ゲージ-ヒッグス統一模型における異常磁気モーメント•電気双極子モーメントの性質

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References

- Y.A., N. Maru, C.S. Lim, Phys. Rev. D 79, 075018 (2009),
- Y.A., N. Maru, C.S. Lim, arXiv:0904.1695 [hep-ph],
- Y.A., N. Maru, C.S. Lim, arXiv:0905.1022 [hep-ph].

1 Introduction

- Gauge-Higgs Unification
- Anomalous Magnetic Moment ($g-2$) and Electric dipole moment

Gauge-Higgs unification

Higgs $\cdots$ extra component of higher dim'l gauge fields

$$A_M = \begin{pmatrix} A_\mu \\ A_y \\ \vdots \end{pmatrix} \begin{matrix} \longleftarrow \text{4D gauge fields} \\ \left.\right\} \leftarrow \text{Standard Model Higgs} \end{matrix}$$

operator $(A_y)^2$ is forbidden by local gauge symmetry.

Higgs(A_y) mass is protected from UV divergence

$\Rightarrow$ Higgs mass is a predictable observation in the gauge-Higgs unification

Any other calculable quantity $\Rightarrow$ fermion $g-2$, EDM

Anomalous magnetic moment and Electric dipole moment

interactions between spin $\vec{\sigma}$ and electromagnetic fields $\vec{E}, \vec{B}$

$$\mu\vec{\sigma}\cdot B \qquad d\vec{\sigma}\cdot E$$

μ : magnetic moment d : electric dipole moment

- Anomalous magnetic moment $(g-2)$

  quantum corrections of magnetic moment

  electroweak precision measurement (muon $g-2$)

$$a_\mu(\mathrm{EXP}) = 11659208.0(6.3)\times 10^{-10}$$

$$a_\mu(\mathrm{SM}) = 11659179.0(6.5)\times 10^{-10}$$

- **Electric dipole moment (EDM)**

  $\mathcal{P}, C\mathcal{P}$ **odd quantity (neutron EDM):**

$$d_{\mathrm{N}}(\mathrm{EXP}) < 2.9 \times 10^{-26}[e \cdot \mathrm{cm}](\mathrm{CL} = 90\%)$$

$$d_{\mathrm{N}}(\mathrm{SM}) = 10^{-31} \sim 10^{-34}[e \cdot \mathrm{cm}]$$

$\Rightarrow$ **Both observations are studied precisely.**

Issues

- **UV divergence of fermion $g-2$ (EDM) in the G-H U.**
- $\mathcal{P}, C\mathcal{P}$ **violation in the $5D$ G-H U.**
- **Constraints of compactification scale**

2 UV divergence of fermion $g-2$ (EDM) in the gauge-Higgs unification

- $SU(3)$ gauge-Higgs unification model
- Operator Analysis
- cancellation of divergences
- constraint on compactification scale

The Model

- (D+1)-dim $SU(3)$ gauge theory on S^1/Z_2 with Z_2 odd bulk mass term

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}F_{MN}F^{MN} + \bar{\Psi}\left[i\not{D} - M\epsilon(y)\right]\Psi$$

$$F_{MN} = \partial_M A_N - \partial_N A_M - ig_5[A_M, A_N], \not{D} = (\partial_L - ig_5 A_L)\Gamma^L, \Psi = (\Psi_1, \Psi_2, \Psi_3)^T$$

- Boundary Conditions

$$A_\mu = \begin{pmatrix} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ (-,-) & (-,-) & (+,+) \end{pmatrix}, A_y = \begin{pmatrix} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ (+,+) & (+,+) & (-,-) \end{pmatrix}, \psi = \begin{pmatrix} \psi_{1L}(+,+) + \psi_{1R}(-,-) \\ \psi_{2L}(+,+) + \psi_{2R}(-,-) \\ \psi_{3L}(-,-) + \psi_{3R}(+,+) \end{pmatrix}$$

(+,+) has mass less mode.

- mass less mode

$SU(3)$ breaks into $SU(2) \times U(1)$ and chiral fermion appear.

$$A_\mu = \begin{pmatrix} \gamma, Z & W^\pm & 0 \\ W^\pm & \gamma, Z & 0 \\ 0 & 0 & \gamma, Z \end{pmatrix}_\mu, A_y = \begin{pmatrix} 0 & 0 & \phi^\pm \\ 0 & 0 & h + i\phi^0 \\ \phi^\pm & h - i\phi^0 & 0 \end{pmatrix}_y, \psi = \begin{pmatrix} \begin{pmatrix} \nu \\ \mu \end{pmatrix}_L \\ \mu_R \end{pmatrix}$$

Operator analysis

SM		gauge-Higgs
$\langle H\rangle\bar{\psi}_L\sigma^{\mu\nu}\psi_R F_{\mu\nu}$	$\subset$	$\bar{\psi}_L\sigma^{MN}\underbrace{\langle D_L\rangle\Gamma^L\psi_R}_{=M\epsilon(y)}F_{MN}$
	$=$	$\bar{\psi}_L\sigma^{MN}M\epsilon(y)\psi_R F_{MN}$

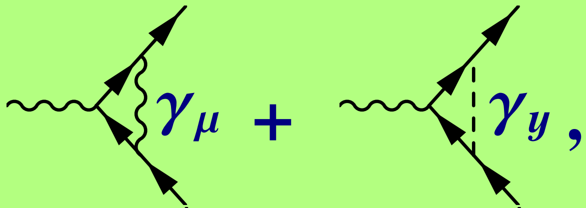
$M\epsilon(y)\bar{\psi}_L\sigma^{MN}\psi_R$ is ***not*** $SU(2)_L$ gauge invariant operator

$\Rightarrow$ **$g-2$ (EDM) is insensitive to UV divergence**.

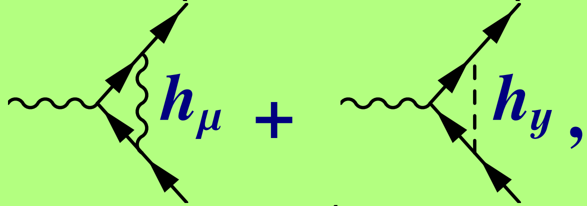
Cancellation of divergences

Y.A., C.S. Lim, N. Maru, arXiv:0904.1695 [hep-ph]

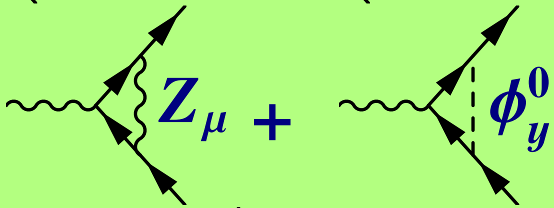
Photon sector $a(\gamma) =$



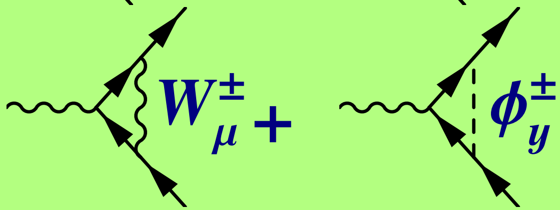
Higgs sector $a(h) =$



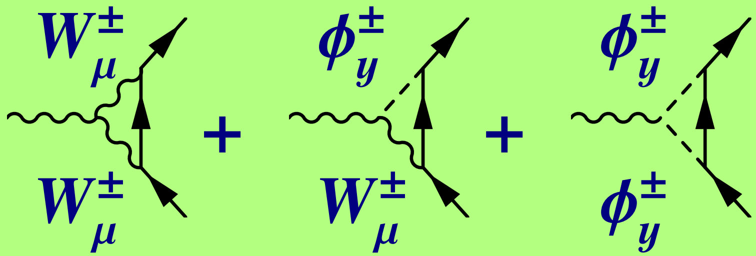
Z boson sector $a(Z+\phi^0) =$



W boson sector $a(W^\pm_\mu+\phi^\pm) =$

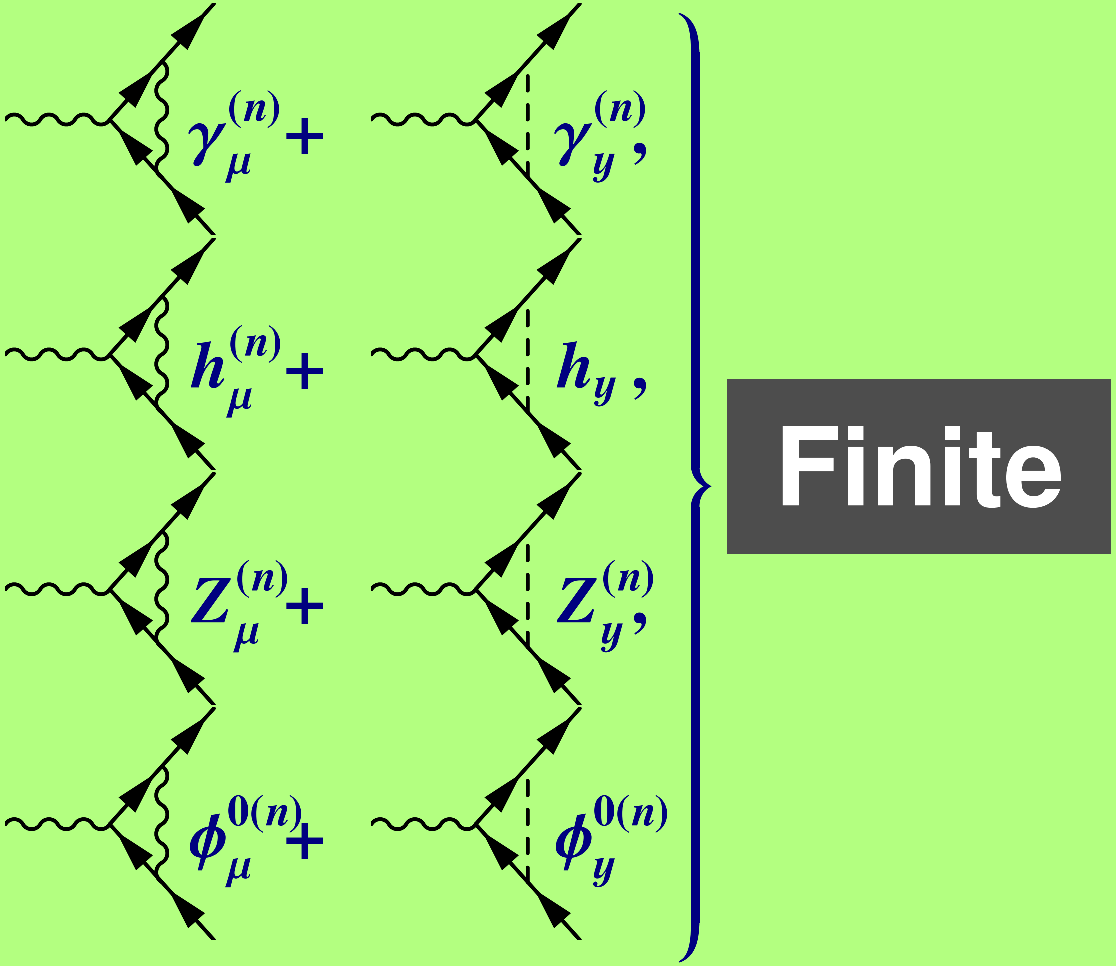


Self interaction $a_{SI}(W^\pm_\mu+\phi^\pm) =$

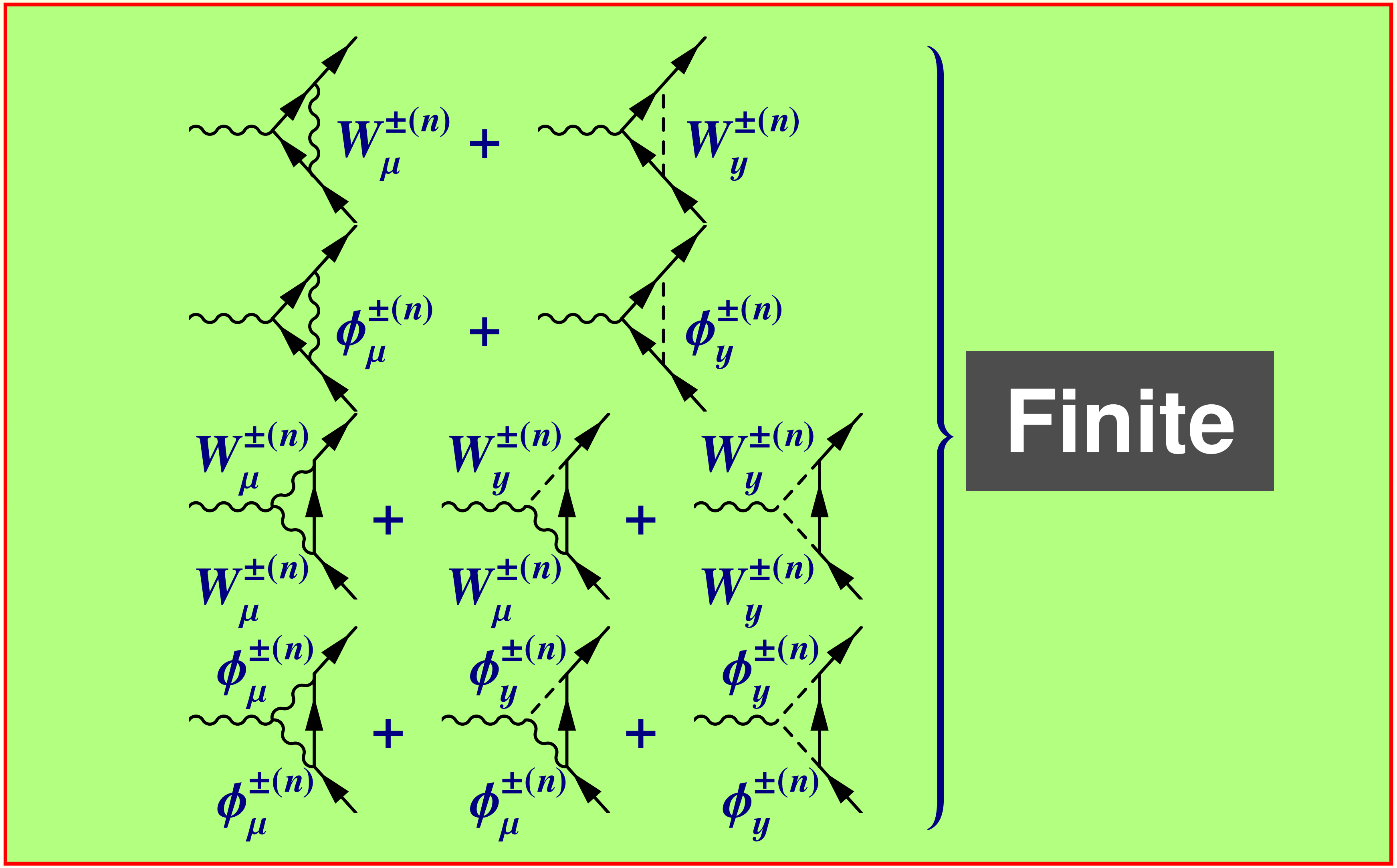


KK mode

•Neutral current



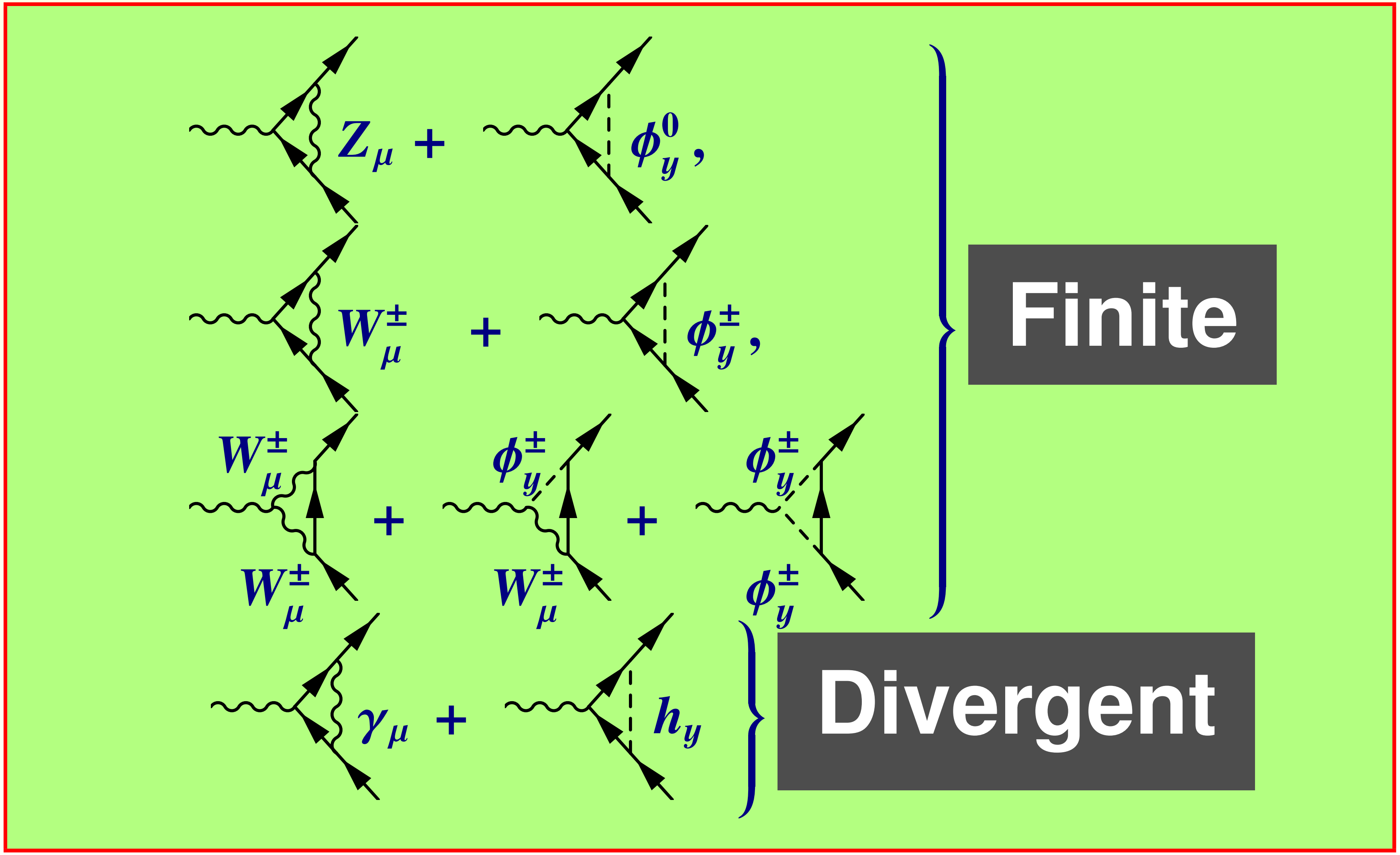
•Charged current



Divergences are completely cancelled between

KK gauge boson and its N-G boson.

Zero mode



divergences are cancelled between gauge boson and its N-G boson,

except for Higgs and photon.

$$\begin{cases} \text{KK mode : KK gauge boson } A_\mu^{(n)} \Longleftrightarrow \text{N-G boson } A_y^{(n)} \\ \text{zero mode : gauge boson } W_\mu^\pm, Z \Longleftrightarrow \text{N-G boson } \phi^\pm, \phi^0 \end{cases}$$

$\Uparrow$

Higgs (-like) mechanism is crucial

Divergence from zero mode photon and higgs remain.

Divergences are weaken in the G-H U., $g-2$ is finite in the $D = 5, 6$.

Contributions of KK mode ($M^4 \times S^1/Z_2$)

Contribution from KK mode are calculated as follows;

$$a(\mathrm{KK}) = 1.4 \times 10^{-5} (Rm_W)^2$$

Combining into muon $g-2$ experimental result:

$$a(\mathrm{KK}) < a(\mathrm{EXP}) - a(\mathrm{SM}) = (2.90 \pm 0.90) \times 10^{-9}$$

$$\Rightarrow \frac{1}{R} > (61 \sim 84) m_W = 4.9 \sim 6.7 \mathrm{TeV}$$

3 $\mathcal{P}, \mathcal{CP}$ violation in the 5D G-H U

- $\mathcal{P}, \mathcal{CP}$ Violation in the 5D G-H U.
- EDM
- Constraint on compactification scale

Y.A., N. Maru, C.S. Lim, arXiv:0905.1022 [hep-ph]

$\mathcal{P}, \mathcal{CP}$ Violation in the 5D G-H U

- $\mathcal{CP}$ violation

$$A_M = \begin{pmatrix} A_\mu \\ A_y \end{pmatrix} \begin{matrix} \leftarrow \text{ 4D gauge fields} \\ \leftarrow \ \mathcal{CP} \text{ odd scalar } \Longleftrightarrow \text{ VEV} \end{matrix}$$

$\Downarrow$

naive guess

A_y VEV breaks $\mathcal{CP}$ symmetry

Also fermion bulk mass term is significant for $\mathcal{CP}$ violation.

- **fermion quadratic term**

$$\bar{\psi}\Big[\underbrace{i\,\partial\!\!\!/}_{\mathcal{CP}\text{even}} - \underbrace{M\epsilon(y)}_{\mathcal{CP}\text{even}} + g_5 \underbrace{\langle A_y \rangle}_{\mathcal{CP}\text{odd}} \gamma_5\Big]\psi$$

1. $\langle A_y \rangle = 0$ case

   There are no $\mathcal{CP}$ odd "source" $\Rightarrow$ $\mathcal{CP}$ conserved

2. $M = 0$ case

   chiral rotation $\psi \to e^{i\frac{\pi}{4}\gamma_5}\psi \Rightarrow A_y : \mathcal{CP}$ even $\Rightarrow$ $\mathcal{CP}$ conserved

$$\Downarrow$$

Both M and A_y VEV is necessary to break $\mathcal{CP}$ symmetry.

•$\mathcal{P}$ violation

orbifold condition ⇒ 4D chiral fermion ⇔ $\mathcal{P}$ violation

⇓

orbifold condition breaks $\mathcal{P}$ violation

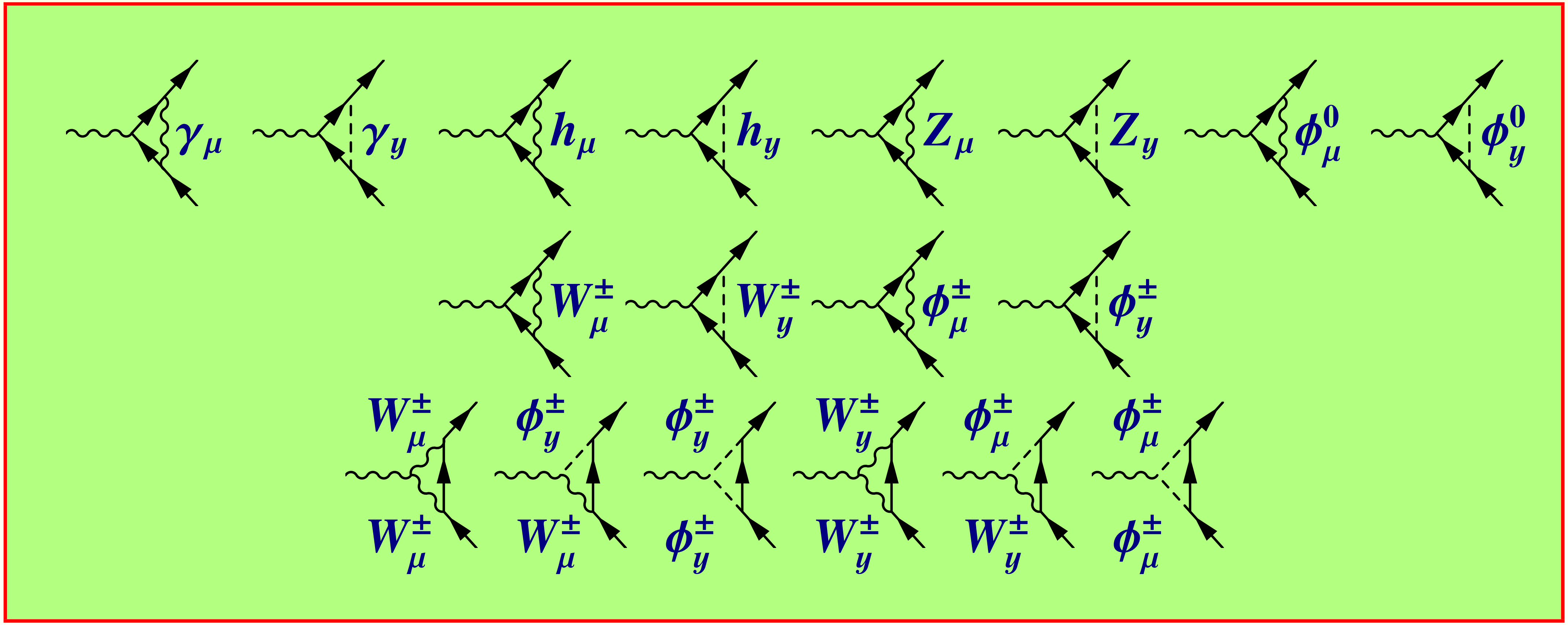
$$\begin{cases} \mathcal{P}\ \text{transformation} & : \psi \to \gamma^0 \psi \\ \text{orbifold condition} & : \psi \to \gamma^5 P \psi \end{cases}$$

diag(P)=(1,1,-1)

⇒ inconsistent with each other ($\gamma^0\gamma^5 = -\gamma^5\gamma^0$)

Electric Dipole Moment

EDM $\cdots$ most striking result of $\mathcal{P},\mathcal{CP}$ violation



Shabalin, Sov. J. Nucl. Phys. 28 (1978) 75

Zero mode (SM particle) has no contributions on EDM,

KK particle has nonzero contribution up to order 1-loop.

Contributions of KK particle

The contributions of KK particles $d_{\rm N}(\rm KK)$ are obtained,

$$d_{\rm N}(\rm KK) = d_{\rm N}(\gamma^{(n)}) + d_{\rm N}(Z^{(n)}) + \cdots$$
$$= -\frac{2}{9} e^3 \frac{(MR)^3}{\pi^3} R^2 m_W \cdot 2.1 \times 10^{-5}$$

Both nonzero M and m_W is significant

$\Uparrow$

$\mathcal{CP}$ violation recovers if we take $M = 0$ or $m_W = 0$

Constraint on compactification scale R

Assuming that neutron EDM $d_{\rm N}(\rm KK)$ is less than the differences between experimental results and theoretical expectation:

$$d_{\rm N}(\rm KK) = -\frac{2}{9}e^3\frac{(MR)^3}{\pi^3}R^2 m_W \cdot 2.1 \times 10^{-5}$$
$$= -2.3 \times 10^{-23}(Rm_W)^2[e \cdot \rm cm] < d_{\rm N}(\rm EXP) - d_{\rm N}(\rm SM)$$

$$\Rightarrow \boxed{\frac{1}{R} > 33 m_W \simeq 2.6\ \text{TeV}}$$

4 Summary

$g-2$

- Divergence from nonzero KK modes are completely cancelled between KK gauge bosons and its N-G bosons
- Divergence from zero mode are cancelled between between gauge bosons and its N-G bosons, except for Higgs and photon

⇒ Higgs(-like) mechanism play crucial role

- In the case of $M^4 \times S^1/Z_2$, constraint of compactification scale was obtained: $M_{\mathrm{KK}} = \frac{1}{R} > 4.9 \sim 6.7$ TeV.

Electric Dipole Moment

- $\mathcal{CP}$ symmetry is broken in the 5D G-H U. by **nonzero bulk mass and A_y VEV**.
- Fermion EDM appears in up to 1-loop.
- Comparing Neutron EDM experimental result, we obtain the constraint of compactification scale: $M_{\mathrm{KK}} = \frac{1}{R} > 2.6$TeV.

Backup

Gordon decomposition

Fermion $g-2(a_\psi)$ and EDM (d_ψ) can be rewritten as

$$\gamma^\mu \to Z\gamma^\mu + a_\psi \frac{p^\mu + p'^\mu}{2m} + \frac{d_\psi}{e}(p^\mu + p'^\mu)\gamma_5$$

$\mathcal{CP}$ transformation

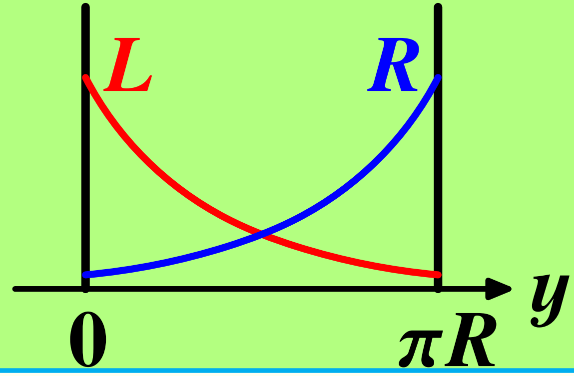
$$\begin{cases} x^\mu \to x_\mu, y \to y \\ \psi(x^\mu, y) \to i\gamma^0\gamma^2\psi^*(x_\mu, y) \\ A^\mu(x^\mu, y) \to -A_\mu^{\mathrm{T}}(x_\mu, y), A_y(x^\mu, y) \to -A_y^{\mathrm{T}}(x_\mu, y) \end{cases}$$

$\mathcal{C}, \mathcal{CP}$ odd EDM

$$\begin{cases} \mathcal{CP}(\bar{\psi}\sigma^{\mu\nu}\gamma_5\psi F_{\mu\nu}) = \bar{\psi}\sigma_{\mu\nu}\gamma_5\psi(-F^{\mu\nu}) \\ \mathcal{P}(\bar{\psi}\sigma^{\mu\nu}\gamma_5\psi F_{\mu\nu}) = -\bar{\psi}\sigma_{\mu\nu}\gamma_5\psi F^{\mu\nu} \end{cases}$$

Yukawa coupling

$$\begin{cases} f_L^{(0)}(y) \propto e^{-My} \\ f_R^{(0)}(y) \propto e^{My} \end{cases}$$



Yukawa coupling is obtained by an overlap integral of zero mode wave functions.

$$Y = g \int_{-\pi R}^{\pi R} dy\, f_L^{(0)} f_R^{(0)} = \frac{2\pi RM}{\sqrt{(1 - e^{-2\pi RM})(e^{2\pi RM} - 1)}} g \sim 2\pi RM e^{-\pi RM} g$$

$$\begin{cases} \text{Muon Yukawa coupling } Y_\mu = Y|_{\pi RM \sim 10} \sim 10^{-4} \\ d \text{ quark Yukawa coupling } Y_d = Y|_{\pi RM \sim 12.5} \sim 10^{-5} \end{cases}$$