高密度QCDにおける
カラー超伝導ボーテックス

（もうすぐ完成）

基研研究会「場の理論と弦理論」

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Color Magnetic Flux Tubes in Dense QCD

(to appear soon)

8. July. 2009 @ YITP

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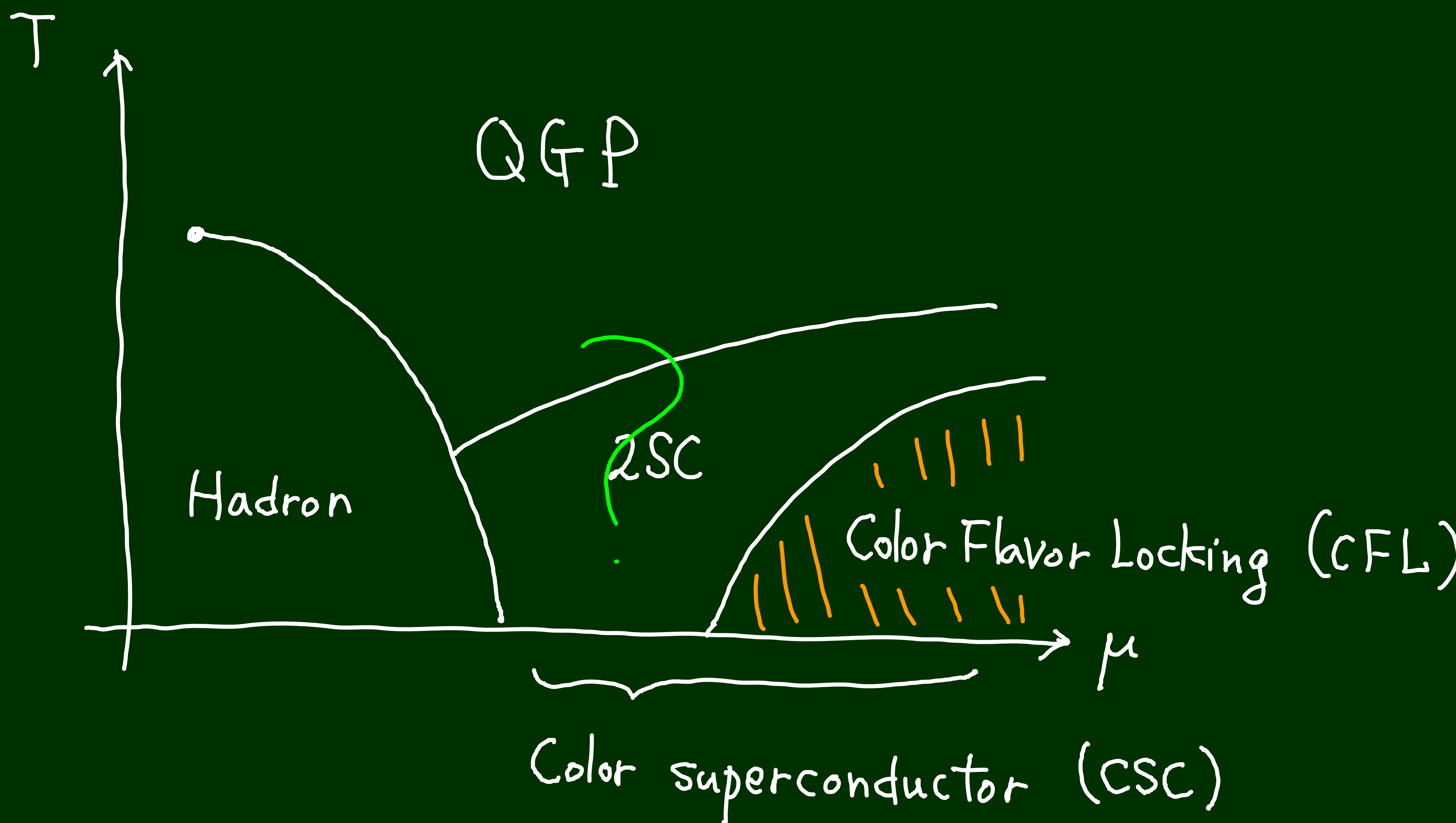
with Muneto Nitta (Univ. Keio)

phase of QCD is expected to be ....



At ultra high density ($\mu \gg \Lambda_{QCD}$), QCD $\rightarrow$ CFL phase.

* CFL : $N_c = N_f = 3$. $(m_u \sim m_d \sim m_s \sim 0 \ll \mu)$

diquark condensate (anti-symmetric)

$$\langle q^a_{fL} C q^b_{gL} \rangle = \epsilon^{abc} \epsilon_{fgh} \bar{\Phi}^h_{cL}$$
$$\langle q^a_{fR} C q^b_{gR} \rangle = \epsilon^{abc} \epsilon_{fgh} \bar{\Phi}^h_{cR}$$

$(\mathbf{\bar{3}}^*, \mathbf{\bar{3}}^*)$
↑ color ↑ flavor

$$\bar{\Phi}_L = -\bar{\Phi}_R \sim \begin{array}{c} \\ R \\ G \\ B \end{array} \begin{array}{c} \begin{array}{ccc} ds & su & ud \end{array} \\ \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \end{array}$$

◁ CFL!

Symmetry

$$\boxed{SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \xrightarrow{\langle\Phi\rangle} SU(3)_{C+L+R}}$$

- $\cancel{U(1)_B}$: super fluid
- $\cancel{SU(3)_c}$: color superconductor
- Chi~~ral~~ symmetry

Asymptotic Freedom $(p \sim p_F \sim \mu \gg \Lambda_{QCD})$

$$\frac{g^2}{4\pi} = \frac{6\pi}{(33-2N_f)\log(\mu/\Lambda_{QCD})} \rightarrow$$ Weak gauge coupling !

CFL in neutron or quark star (?)

⇒ Formation of topological defects

Neutron star

* superfluid vortex ($U(1)_B$ global vortex)

[Forbes-Zhitnisky, Baym-Iida]

* non-Abelian vortex (semi superfluid vortex)

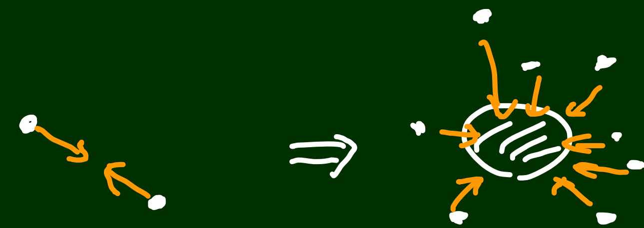
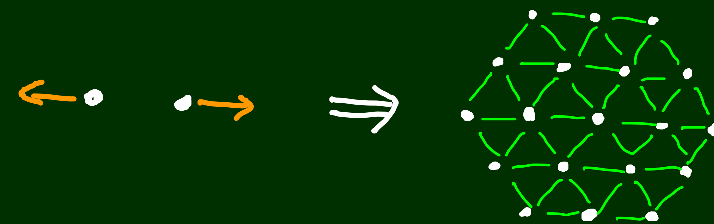
[Balachandran-Digal-Matsuura, Nakano-Nitta-Matsuura]

superfluid vortex + color magnetic flux

$3 \times T_{NA}$ $\qquad\qquad\qquad$ $T_{U(1)_B}$

By studing vortices, we can figure out whether color superconductor is stable or not.

Usual metal

- type I superconductor is unstable 
- type II superconductor is stable. 

In this talk, I will tell you details of vortex in CFL.

Landau-Ginzburg model

diquark

$\Phi \equiv \Phi_L = -\Phi_R$

$$\mathcal{L} = \mathrm{Tr}\left[\frac{1}{4} F_{ij}^2 + D_i \Phi^\dagger D^i \Phi \right] + V \qquad (i,j = 1,2,3)$$

$$V = m^2 \,\mathrm{Tr}[\Phi\Phi^\dagger] + \lambda_1 \left(\mathrm{Tr}[\Phi\Phi^\dagger]\right)^2 + \lambda_2 \,\mathrm{Tr}\left[(\Phi\Phi^\dagger)^2\right]$$

@ weak coupling [Iida-Baym, Giannikis-Ren]

$$\boxed{m^2 = 4N \log \frac{T}{T_c} < 0\,, \quad \lambda_1 = \lambda_2 = \frac{7\zeta(3)}{8(\pi T_c)^2} N\,, \quad N \equiv \frac{\mu^2}{2\pi^2}}$$

CFL vacuum

$$\Phi = v\,\mathbb{1}_3\,, \quad v^2 \equiv \frac{-m^2}{2(3\lambda_1 + \lambda_2)} > 0$$

$$G = SU(3)_c \times SU(3)_F \times U(1)_B \xrightarrow{\langle \Phi \rangle} H = SU(3)_{c+F} \qquad (F = L, R)$$

Vacuum manifold : $M = G/H = U(3)$

Mass spectrum in CFL phase

$$\Phi = v \mathbb{1}_3 + \underbrace{\frac{\phi + i\varphi}{\sqrt{2}}}_{\text{CF singlet}} \mathbb{1}_3 + \underbrace{\frac{\chi^a + i\zeta^a}{\sqrt{2}}}_{\text{CF adjoint}} T^a \qquad (a = 1, 2, \cdots, 8)$$

$$\zeta^a \xrightarrow{\text{Higgs}} A_\mu^a : \text{gluon} \to \text{massive}$$

U(1)$_B$ NG boson

$$\boxed{m_G^2 = 2g^2 v^2, \quad m_\chi^2 = 4\lambda_2 v^2, \quad m_\phi^2 = -2m^2, \quad m_\varphi^2 = 0}$$

Vortices in CFL phase

$$\eta(\nu_3, \nu_8) \equiv \sqrt{\frac{2}{3}}\left(\nu_3 T_3 + \nu_8 T_8\right) \quad \left[T_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & & \\ & 1 & \\ & & -1 \end{pmatrix},\ T_8 = \frac{1}{\sqrt{6}}\begin{pmatrix} -2 & & \\ & 1 & \\ & & 1 \end{pmatrix} \right]$$

$$\begin{cases} \Phi = v\, e^{i\theta(\nu_0 \mathbb{1} - \eta)} \left(\underline{\underline{F(r)}}\, \nu_0 \mathbb{1}_3 - \underline{\underline{G(r)}}\, \eta \right) \\ A_i = \dfrac{1}{g} \dfrac{\epsilon_{ij} x^j}{r^2} \left(1 - \underline{\underline{h(r)}}\right) \eta \end{cases}$$

single valuedness $\longrightarrow$ $e^{i2\pi(\nu_0 \mathbb{1} - \eta)} = 1$

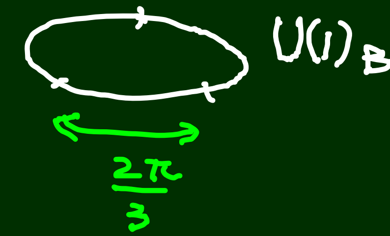
minimal configuration $(\nu_0, \nu_3, \nu_8) = \pm\left(\frac{1}{3}, 0, 1\right), \pm\left(\frac{1}{3}, \pm\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

Family

* Non-Abelian Vortex [Balachandran-Digal-Matsuura, Nakano-Nitta-

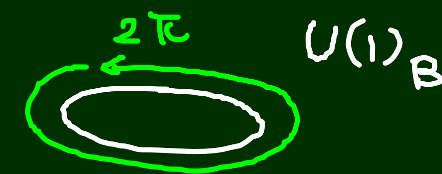
$$(\nu_0, \nu_3, \nu_8) = \left(\frac{1}{3}, 0, 1\right)$$

$$\left(\frac{1}{3}, \pm\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$



* Superfluid vortex [Forbes-Zhitnisky, Baym-Iida]

$$(\nu_0, \nu_3, \nu_8) = (1, 0, 0)$$



* Non-topological vortex [Iida]

$$(\nu_0, \nu_3, \nu_8) = (0, 0, 3)$$

Without loss of generality, we can choose $(\nu_3, \nu_8) = (0, 1)$

In terms of

$$f = \frac{F+2G}{3}, \quad g = \frac{F-G}{3}$$

$$\Phi = \begin{pmatrix} \underbrace{e^{i\theta} f(r)}_{\text{wind}} & & 0 \\ & g(r) & \\ 0 & & \underbrace{g(r)}_{\text{unwind}} \end{pmatrix}$$

$$F_{12} = \frac{\sqrt{6}}{3gr} h'(r)\, T_8$$

Why non-Abelian?

$$SU(3)_{C+F} \text{ in CFL} \xrightarrow{\text{Vortex}} U(1)\times SU(2)$$

$$\boxed{\begin{array}{c}\text{NG mode}\\ \mathbb{C}P^2\end{array}}$$

$$v\begin{pmatrix} fe^{i\theta} & & \\ & g & \\ & & g \end{pmatrix}$$

$$\mathbb{Z}_3$$

$$v\begin{pmatrix} g & & \\ & fe^{i\theta} & \\ & & g \end{pmatrix} \qquad v\begin{pmatrix} g & & \\ & g & \\ & & fe^{i\theta} \end{pmatrix}$$

E.o.M for $\{f, g, h\}$

$$f'' + \frac{f'}{r} - \frac{(2h+1)^2}{9r^2} f - \frac{m_\phi^2}{6} f(f^2 + 2g^2 - 3) - \frac{m_\chi^2}{3} f(f^2 - g^2) = 0$$

$$g'' + \frac{g'}{r} - \frac{(h-1)^2}{9r^2} g - \frac{m_\phi^2}{6} g(f^2 + 2g^2 - 3) + \frac{m_\chi^2}{6} g(f^2 - g^2) = 0$$

$$h'' - \frac{h'}{r} - \frac{m_G^2}{3}\left(g^2(h-1) + f^2(2h+1)\right) = 0$$

$$\text{B.C.} \begin{cases} \{f, g, h\} \longrightarrow \{1, 1, 0\} & \text{as } r \to \infty \\ \{f, g', h\} \longrightarrow \{0, 0, 1\} & \text{as } r \to 0 \end{cases}$$

[Balachandran-Digal-Matsuura] numerically solved these equations with an approximation $g(r) \equiv 1$.

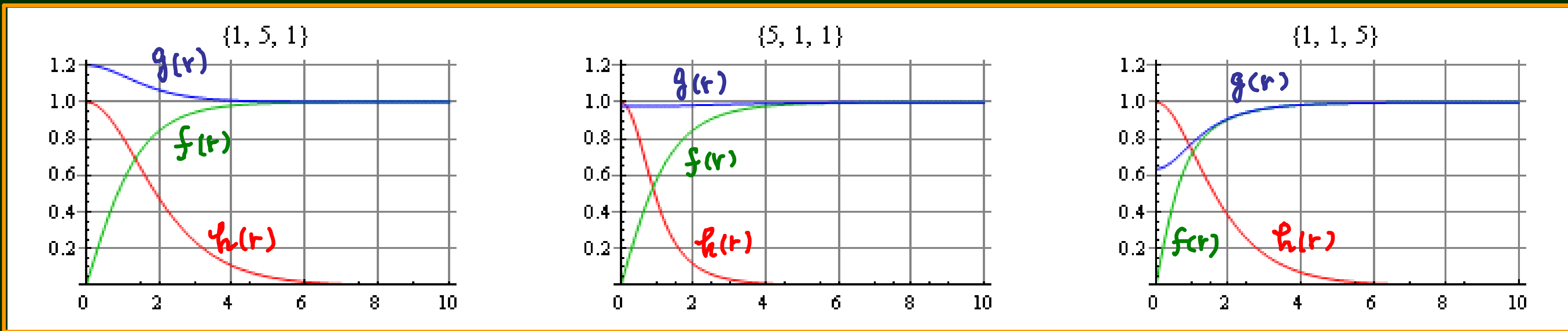
But $g(r) = 1$ can be never true !!

[Only when $m_\phi = m_\chi$, $g(r) \sim 1$.]

Espcially, $G(r) = f(r) - g(r)$ is an order parameter for the breaking of $SU(3)_{C+F} \longrightarrow U(1) \times SU(2)$.

We have obtained EXACT vortex solutions without any approximation.

Exact Solutions for $\{m_\sigma, m_\phi, m_\chi\}$.



* $0 < g(0) < \sqrt{\frac{3}{2}}$
* $m_\chi \gg m_\phi \qquad g(r) \to f(r)$
* $m_\phi \gg m_\chi \qquad g(r) \to \sqrt{\frac{3 - f^2(r)}{2}}$

$\boxed{\text{$g(r)$ is sensitive to $\{m_\phi, m_\chi\}$.}}$

We checked

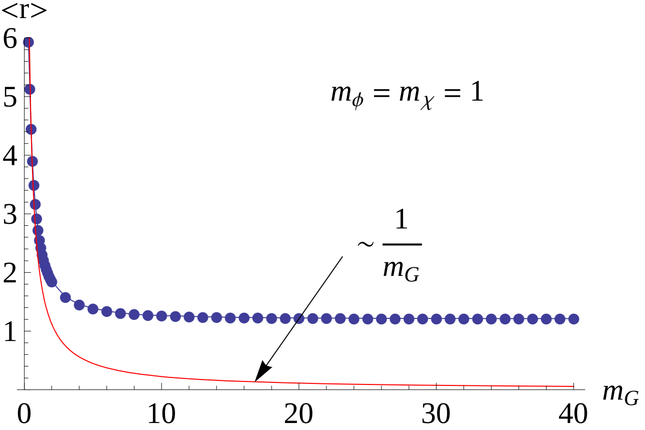
* Tension : log divergent (superfluid vortex)

* Color magnetic flux $\longrightarrow$ well squeezed

$\langle$Finite width of flux tube$\rangle$

$$\langle R \rangle \equiv \sqrt{\frac{\int d^2x \, F_{12}^8 \, r^2}{\int d^2x \, F_{12}^8}} \leftarrow \text{finite}$$

depend on $\{m_G, m_\phi, m_\chi\}$.


<r>
6
5
4
3
2
1
0
10
20
30
40
m_G
$m_\phi = m_\chi = 1$
$\sim \frac{1}{m_G}$

– Toward Intervortex Force –

* usual superconductor (photon, scalar)

type I ($m_\gamma > m_s$), BPS ($m_\gamma = m_s$), type II ($m_\gamma < m_s$)

* color superconductor

$\underbrace{m_G\ ,\ m_\chi}_{\text{SU(3) superconductor}}$ and $\underbrace{m_\phi,\ m_\varphi = 0}_{\text{super fluid}}$

Vortices have internal orientation $\mathbb{C}P^2$.

$\Longrightarrow$ Very complicated !!! (never BPS)

Asymptotics $\quad r \gg \max\{m_\phi^{-1}, m_\chi^{-1}, m_G^{-1}\}$

$$F = 3 + \delta F , \quad G = \delta G , \quad h = \delta h \equiv m_G r \, \delta\tilde{h}$$

Linearized equations

$$\left(\Delta - m_\phi^2 - \frac{1}{9r^2}\right)\delta F = \frac{1}{3r^2}$$

← decouple $U(1)_B$

$$\left.\begin{aligned}\left(\Delta - m_\chi^2 - \frac{1}{9r^2}\right)\delta G &= \frac{2m_G}{3r}\,\delta\tilde{h} \\ \left(\Delta - m_G^2 - \frac{1}{r^2}\right)\delta\tilde{h} &= \frac{2m_G}{3r}\,\delta G\end{aligned}\right\}$$

couple $SU(3)_c$

$$\left(\Delta \equiv \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right)$$

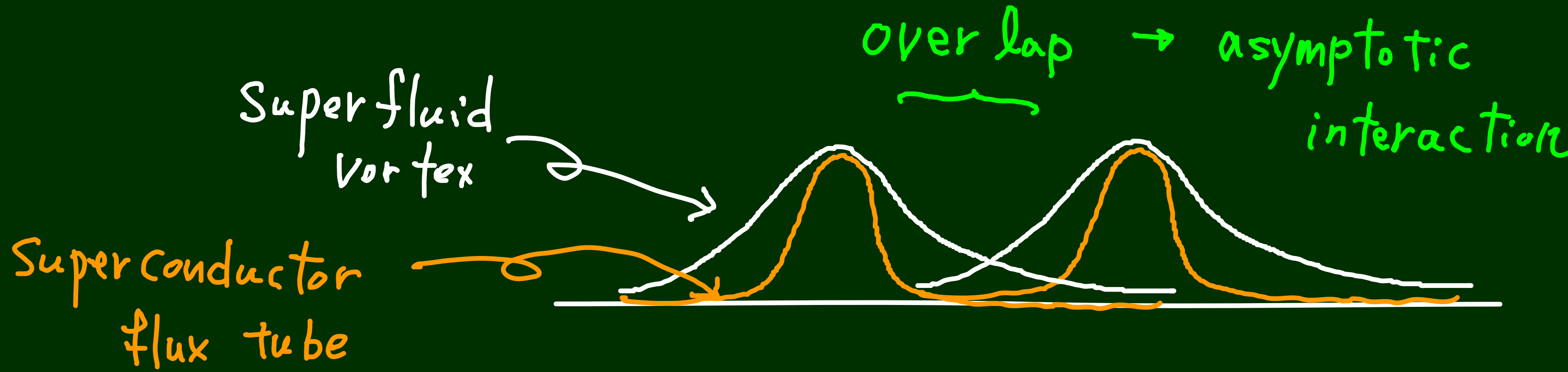
$$\delta F \cong -\frac{1}{3m_\phi^2 r^2} + O(r^{-4}) + O(e^{-m_\phi r})$$

long tail

short tails

$$m_G > m_\chi \quad \begin{cases} \delta G \cong q\sqrt{\dfrac{\pi}{2m_\chi r}}\, e^{-m_\chi r} \\ \delta h \cong -\dfrac{2}{3} q \dfrac{m_G^2}{m_G^2 - m_\chi^2}\sqrt{\dfrac{\pi}{2m_\chi r}}\, e^{-m_\chi r} \end{cases}$$

$$m_\chi > m_G \quad \begin{cases} \delta G \cong -\dfrac{2}{3}\mu \dfrac{1}{(m_\chi^2 - m_G^2) r^2}\sqrt{\dfrac{\pi m_G r}{2}}\, e^{-m_G r} \\ \delta h \cong \mu \sqrt{\dfrac{\pi m_G r}{2}}\, e^{-m_G r} \end{cases}$$



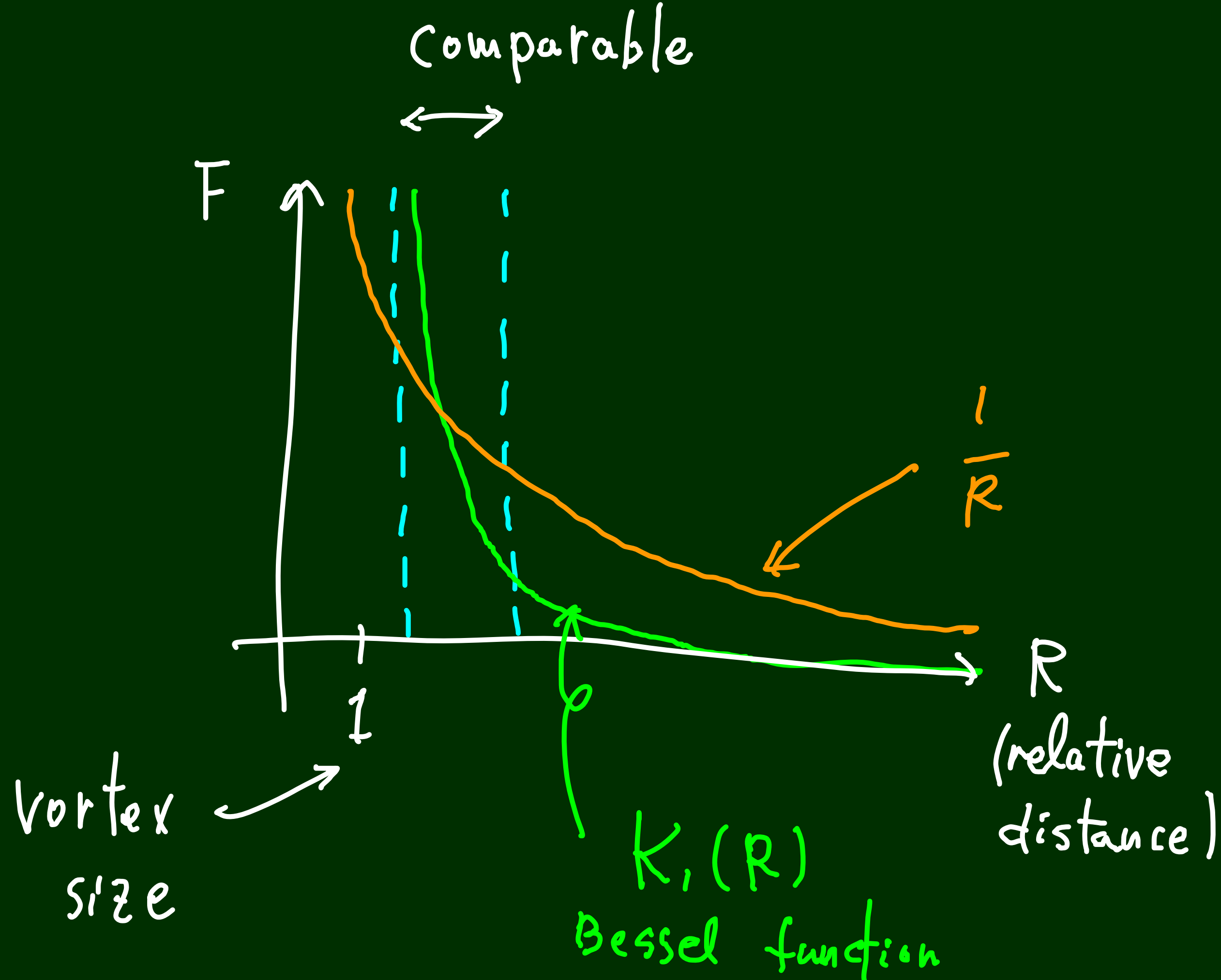
Very long range → Universal repulsion [Nakano-Nitta-Matsuura]

(Massless NG boson, $U(1)_B$ superfluid)

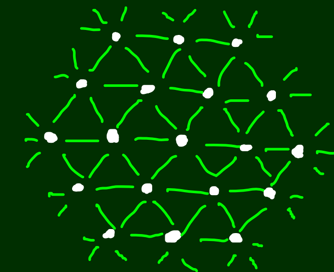
Short range → color superconductivity

- massive particles. m_q, m_χ, m_ϕ
- internal orientations

Naive expectation

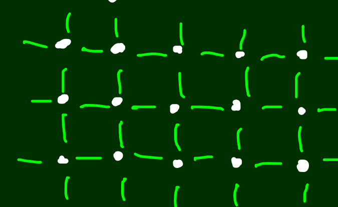


low density vortices



high density vortices

- remain stable ?
- change lattice ?



- get unstable ?

— Conclusion —

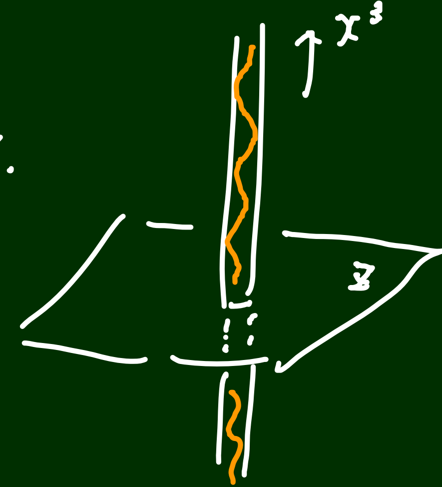
We have studied non-Abelian vortices in CFL phase of dense QCD.

* Exact numerical solutions
* Measure finite width of color magnetic flux tube
* Strange assymptotic tails

$\longrightarrow$ short range interactions and stability of CSC.

– Future works –

* low energy effective action of NA vortex.



$1+1$ dim $\mathbb{C}P^2$ NLσm

* $m_s \gg m_u, m_d \sim 0 \longrightarrow SU(3)_F \to U(1)_F^2$

* $U(1)_{EM}$: gauging $T_8 \in SU(3)_{C+F}$

$$(Q_u, Q_d, Q_s) = \left(\frac{2}{3}e, -\frac{1}{3}e, -\frac{1}{3}e\right) \longrightarrow (Q_{ds}, Q_{su}, Q_{ud}) = \left(-\frac{2}{3}e, \frac{1}{3}e, \frac{1}{3}e\right)$$

Gauging $U(1) \subset SU(3)_{C+F} \Longrightarrow U(1)^2$

* $\langle qq \rangle$ Condensate $\xrightarrow[\text{vortex}]{}$ monopole confinement

Quark - Hadron continuity [Schafer-Wilczek]

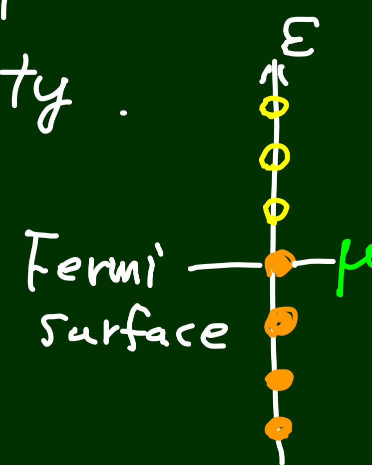
	Hadron	Quark
Symm.	$SU(3)_L \times SU(3)_R \to SU(3)_{L+R}$	$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_{c+L+R}$
NG	8 pions	8 pions + 1
Vector	9 vector mesons	8 gluons $\to$ massive
fermion	9 baryons	9 quarks
Skyrmion	baryon	quark (quariton)
Confinment	monopole condensation	quark condensation

Quarks are fermions.

At low temperature and high density, quarks form degenerate fermi system (quark matter).

$$\Rightarrow \text{ all states } \begin{cases} \varepsilon \leq \varepsilon_F = \mu & \text{are occupied} \\ \varepsilon > \varepsilon_F = \mu & \text{are empty.} \end{cases}$$

Because of Pauli-blocking, interactions mainly modify states near Fermi surface.



Interaction : $p \sim p_F \sim \mu \gg \Lambda_{QCD}$ (~ 200 MeV)

Asymptotic Free : $\dfrac{g^2}{4\pi} = \dfrac{6\pi}{(33-2N_f)\log(\mu/\Lambda_{QCD})}$

$\rightarrow$ Weak coupling

▷ Quarks near Fermi surface are almost free with weak QCD interactions.

$\begin{cases} \text{No interaction} \longrightarrow \text{No cost for cooper pairing} \\ \underbrace{\text{Any weak interaction}}_{\text{one gluon exchange}} \longrightarrow \text{Cooper instability (BCS)} \end{cases}$

color superconductor

Di-quark condensation (massless case)

[Alford-Rajagopal-Wilczek]

$$\left.\begin{aligned}\langle q^{a}_{iL} C q^{b}_{jL}\rangle &= \epsilon^{abc}\epsilon_{ijk}\Phi_{Lc}{}^{k}\\ \langle q^{a}_{iR} C q^{b}_{jR}\rangle &= \epsilon^{abc}\epsilon_{ijk}\Phi_{Rc}{}^{k}\end{aligned}\right\} \xrightarrow{\text{ground state}} \Phi_L = -\Phi_R \sim \begin{pmatrix}1 & & \\ & 1 & \\ & & 1\end{pmatrix}$$

$$\left[\begin{aligned} a, b &= R, G, B \\ i, j &= u, d, s \end{aligned}\right]$$

Color-Flavor Locking (CFL)

$$\boxed{SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \longrightarrow SU(3)_{c+L+R}}$$

$c+L$ and $c+R$ $\longrightarrow$ ~~Chiral symmetry~~