calabi-Yau,
non-geometric spaces, ...**

[GG: Hitchin '02; DG: Hull '04]

T-duality on the internal space!

A symmetry of the compactified theory



T-duality group $O(d,d;\mathbb{Z})$ toroidal compactifications

T-duality on the internal space!

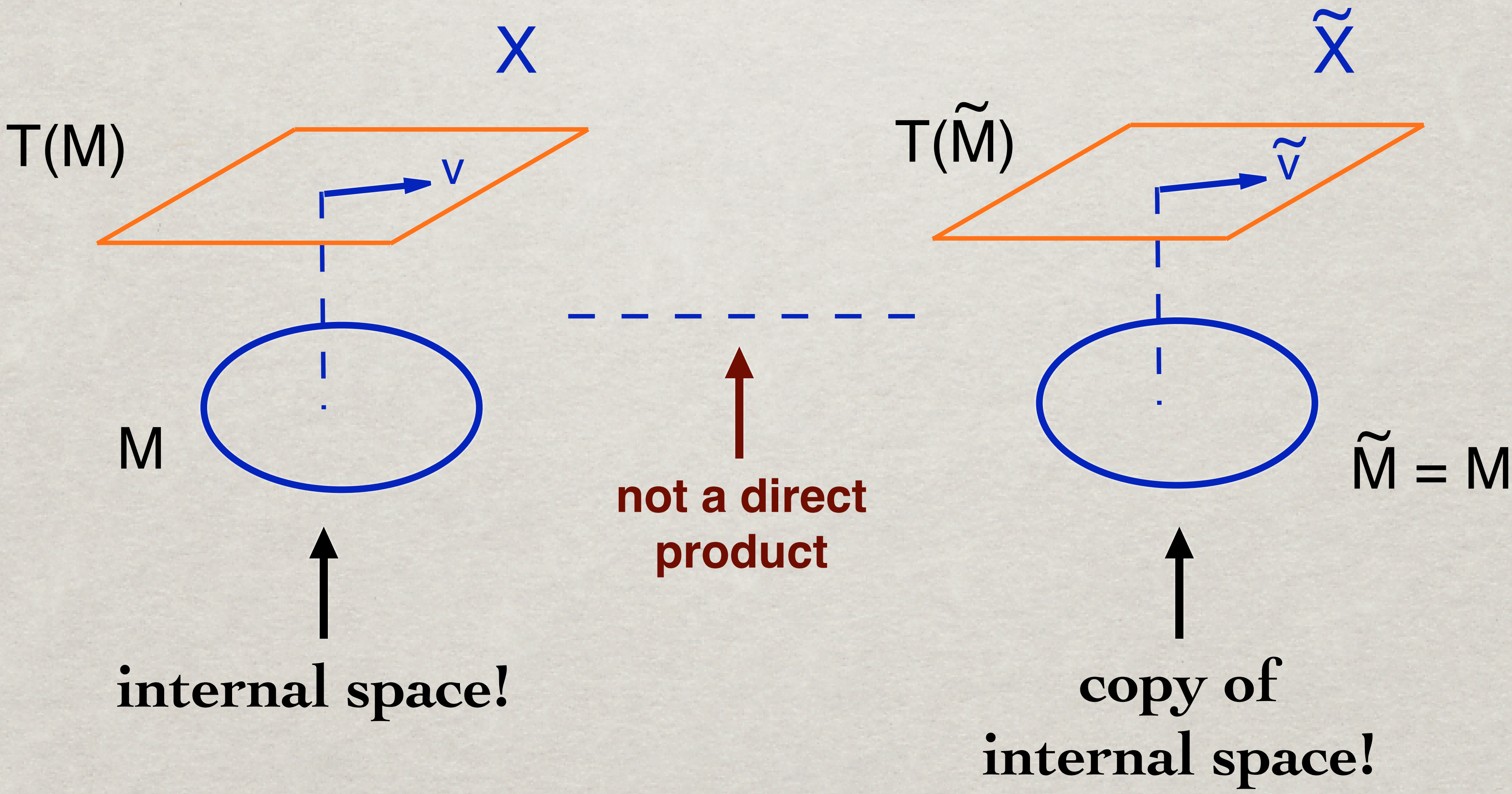
$$\begin{tikzcd} X \arrow[rr, leftrightarrow] \arrow[dr] & & \tilde{X} \arrow[dl] \\ & \mathbb{X} \equiv \{X, \tilde{X}\} \in T(M) \oplus T(\tilde{M}) \arrow[dl, "\Pi"] \arrow[dr, "\tilde{\Pi}"'] & \\ \text{original model} & & \text{dual model} \end{tikzcd}$$

Doubled Geometry

to describe the internal space!

Doubled Geometry

ABSTRACT description of the internal space!



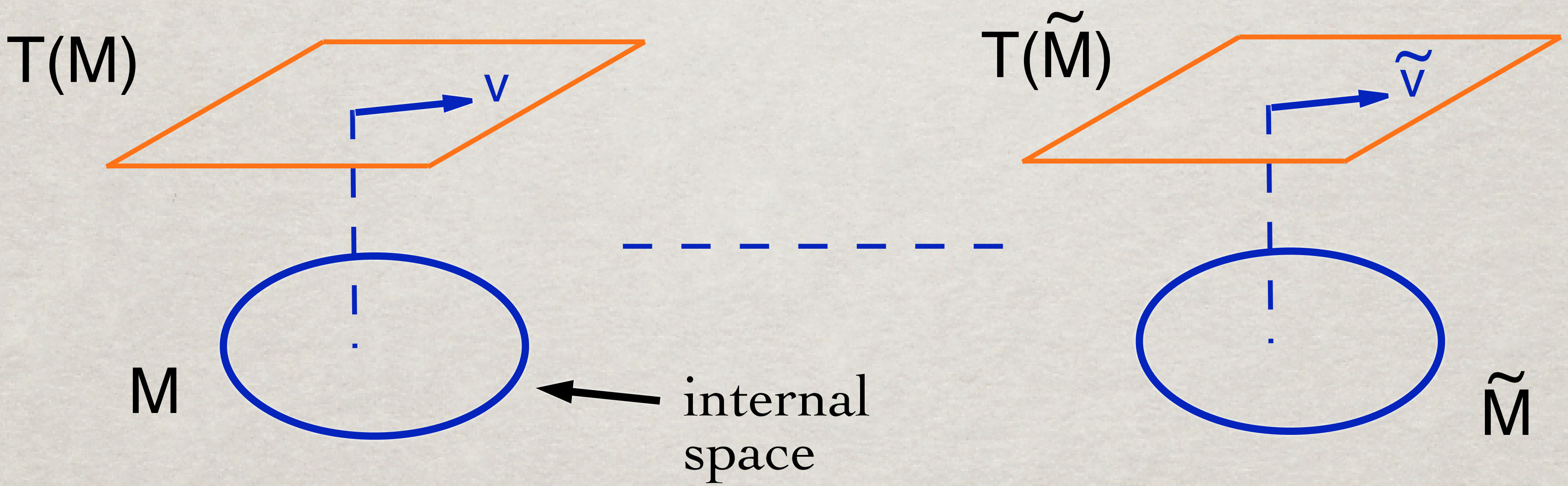
Doubled Geometry describing the internal space!

- **Consider the space**

$$T(M) \oplus T(\tilde{M}) = \{v + \tilde{v}\ ;\ v \in T(M),\ \tilde{v} \in T(\tilde{M})\}$$

$\Gamma \setminus G$

$\mathbb{X} \equiv \{X, \tilde{X}\}$



with an O(d,d) structure imposed by a *self-duality constraint*:

$$\boxed{\mathcal{P} = L^{-1}\mathcal{M}\,{}^*\mathcal{P}}$$

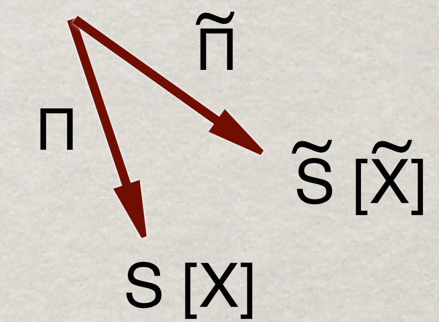
T-duality implemented in Doubled Geometry

(describing the internal space!)

Sigma model defined on doubled geometry:

$$S = \tfrac{1}{4} \int M_{mn}\, d\mathbb{X}^m \wedge {*d\mathbb{X}^n} + \tfrac{1}{4}\,\tfrac{1}{3} \int H_{mnr}\, d\mathbb{X}^m \wedge d\mathbb{X}^n \wedge d\mathbb{X}^r$$

where $\mathbb{X}^m = \{X^\mu, \tilde{X}^\mu\}$



Mutually dual models are found by:

- choosing different coordinate frames

T-duality group is in diffeom group: O(d,d) ⊂ GL(2d)

- eliminating half of d.o.f, imposing self-duality: $dX = {*d\tilde{X}}$

Structures on Doubled Geometry

$[T_a, T_b] = t_{ab}{}^c\, T_c$ **Lie algebra structure on $\Gamma \backslash G$**

L **neutral metric: signature (d,d)**

$\mathcal{M}$ **positive definite metric**

$S \equiv \Pi - \tilde{\Pi}$ **almost product structure**

$T \equiv L^{-1}\mathcal{M}$ **almost product structure**

$\mathcal{P} = L^{-1}\mathcal{M} * \mathcal{P}$

$\mathcal{J} \equiv TS$ **almost complex structure**

$S, T, \mathcal{J}$ **satisfy a *para-quaternion* algebra:**

$$\mathcal{J} = TS = -ST \qquad -\mathcal{J}^2 = S^2 = T^2 = \mathbb{1}$$

Metric compatibility

$L(SX,SY) = -L(X,Y)$ **almost para-Hermitian structure**

$L(TX,TY) = L(X,Y)$ **pseudo-Riemannian almost product structure**

$L(\mathcal{J}X,\mathcal{J}Y) = -L(X,Y)$ **almost complex anti-Hermitian structure**

$\mathcal{M}(SX,SY) = \mathcal{M}(X,Y)$ **Riemannian almost product structure**

$\mathcal{M}(TX,TY) = \mathcal{M}(X,Y)$ **Riemannian almost product structure**

$\mathcal{M}(\mathcal{J}X,\mathcal{J}Y) = \mathcal{M}(X,Y)$ **almost Hermitian structure**

☞ Doubled Geometry is a *neutral hypercomplex manifold*

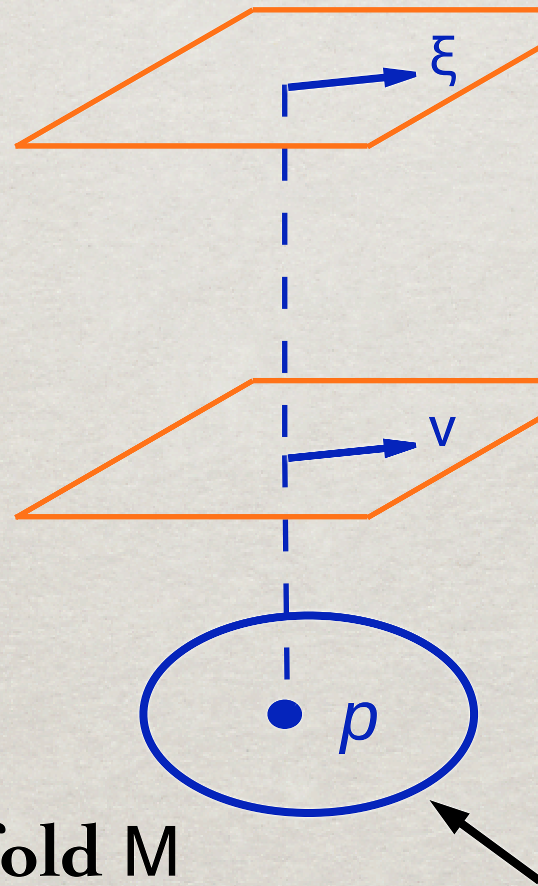
- Neutral metric L
- Three structures S, T, $\mathcal{J}$ satisfying para-quaternion algebra
- The metrics L and $\mathcal{M}$ are ***twin metrics*** with respect to T

Generalised Geometry

Cotangent space at point $p \in M$

$T^*_p(M)$

spanned by cotangent vectors $\xi \in T^*_p(M)$, **e.g.** $\{dx^\mu\}$



Tangent space at point $p \in M$

$T_p(M)$

spanned by tangent vectors $v \in T_p(M)$, **e.g.** $\{\partial/\partial x^\mu\}$

Manifold M

internal space

Generalised Geometry **describing the internal space!**

complex geometry:
structures on T(M)

symplectic geometry:
structures on T*(M)

T(M)

T*(M)

T(M) $\oplus$ T*(M)

choice of section

complex geometry

symplectic geometry

Structures on Generalised Geometry

L	neutral metric: signature (d,d)
$\mathcal{M}$	positive definite metric
$S \equiv \Pi - \tilde{\Pi}$	generalised almost product structure (local split into tangent and cotangent space)
$T \equiv L^{-1}\,\mathcal{M}$	almost product structure

Structure group O(d,d)

Comparison

Doubled Geometry	Generalised Geometry
Almost product structures S and T	Almost product structures S and T
Neutral metric L and pos def metric $\mathcal{M}$	Neutral metric L and pos def metric $\mathcal{M}$
T-duality group naturally encoded	T-duality group naturally encoded

Comparison

Doubled Geometry	Generalised Geometry
Space M is doubled	Space M not doubled
$T(M) \oplus T(\tilde{M})$	$T(M) \oplus T^*(M)$
O(d,d) transition functions	GL(d) transition functions
Neutral hypercomplex!	

$T(M) \oplus T(\tilde{M})$ $\qquad$ $T(M) \oplus T^*(M)$

Summary

- DG is a neutral hypercomplex manifold

- Although DG and GG structures are defined on different spaces, there are many similarities

- DG appears to be more restricted than GG, but less restricted than Gen Complex Geometry

The End