Super Galilean conformal algebra in AdS/CFT

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based on

``Super Galilean conformal algebra in AdS/CFT,'' arXiv:0905.0188 [hep-th].

Introduction

AdS/CFT correspondence $\quad psu(2,2|4) \supset so(2,4) \times su(4)$

IIB string in $AdS_5 \times S^5$ ⟺ N=4 super Yang-Mills in 4-dim

weak *strong*

Applications AdS/CMP

superconductor [Gubser '0801, Hartnoll-Herzog-Horowitz '0810, …]

quantum Hall effect [Keski-Vakkuri-Kraus '0805, Davis-Kraus-Shah '0809, …]

fermions at unitarity [Son '0804, Balasubramanian-McGreevy '0804,…]

Non-relativistic CFT

- Schrödinger symmetry
- **Galilean conformal symmetry**

conformal
U
Poincare —NR limit→ Galilei

Schrödinger symmetry

Galilean symmetry $\{ H, P_i, J_{ij}, G_i \}$

dilatation D, Galilean special conformal K

central charge M

- symmetry of (free) Schrödinger equation [Hagen'72, Niederer'72]
- subalgebra of conformal algebra in one higher dimensions
- relativistic FT $\xrightarrow{\text{NR limit}}$ Schrödinger-invariant FT (for example [Jackiw-Pi'90])
- $z=2$ case

z: dynamical exponent

anisotropic scale transformation D

$$x^i \longrightarrow \lambda x^i \ , \quad t \longrightarrow \lambda^z t$$

Galilean conformal symmetry

Galilean symmetry $\{ H, P_i, J_{ij}, G_i \}$

dilatation D, Galilean special conformal K

acceleration K_i

→ $z=1$ case

PLAN

- Inönü-Wigner contraction of conformal algebra (review)
- bulk realization of GCA
- supersymmetric GCA

GCA from conformal algebra

conf$_{d+1}$

$$\{\tilde{K}_\mu,\ \tilde{D},\ \tilde{J}_{\mu\nu},\ \tilde{P}_\mu\}$$

$$\mu = 0, 1, \cdots, d$$

$$[\tilde{D}, \tilde{P}_\mu] = \tilde{P}_\mu\ , \quad [\tilde{D}, \tilde{K}_\mu] = -\tilde{K}_\mu\ , \quad [\tilde{P}_\mu, \tilde{K}_\nu] = \frac{1}{2}\tilde{J}_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\tilde{D}\ ,$$

$$[\tilde{J}_{\mu\nu}, \tilde{P}_\rho] = \eta_{\nu\rho}\tilde{P}_\mu - \eta_{\mu\rho}\tilde{P}_\nu\ , \quad [\tilde{J}_{\mu\nu}, \tilde{K}_\rho] = \eta_{\nu\rho}\tilde{K}_\mu - \eta_{\mu\rho}\tilde{K}_\nu\ ,$$

$$[\tilde{J}_{\mu\nu}, \tilde{J}_{\rho\sigma}] = \eta_{\nu\rho}\tilde{J}_{\mu\sigma} + \text{3-terms}\ ,$$

IW contraction

cf: [Lukierski-Stichel-Zakrzewski '0511]

$$\mu = (0, i)$$

$$\tilde{P}_0 = H\ , \quad \tilde{K}_0 = K\ , \quad \tilde{D} = D\ , \quad \tilde{J}_{ij} = J_{ij}\ , \quad \tilde{P}_i = \omega P_i\ , \quad \tilde{K}_i = \omega K_i\ , \quad \tilde{J}_{i0} = \omega G_i$$

and $\omega \to \infty$

Galilean conformal algebra

$$\{H,\ K,\ D,\ J_{ij},\ P_i,\ K_i,\ G_i\}$$

$$[D, H] = H\ , \quad [D, K] = -K\ , \quad [H, K] = -\frac{1}{2}D\ ,$$

$$[D, P_i] = P_i\ , \quad [D, K_i] = -K_i\ , \quad [H, K_i] = -\frac{1}{2}G_i\ ,$$

$$[H, G_i] = P_i\ , \quad [K, G_i] = K_i\ , \quad [K, P_i] = -\frac{1}{2}G_i\ ,$$

$$[J_{ij}, F_k] = \delta_{jk}F_i - \delta_{ik}F_j\ , \quad F_i = \{P_i, K_i, G_i\}\ ,$$

$$[J_{ij}, J_{kl}] = \delta_{jk}J_{il} + \text{3-terms}\ .$$

$\{H, K, D\}$: $\mathrm{sl}(2, R) \cong \mathrm{so}(2, 1)$

$\{J_{ij}\}$: $\mathrm{so}(d)$

GCA_{1+d}

$\downarrow \{P_i,\ K_i,\ G_i\}$

so(2,1) + so(d)

$\{H, K, D\}$ $\{J_{ij}\}$

bulk realization of GCA

[Lukierski-Stichel-Zakrzewski '0511]

$$\mathrm{conf}_{d+1} \xrightarrow{\text{contraction}} \mathrm{GCA}_{d+1}$$

bulk realization of GCA

[Lukierski-Stichel-Zakrzewski '0511]

$$\mathbf{P}_\mu = \tilde{P}_\mu + \tilde{K}_\mu$$
$$\mathbf{P}_{d+1} = \tilde{D}$$
$$\mathbf{J}_{\mu\nu} = \tilde{J}_{\mu\nu}$$
$$\mathbf{J}_{\mu d+1} = \tilde{K}_\mu - \tilde{P}_\mu$$

conf_{d+1} —contraction→ GCA_{d+1}

$\cong$

AdS_{d+2}

bulk realization of GCA

[Lukierski-Stichel-Zakrzewski '0511]

$$\mathbf{P}_\mu = \tilde{P}_\mu + \tilde{K}_\mu$$
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conf_{d+1} —contraction→ GCA_{d+1}

$\cong$ $\cong$

AdS_{d+2} —contraction→ ???

bulk realization of GCA

[Lukierski-Stichel-Zakrzewski '0511]

$$\mathrm{conf}_{d+1} \xrightarrow{\text{contraction}} \mathrm{GCA}_{d+1}$$

$$\cong \qquad\qquad\qquad \cong$$

$$\mathrm{AdS}_{d+2} \xrightarrow{\text{contraction}} ???$$

$$\mathbf{P}_\mu = \tilde{P}_\mu + \tilde{K}_\mu$$
$$\mathbf{P}_{d+1} = \tilde{D}$$
$$\mathbf{J}_{\mu\nu} = \tilde{J}_{\mu\nu}$$
$$\mathbf{J}_{\mu d+1} = \tilde{K}_\mu - \tilde{P}_\mu$$

$$\mathbf{P}_{\bar{a}} \to \mathbf{P}_{\bar{a}} \ , \quad \mathbf{J}_{ij} \to \mathbf{J}_{ij} \ , \quad \mathbf{J}_{\bar{a}\bar{b}} \to \mathbf{J}_{\bar{a}\bar{b}} \ , \quad \mathbf{P}_i \to \omega \mathbf{P}_i \ , \quad \mathbf{J}_{i\bar{a}} \to \omega \mathbf{J}_{i\bar{a}}$$

$$\bar{a} = 0, d+1$$

bulk realization of GCA

[Lukierski-Stichel-Zakrzewski '0511]

$$\mathbf{P}_\mu = \tilde{P}_\mu + \tilde{K}_\mu$$
$$\mathbf{P}_{d+1} = \tilde{D}$$
$$\mathbf{J}_{\mu\nu} = \tilde{J}_{\mu\nu}$$
$$\mathbf{J}_{\mu d+1} = \tilde{K}_\mu - \tilde{P}_\mu$$

conf_{d+1} —contraction→ GCA_{d+1}

$\cong$ $\cong$

AdS_{d+2} —contraction→ Newton-Hooke string algebra

[Gomis-Gomis-Kamimura '0507]

$$\mathbf{P}_{\bar{a}} \to \mathbf{P}_{\bar{a}} \ , \quad \mathbf{J}_{ij} \to \mathbf{J}_{ij} \ , \quad \mathbf{J}_{\bar{a}\bar{b}} \to \mathbf{J}_{\bar{a}\bar{b}} \ , \quad \mathbf{P}_i \to \omega \mathbf{P}_i \ , \quad \mathbf{J}_{i\bar{a}} \to \omega \mathbf{J}_{i\bar{a}}$$

$$\bar{a} = 0, d+1$$

GCA is realized as NH string algebra in the bulk

GCA and Newton-Hooke string algebra

AdS_{d+2}

[Gomis-Gomis-Kamimura '0507]

$$\mathbf{P}_{\bar a} \to \mathbf{P}_{\bar a}\ , \quad \mathbf{J}_{ij} \to \mathbf{J}_{ij}\ , \quad \mathbf{J}_{\bar a \bar b} \to \mathbf{J}_{\bar a \bar b}\ , \quad \mathbf{P}_i \to \omega \mathbf{P}_i\ , \quad \mathbf{J}_{i\bar a} \to \omega \mathbf{J}_{i\bar a}$$

$$\bar a = 0, d+1$$

Newton-Hooke string algebra

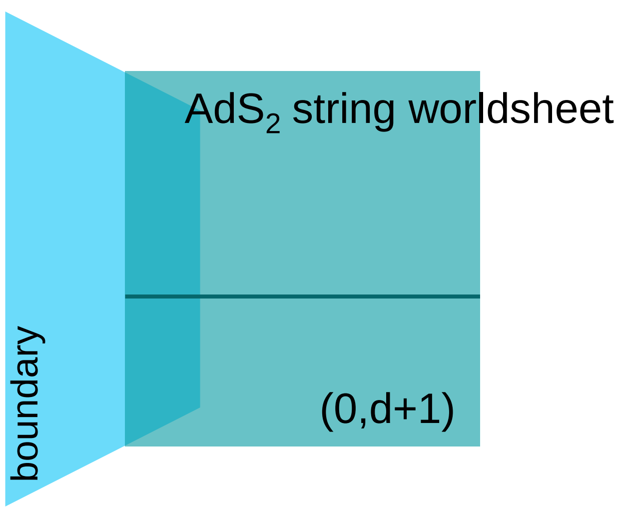
$$[\mathbf{P}_{\bar a}, \mathbf{P}_{\bar b}] = \mathbf{J}_{\bar a \bar b}\ , \quad [\mathbf{J}_{\bar a \bar b}, \mathbf{P}_{\bar c}] = \eta_{\bar b \bar c}\mathbf{P}_{\bar a} - \eta_{\bar a \bar c}\mathbf{P}_{\bar b}\ , \quad [\mathbf{J}_{\bar a \bar b}, \mathbf{J}_{\bar c \bar d}] = \eta_{\bar b \bar c}\mathbf{J}_{\bar a \bar d} + \text{3-tems}\ ,$$

AdS$_2$

$$[\mathbf{P}_{\bar a}, \mathbf{P}_i] = \mathbf{J}_{\bar a i}\ , \quad [\mathbf{J}_{i\bar a}, \mathbf{P}_{\bar b}] = \eta_{\bar a \bar b}\mathbf{P}_i\ , \quad [\mathbf{J}_{\bar a \bar b}, \mathbf{J}_{i\bar c}] = \eta_{\bar a \bar c}\mathbf{J}_{\bar b i} - \eta_{\bar b \bar c}\mathbf{J}_{\bar a i}\ , \quad [\mathbf{J}_{ij}, \mathbf{J}_{k\bar c}] = \eta_{jk}\mathbf{J}_{i\bar c} - \eta_{ik}\mathbf{J}_{j\bar c}$$

$$[\mathbf{J}_{ij}, \mathbf{P}_k] = \eta_{jk}\mathbf{P}_i - \eta_{ik}\mathbf{P}_j\ , \quad [\mathbf{J}_{ij}, \mathbf{J}_{kl}] = \eta_{jk}\mathbf{J}_{il} + \text{3-terms}\ .$$

so(d)



close-up of AdS$_2$ string worldsheet

NR string
[GGK'0507, MS-Yoshida'0703,'0709,'0712]

32 susy NH algebras
[GGK '0507, MS-Yoshida '0605]

GCA$_{1+d}$	NH string algebra
↓ $\{P_i,\ K_i,\ G_i\}$	↓ $\{\mathbf{P}_i,\ \mathbf{J}_{\bar{a}i}\}$
so(2,1) + so(d)	so(2,1) + so(d)
$\{H, K, D\}$ $\{J_{ij}\}$	$\{\mathbf{P}_{\bar{a}},\ \mathbf{J}_{\bar{a}\bar{b}}\}$ $\{\mathbf{J}_{ij}\}$

Close-up of AdS brane in AdS_{d+1}

NH brane algebra $\underset{\sim}{=}$ semi-GCA

[Brugues-Gomis-Kamimura'0603] [Alishahiha-Davody-Vahedi'0903]

semi-GCA is realized as NH brane algebra in the bulk

supersymmetric GCA_{1+3} from psu(2,2|4)

psu(2,2|4) —contraction→ super GCA_{1+3} [MS, de Azcarraga-Lukierski '0905]

∪

- AdS_5 x S^5 — $so(6)$: $\{P_{a'}, J_{a'b'}\}$ $\quad a' = 5, \cdots, 9$
- a pair of 16 component Majorana-Weyl spinors $\mathcal{Q} = \begin{pmatrix} \mathcal{Q}_1 \\ \mathcal{Q}_2 \end{pmatrix}$

We are interested in

dynamical susy $\{Q_+, Q_+\} \sim H + \cdots$

⬇

susy on AdS_2

$$\tilde{Q}_\pm = \mathcal{Q}\ell_\pm$$

$$\ell_\pm = \frac{1}{2}(1 \pm \Gamma^{04}\sigma)$$

$$\sigma = \sigma_1, \sigma_3$$

1/2 BPS brane configurations in AdS_5 x S^5 [MS-Yoshida '0605]

ρ	p=1	p=3	p=5	p=7
σ_1, σ_3	(2,0)		(2,4), (4,2)	
$i\sigma_2$		(3,1), (1,3)		(5,3), (3,5)
1	(1,1)		(1,5), (3,3), (5,1)	

$$\tilde{Q}_\pm = \ell_\pm \mathcal{Q} \qquad \ell_\pm = \frac{1}{2}(1 \pm \Gamma^{\bar{a}_0 \cdots \bar{a}_p}\rho)$$

IW contraction

$$\tilde{Q}_+ = Q_+ \ , \quad \tilde{Q}_- = \omega Q_- \ , \quad \tilde{P}_{a'} = \omega P_{a'} \ , \quad \tilde{J}_{a'b'} = J_{a'b'}$$ and $\omega \to \infty$

32 susy GCA_{1+3}

dynamical susy

$$\{\underbrace{H,\ K,\ D,\ P_i,\ K_i,\ G_i,\ J_{ij},}_{\text{GCA}}\ \underbrace{P_{a'},\ J_{a'b'},}_{\text{iso(5)}}\ Q_+,\ Q_-\}$$

$$[J_{a'b'}, J_{c'd'}] = \delta_{b'c'} J_{a'd'} + \text{3-terms} \ , \quad [J_{a'b'}, P_{c'}] = \delta_{b'c'} P_{a'} - \delta_{a'c'} P_{b'}$$

$$[H, Q_\pm] \sim Q_\pm p_+ \ , \quad [K, Q_\pm] \sim Q_\pm p_- \ , \quad [D, Q_\pm] \sim Q_\pm \ , \quad [J_{ij}, Q_\pm] \sim Q_\pm$$

$$[P_i, Q_+] \sim Q_- p_+ \ , \quad [K_i, Q_+] \sim Q_- p_- \ , \quad [G_i, Q_+] \sim Q_- \ ,$$

$$[J_{a'b'}, Q_\pm] \sim Q_\pm \ , \quad [P_{a'}, Q_+] \sim Q_- \ ,$$

$$\{Q_+^T, Q_+\} \sim h_+ \ell_+ \left(p_- H + p_+ K + D + J_{ij} + J_{a'b'}\right) \ , \qquad p_\pm = \frac{1}{2}(1 \pm \Gamma^{0123} i\sigma_2)$$

$$\{Q_+^T, Q_-\} \sim h_+ \ell_- \left(p_- P_i + p_+ K_i + G_i + P_{a'}\right) \ ,$$

1/2 susy subalgebra $\{\underbrace{H,\ K,\ D,}_{\text{so(2,1)}}\ \underbrace{J_{ij},}_{\text{so(3)}}\ \underbrace{J_{a'b'},}_{\text{so(5)}}\ Q_+\}$

32 susy GCA_{1+3}

$$\{P_i,\ K_i,\ G_i,\ P_{a'},\ Q_-\}$$

so(2,1) + so(3) + so(5) + 16 susy

$$\{H, K, D\} \quad \{J_{ij}\} \quad \{J_{a'b'}\} \quad \{Q_+\}$$

16 susy GCA_{1+3} from su(2,|2)

$$\mathcal{Q} \equiv \mathcal{Q} q_+ \qquad q_+ = \frac{1}{2}(1+\Gamma^{5678})$$

su(2,2|2) ← psu(2,2|4)

IW contraction

16 susy GCA_{1+3}

[MS, de Azcarraga-Lukierski ‘0905]

$$\{\underbrace{H,\ K,\ D,\ J_{ij},\ P_i,\ K_i,\ G_i}_{\text{GCA}},\ \underbrace{P_9,\ J_I^{(+)}}_{\text{u(1) x su(2)}},\ Q_+,\ Q_-\}$$

super GCA_{1+2}

[MS '0905]

M2 (AdS_2 x S^1) in AdS_4 x S^7

32 susy GCA from osp(8|4)

$$\{\underbrace{H,\ K,\ D,\ P_i,\ K_i,\ G_i,\ J_{ij},}_{\text{GCA}}\ \underbrace{P_4,\ P_m,\ J_{mn},\ J_{4m},}_{\text{close-up of } S^1 \text{ in } S^7}\ Q_+,\ Q_-\}$$

16 susy GCA from osp(4|4)

$$\{\underbrace{H,\ K,\ D,\ J_{ij},\ P_i,\ K_i,\ G_i,}_{\text{GCA}}\ \underbrace{J_I^{(+)},\ J_I'^{(+)}}_{\text{so(3) x iso(2)}},\ Q_+,\ Q_-\}$$

$$q_+ = \frac{1}{2}(1+\Gamma^{789\natural})$$

8 susy GCA from osp(2|4)

$$\{\underbrace{H,\ K,\ D,\ J_{ij},\ P_i,\ K_i,\ G_i,}_{\text{GCA}}\ \underbrace{R,}_{\text{u(1)}}\ Q_+,\ Q_-\}$$

$$q_+ = \frac{1}{2}(1+\Gamma^{5678})\frac{1}{2}(1+\Gamma^{789\natural})$$

super semi-GCA$_{1+5}$

[MS '0905]

32 susy semi-GCA from osp(8*|4)

M2 (AdS$_3$) in AdS$_7$ x S^4

$$\{\underbrace{P_\alpha,\ K_\alpha,\ D,\ P_i,\ K_i,\ G_{i\alpha},\ J_{ij},\ J_{05},}_{\text{semi-GCA}}\ \underbrace{P_{a'},\ J_{a'b'},}_{\text{iso(4)}}\ Q_+,\ Q_-\}$$

1/2 susy subalgebra

$$\{\underbrace{P_\alpha,\ K_\alpha,\ D,\ J_{05},}_{\text{so(2,2)}}\ \underbrace{J_{ij},\ J_{a'b'},}_{\text{so(4) + so(4)}}\ Q_+\}$$

AdS$_3$

16 susy semi-GCA from osp(8*|2)

$$\{\underbrace{P_\alpha,\ K_\alpha,\ D,\ P_i,\ K_i,\ G_{i\alpha},\ J_{ij},\ J_{05},}_{\text{semi-GCA}}\ \underbrace{J_I^{(+)}}_{\text{su(2)}},\ Q_+,\ Q_-\}$$

Summary

✓ GCA is a boundary realization of NH string algebra

✓ semi-GCA is a boundary realization of NH brane algebra

✓ super GCA_{1+3} from SCA_4, psu(2,2|4) and su(2,2|2)
string(AdS_2) in AdS_5

✓ super GCA_{1+2} from SCA_3, osp(8|4), osp(4|4) and osp(2|4)
M2(AdS_2 x S^1) in AdS_4 x S^7

✓ super semi-GCA_{1+5} from SCA_6, osp(8*|4) and osp(8*|2)
M2(AdS_3) in AdS_7

- super GCAs for other 1/2 BPS branes

- NRCFT with GCA and NR string
  Wilson line in SYM_4

- infinitely extended super GCA

and so on

Thank you very much.