Monte Carlo studies of the six-dimensional IKKT model

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Development of Quantum Field Theory and String Theory,

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with T. Aoyama, M. Hanada and J. Nishimura

Contents

1 Introduction

Matrix models as a constructive definition of superstring theory

IKKT model (IIB matrix model)

⇒ Promising candidate for the constructive definition of superstring theory.
Ishibashi, Kawai, Kitazawa and Tsuchiya, hep-th/9612115.

$$S = N\left(-\frac{1}{4}\mathrm{tr}\,[A_\mu, A_\nu]^2 + \frac{1}{2}\mathrm{tr}\,\bar{\psi}_\alpha(\Gamma_\mu)_{\alpha\beta}[A_\mu, \psi_\beta]\right).$$

- **A_μ (10d vector) and ψ_α (10d Majorana-Weyl spinor) ⇒ $N \times N$ matrices .**

- **Evidences for spontaneous breakdown of SO(10) symmetry to SO(4).**
  Nishimura and Sugino, hep-th/0111102, **Kawai, et. al.** hep-th/0204240,0211272,0602044,0603146.

- **Complex fermion determinant:**

  * **Crucial for rotational symmetry breaking.**
    Nishimura and Vernizzi, hep-th/0003223.

  * **Difficulty of Monte Carlo simulation.**

2 6d IKKT model

T .Aoyama, T. Azuma, M. Hanada and J. Nishimura

$$\boxed{S = \underbrace{-\frac{N}{4}\mathrm{tr}\,[A_\mu, A_\nu]^2}_{=S_B} + \underbrace{\frac{N}{2}\mathrm{tr}\,\bar{\psi}\Gamma_\mu[A_\mu, \psi]}_{=S_F}.}$$

- A_μ (6d vector) and ψ (6d Weyl spinor) are $N \times N$ matrices .

  $\Gamma_1 = i\sigma_1 \otimes \sigma_2,\ \Gamma_2 = i\sigma_2 \otimes \sigma_2,\ \Gamma_3 = i\sigma_3 \otimes \sigma_2,\ \Gamma_4 = i1 \otimes \sigma_1,\ \Gamma_5 = i1 \otimes \sigma_3,\ \Gamma_6 = 1 \otimes 1.$

- SO(6) rotational symmetry and SU(N) gauge symmetry.
- Presence of $\mathcal{N} = 2$ supersymmetry.
- $Z = \int dAe^{-S_B}(\det \mathcal{M}) = \int dAe^{-S_0}e^{i\Gamma}$. CPU cost is $O(N^6)$.

  4d $\rightarrow$ $\det \mathcal{M}$ is real positive

  6d and 10d $\rightarrow$ $\det \mathcal{M}$ is complex.

  Complex phase is important in SO(6) breakdown.

- **Previous works on this model:**

  * **Simulation of phase-quenched 6d and 10d IKKT model**

    ⇒ no symmetry breakdown of SO(6) (and SO(10)).

    J. Ambjorn, K. N. Anagnostopoulos, W. Bietenholz, T. Hotta and J. Nishimura, hep-th/0005147

  * **Simulation of one-loop effective action (CPU cost is O(N^3)).**

    K.N. Anagnostopoulos and J. Nishimura, hep-th/0108041.

  * **Gaussian expansion method ⇒ symmetry breakdown of SO(6) to SO(3).**

    T. Aoyama, J. Nishimura and T. Okubo

3 Monte Carlo simulation

Factorization method

An approach to the complex action problem in Monte Carlo simulation.
Anagnostopoulos and Nishimura, hep-th/0108041,

Partition function:

$$Z = \int dA e^{-S_B} (\det \mathcal{M})^{N_f} = \int dA e^{-S_0} e^{i\Gamma},$$
$$Z_0 = \int dA e^{-S_0} = \int dA e^{-S_B} |\det \mathcal{M}|^{N_f}.$$

Distribution function

$$\rho_i(x) \stackrel{\text{def}}{=} \langle \delta(x - \tilde{\lambda}_i) \rangle = \frac{1}{C} \rho_i^{(0)}(x) w_i(x), \text{ where}$$

$$\tilde{\lambda}_i = \lambda_i / \langle \lambda_i \rangle_0, \quad C = \langle \cos \Gamma \rangle_0,$$
$$\rho_i^{(0)}(x) = \langle \delta(x - \tilde{\lambda}_i) \rangle_0, \quad w_i(x) = \langle \cos \Gamma \rangle_{i,x},$$
$$\langle * \rangle_0 = [\text{V.E.V. for the phase-quenched partition function } Z_0]$$
$$\langle * \rangle_{i,x} = [\text{V.E.V. for the partition function } Z_{i,x} = \int dA e^{-S_0} \delta(x - \tilde{\lambda}_i)].$$

The position of the peak x_p for the distribution function $\rho_{i,V}(x)$:

$$0 = \frac{\partial}{\partial x} \log \rho_{i,V}(x) = f_i^{(0)}(x) - \langle \lambda_i \rangle_0 V'(\langle \lambda_i \rangle_0 x), \text{ where } f_i^{(0)}(x) \stackrel{\text{def}}{=} \frac{\partial}{\partial x} \log \rho_i^{(0)}(x).$$

Monte Carlo evaluation of $\langle\tilde{\lambda}_i\rangle$

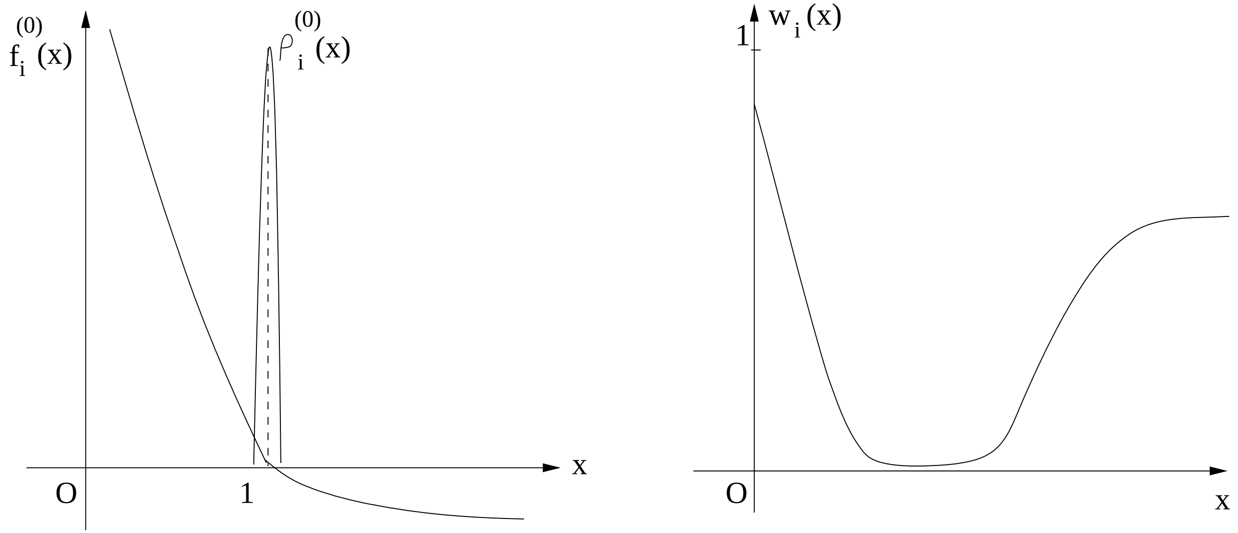
$w_i(x) > 0 \Rightarrow \langle\tilde{\lambda}_i\rangle$ **is the minimum of** $\mathcal{F}_i(x)$**:**

$$\mathcal{F}_i(x) = (\text{free energy density}) = -\frac{1}{N^2}\log\rho_i(x).$$

We solve $\mathcal{F}_i'(x) = 0$**, namely** $\frac{1}{N^2}f_i^{(0)}(x) = -\frac{d}{dx}\left\{\frac{1}{N^2}\log w_i(x)\right\}$.

Do both $\frac{1}{N^2}\log w_i(x)$ **and** $\frac{1}{N^2}f_i^{(0)}(x)$ **scale at large** N **as**

$$\frac{1}{N^2}\log w_i(x) \to \Phi_i(x), \quad \frac{1}{N^2}f_i^{(0)}(x) \to F_i(x)?$$



Behavior of $\Phi_i(x)$

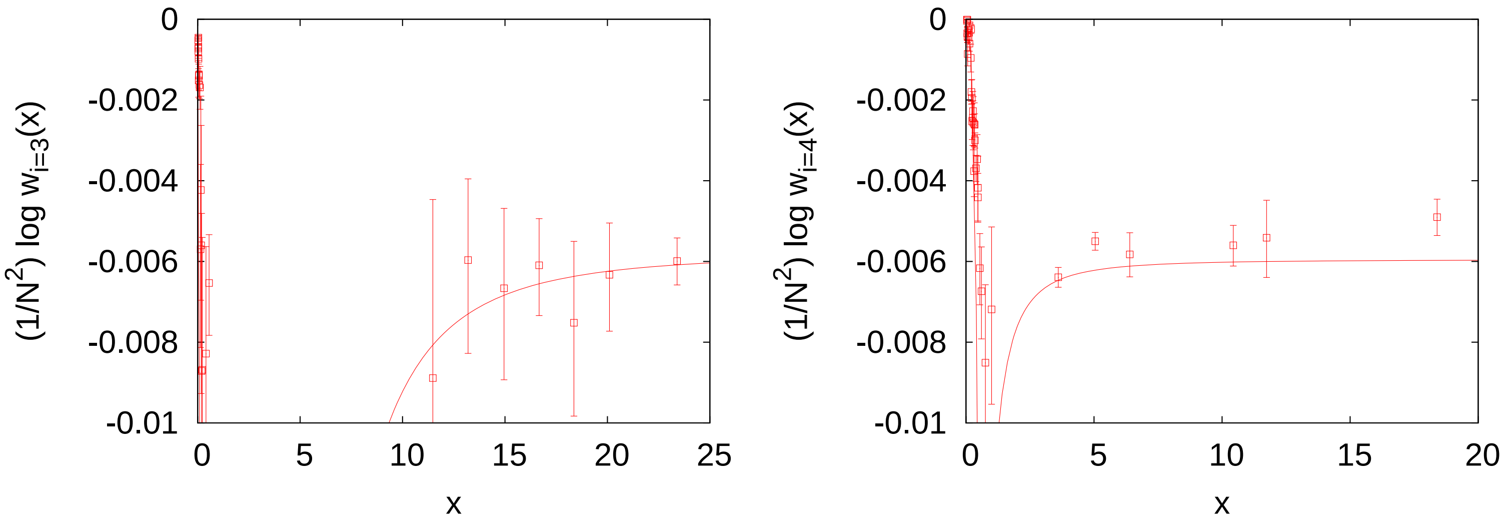
Asymptotic behavior of $\Phi_i(x) = \frac{1}{N^2}\log w_i(x)$ at $x \ll 1$ and $x \gg 1$.

When we fix the i-th largest eigenvalue $\rightarrow$

- $x \ll 1$ $(i = 2, \cdots, 6)$: $(7 - i)$ **directions are shrunk** $\Rightarrow$ $(i - 1)$**-dim. configuration**
- $x \gg 1$ $(i = 1, \cdots, 5)$: $(6 - i)$ **directions are shrunk** $\Rightarrow$ i**-dim. configuration**

Expected power behaviors:

$$\Phi_i(x) \propto \begin{cases} c_{i,0}x^{7-i} + \cdots & (x \ll 1, i = 2, 3, 4) \\ \frac{d_{i,0}}{x^{6-i}} + \cdots & (x \gg 1, i = 1, 2, 3) \end{cases}$$



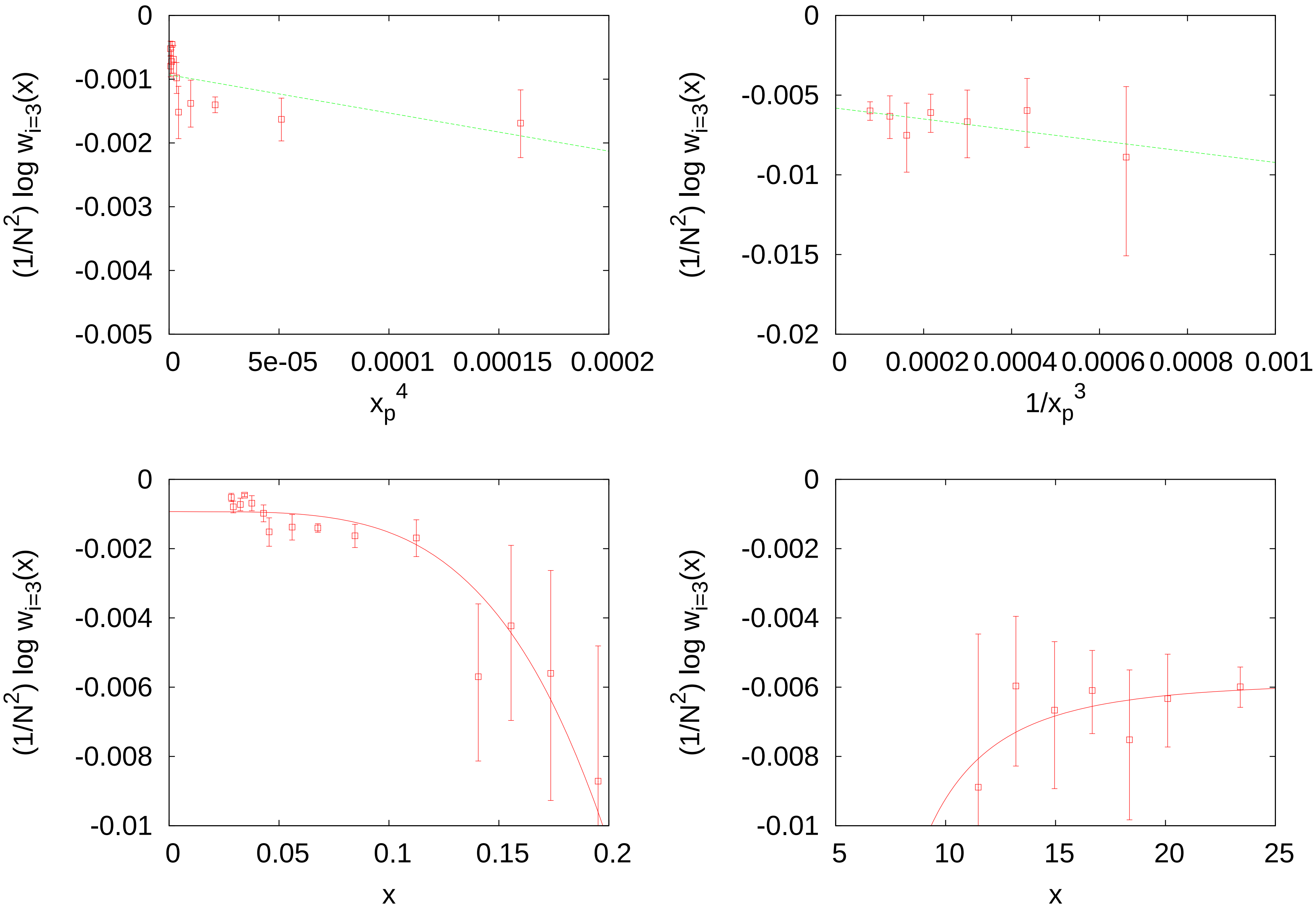


Figure 1: $\frac{1}{N} \log w_{i=3}(x)$ for $N = 8$

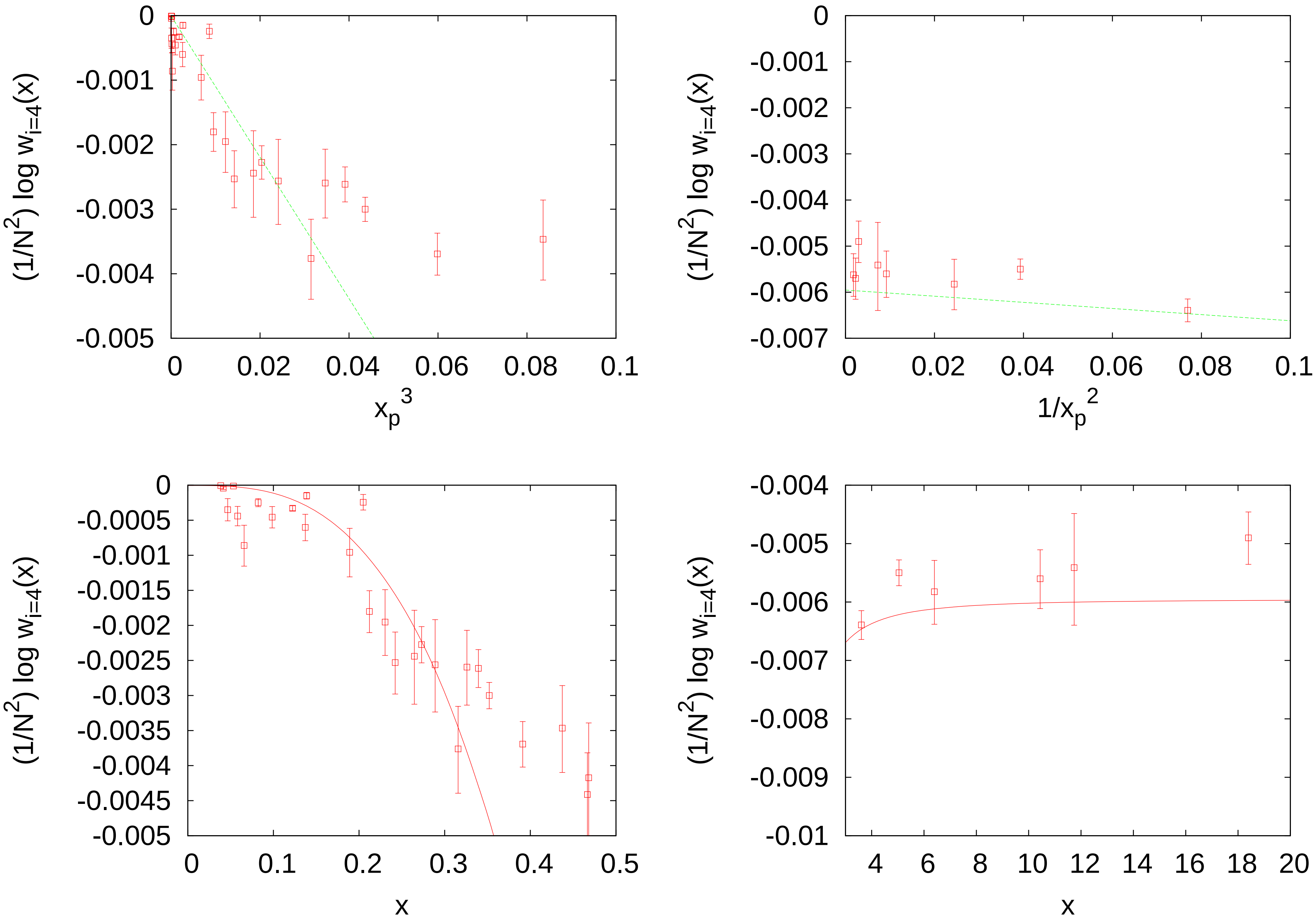


Figure 2: $\frac{1}{N} \log w_{i=4}(x)$ for $N = 8$

Behavior of $\frac{1}{N^2} f_i^{(0)}(x)$

Leading behavior at small x ($x \ll 1$) $\rightarrow$ $(7-i)$ directions are shrunk.

- $i = 2, \cdots, 6$: $\rho_i^{(0)}(x) \simeq (\sqrt{x})^{N^2(7-i)} \Rightarrow \frac{1}{N^2} f_i^{(0)}(x) = \frac{7-i}{2x}$

- $i = 1$: **Eigenvalues of A_μ are collapsed to zero.**

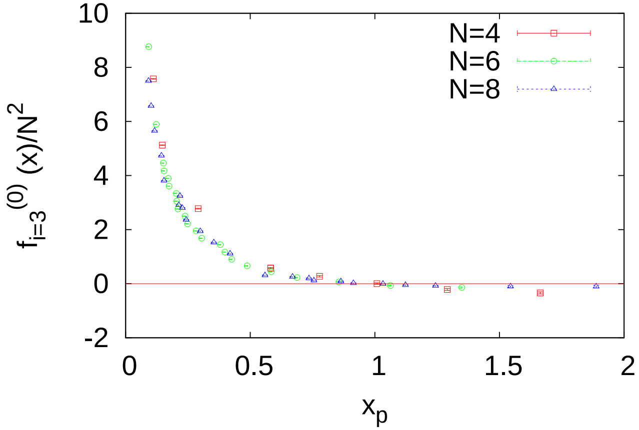
  $\Rightarrow$ **Add the effect of fermionic determinant (polynomial of A_μ with degree $4N^2$).**

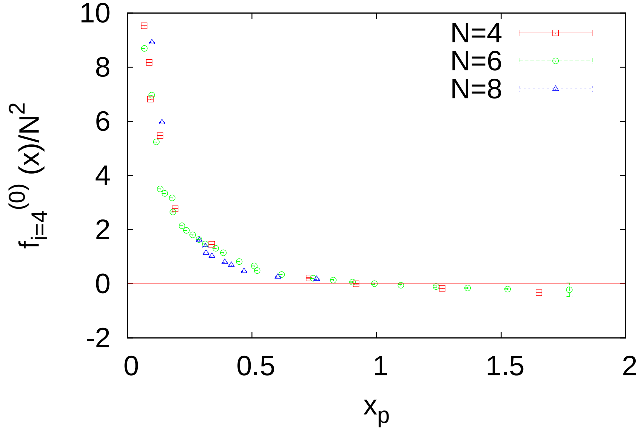
  $\Rightarrow \rho_{i=1}^{(0)}(x) \simeq (\sqrt{x})^{10N^2} \Rightarrow \frac{1}{N^2} f_i^{(0)}(x) = \frac{5}{x}$

At large x: $\frac{1}{N^2} f_i^{(0)}(x) \rightarrow 0$.

Ansatz for all x: $$\frac{1}{N^2} f_i^{(0)}(x) = \begin{cases} \frac{5}{x} \exp(-b_{i=1} x) & i = 1 \\ \frac{7-i}{2x} \exp(-b_i x) & i = 2, \cdots, 6 \end{cases}$$

For $N = 8$ numerical data, we have $b_{i=3,4} \simeq 5$.





Solutions of the equation $\frac{1}{N^2} f_i^{(0)}(x) = -\frac{d}{dx}\left\{\frac{1}{N^2}\log w_i(x)\right\}$ **:**

Double-peak structure for $i = 3, 4 \rightarrow$ **two solutions** x_s **and** x_l $(x_s < x_l = +\infty)$

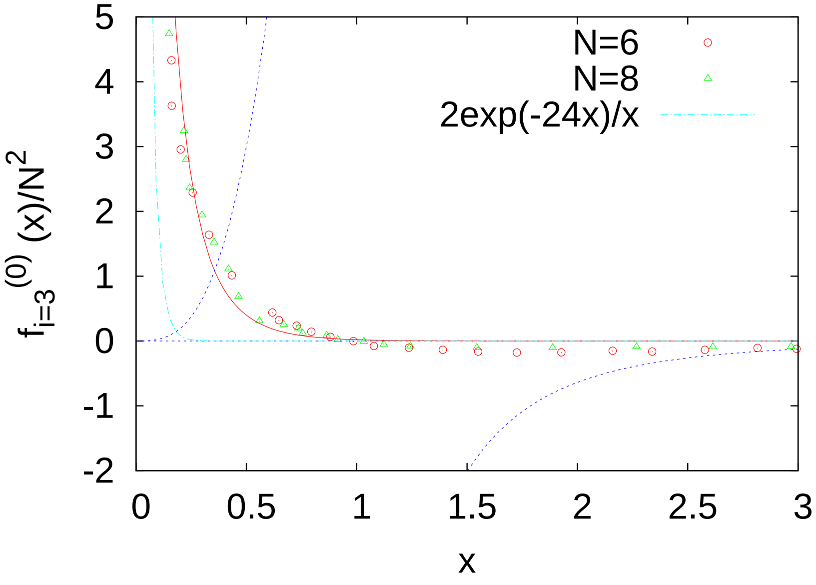
Result of Gaussian Expansion Method (GEM) **T. Aoyama, J. Nishimura and T. Okubo**

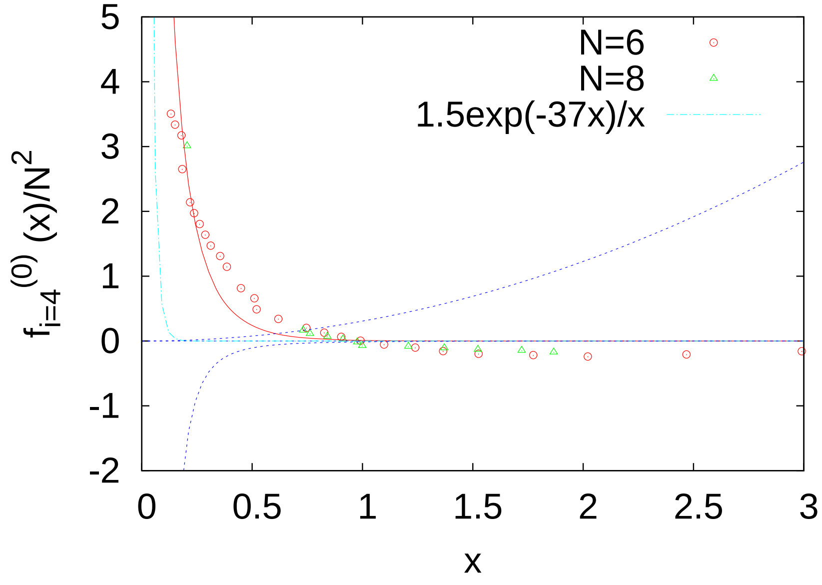
- **Symmetry breakdown SO(6) → SO(3):**

  x_l **should dominate for** $i = 3$. x_s **should dominate for** $i = 4$.

- **For collapsed directions,** $x_s = 0.18$.

  To have this solution, the coeffcient b_i **should be** $b_{i=3} \simeq 24, \quad b_{i=4} \simeq 37$.





- **Strong finite-N effect → For $i = 4$ we expect $x_s \simeq 0.18$ ($b_{i=4} \simeq 37$) at large N.**
- **For $b_{i=3} \simeq 24, b_{i=4} \simeq 37$ (expected behavior at large N) →**
  x_l dominates for $i = 3$ while x_s dominates for $i = 4$.

Evidence for symmetry breakdown SO(6) → SO(3)

4 Conclusion

Monte Carlo simulation of 6d IKKT model to study the spontaneous breakdown of SO(6) symmetry.

Can we understand the emergence of the spacetime?

- **Gaussian Expansion Method: SO(6) $\to$ SO(3).**
- **Numerical evidence for symmetry breakdown SO(6) $\to$ SO(3).**

Future works

- **In 6d IKKT model, finite-N effect is strong $\to$ simulation of larger N.**
  Use of Rational Hybrid Monte Carlo (RHMC) simulation.
- **Ultimately, studies of 10d IKKT model.**