Baryonium in Confining Gauge Theories

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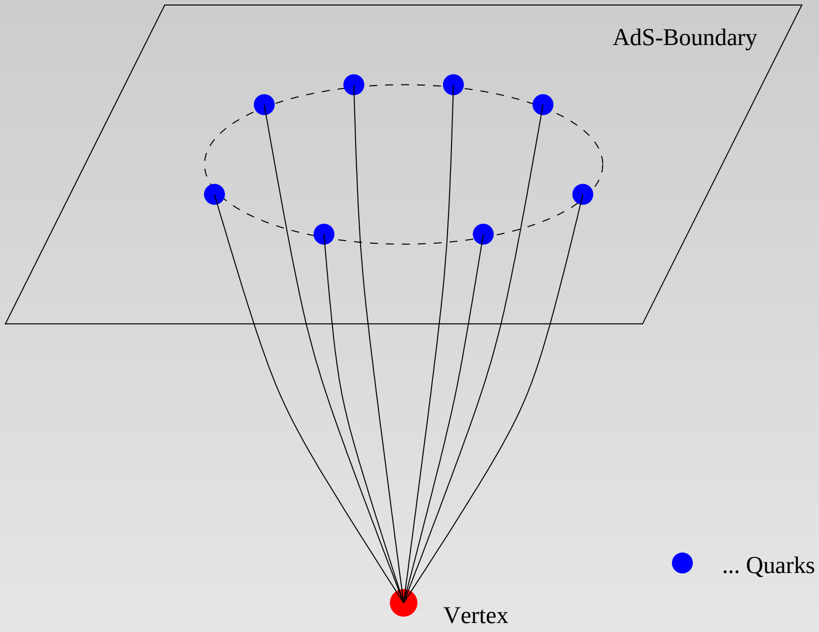
Baryon in gravity dual



A. Brandhuber, et al. JHEP 9807:020,1998

Baryon in SU(N) gauge theory
$\cdots$ the strings stretched between the boundary of the AdS_5 and the D5-brane wrapped on the S_5

E. Witten ('98)

In the type IIB string theory, there is a self dual field strength G_5 and there are N units of flux on the S^5

$$\int_{S^5} \frac{G_5}{2\pi} = N$$

On the D5-brane, there is a $U(1)$ gauge field A which couples to G_5 as

$$\int_{R\times S^5} A \wedge \frac{G_5}{2\pi}$$

By adding (-1) unit of charge to each string endpoint, the total charge in S^5 vanishes.

deformed AdS

We consider the deformed AdS background by including the R-R scalar χ

$$S = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{g}\left(R - \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}e^{2\phi}(\partial\chi)^2 - \frac{1}{4\cdot 5!}F_5^2\right)$$

the ansatz for supersymmetry: $\chi = -e^{\phi} + \chi_0$ (A. Kehagius and K. Sfetsos ('99))
The solution is given as follows.

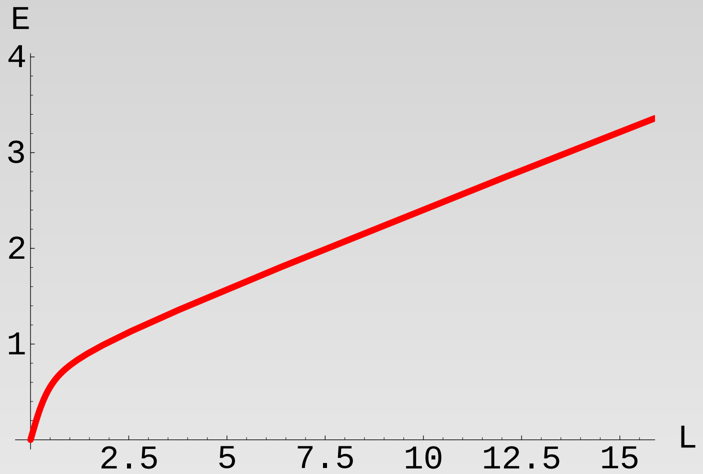
$$ds^2 = e^{\frac{\phi}{2}}\left(\frac{r^2}{R^2}(-dt^2 + (dx^i)^2) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2\right)$$

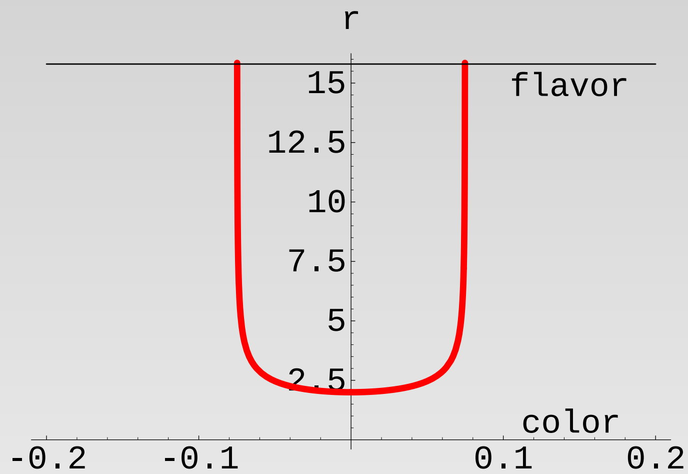
$$e^{\phi} = 1 + \frac{q}{r^4} \quad q \equiv \frac{\chi_0 R}{4} \quad R = (4\pi g_s \alpha'^2 N)^{1/4}$$

$q \cdots$ VEV of gauge field condensate $\langle F^{\mu\nu}F_{\mu\nu}\rangle$ (H. Liu and A.A. Tseytlin('99))

The dual gauge theory of this background is in the confinement phase.

The $q\bar{q}-$potential is given by the energy of the U-shaped string.





The U-shaped configuration

$\mathbf{X}_{||} = (\sigma, 0, 0), \quad \Omega_5 = \text{constant}$

Baryon

We set the world volume coordinates of D5-brane as $\xi^a = (t, \theta, \theta_2, \ldots, \theta_5)$. r, and A_t depend only on θ. (C.G.Callan, et al. ('99) Y. Imamura ('99) M.I and K. Ghoroku ('08))
Then, the embedded configuration of the D5 brane is

$$S = T_5 \Omega_4 R^4 \int dt d\theta \sin^4 \theta (-\sqrt{e^{\Phi}(r^2 + r'^2) - F_{\theta t}^2} + 4A_t)$$

where, $\Omega_4 = 8\pi^2/3$ is the volume of the unit four sphere.
We define the dimensionless displacement as $D = \frac{1}{T_5 \Omega_4 R^4} \frac{\delta S}{\delta F_{t\theta}}$
Then, the gauge field equation of motion and its solution are given.

$$\partial_\theta D = -4 \sin^4 \theta$$

$$D(\nu, \theta) = \frac{1}{T_5 \Omega_4 R^4} \frac{\delta S}{\delta F_{t\theta}} = \left[\frac{3}{2}(\nu\pi - \theta) + \frac{3}{2} \sin\theta \cos\theta + \sin^3\theta \cos\theta \right]$$

We eliminate the gauge filed in favor of D.

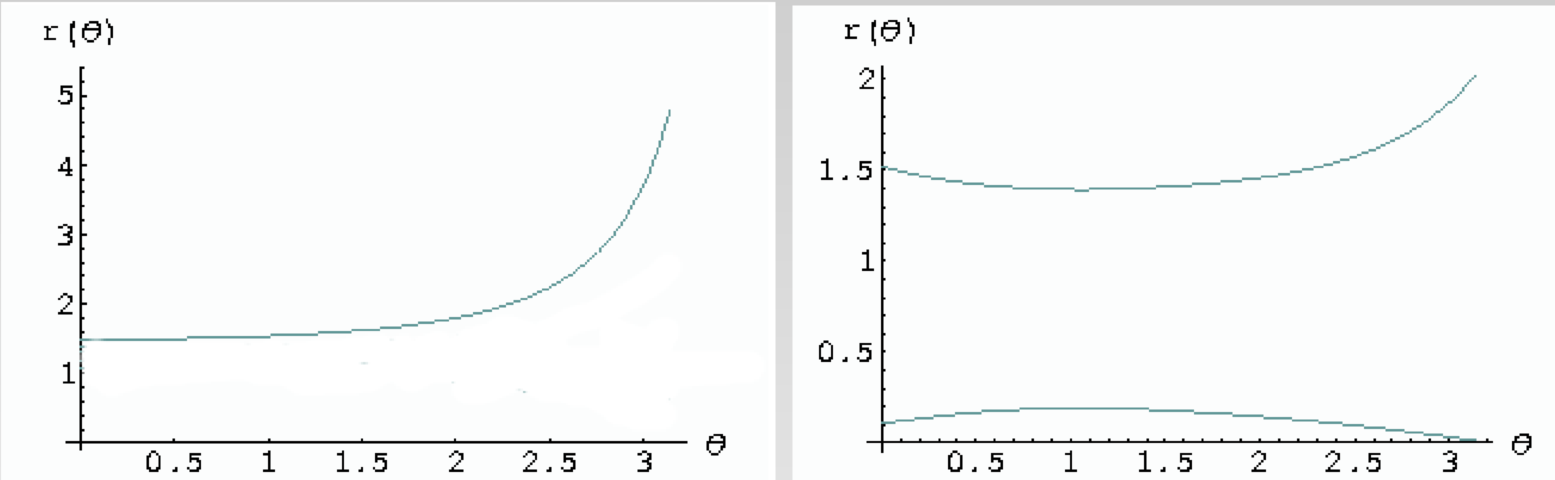
$$U_{D5} = \frac{N}{3\pi^2\alpha'} \int d\theta \; e^{\Phi/2} f \sqrt{r^2 + r'^2} \, \sqrt{V_\nu(\theta)}$$

$$V_\nu(\theta) = D(\nu, \theta)^2 + \sin^8\theta$$

where we used $T_5 \Omega_4 R^4 = N/(3\pi^2\alpha')$.
Then, we get the equation of motion

$$\partial_\theta \left(\frac{r'}{\sqrt{r^2 + (r')^2}} \sqrt{V_v(\theta)} \right) - \frac{1 - q/r^4}{1 + q/r^4} \frac{r}{\sqrt{r^2 + (r')^2}} \sqrt{V_v(\theta)} = 0$$

The solutions for $q = 2.8$



These solutions have cusps.

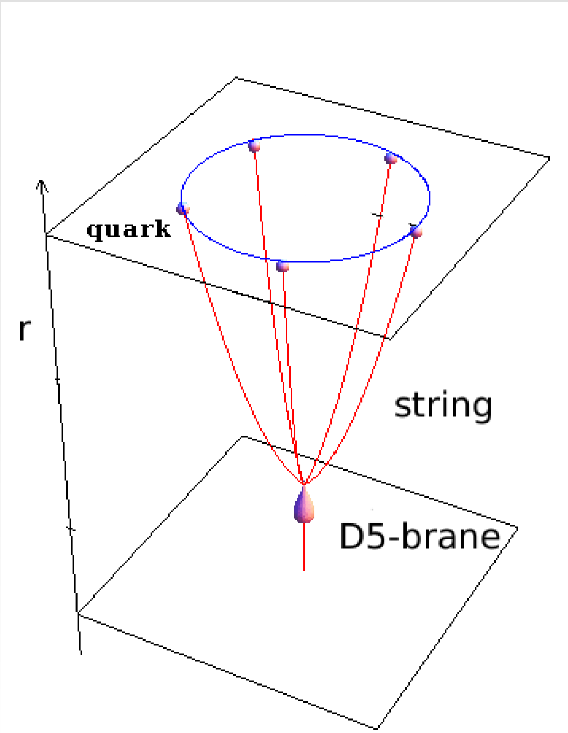
left: one cusp for $\nu = 0$ at $\theta = \pi$
right: two cusps for $\nu \neq 0$ at $\theta = \pi$ and $\theta = 0$

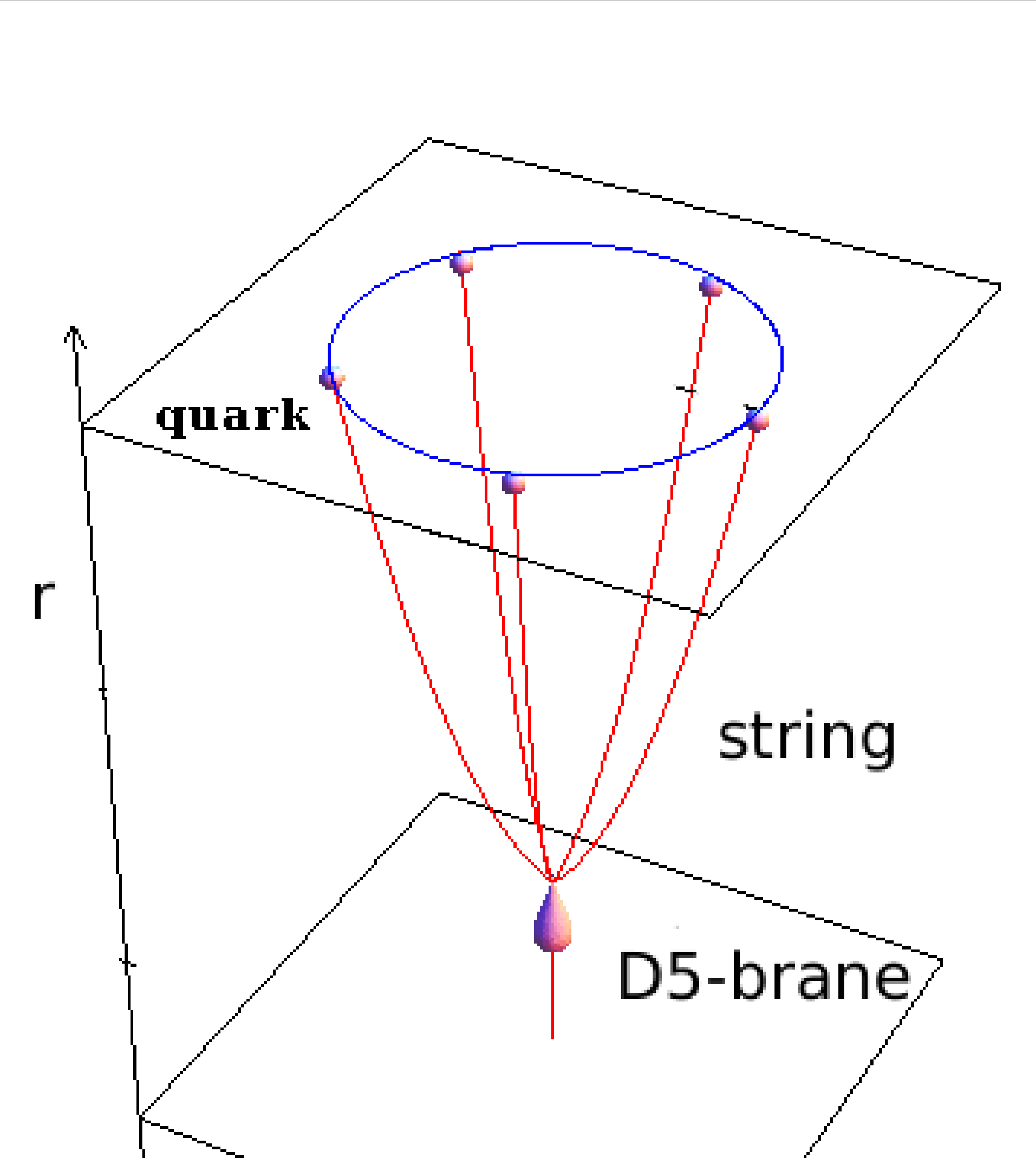
For simplicity, we consider the case of $\nu = 0$ in this section.

The D5-brane configuration is singular at $r(\theta = \pi)$ for $\nu = 0$.
The flux number at this cusp is given as

$$\frac{\delta S}{\delta F_{t\theta}}|_{\theta=\pi} = \frac{2N}{3\pi} D(\pi) = N \qquad \text{(here we set } 2\pi\alpha' \equiv 1)$$

Then, this singularity is canceled out by the boundary term of the N F-strings stretching from this cusp. We can get the baryon.




quark
r
string
D5-brane

Baryonium

We set its world volume coordinates as $\xi^a = (t, \theta, \theta_2, \ldots, \theta_5)$. r, A_t and x depend only on θ. (C.G.Callan, et al. ('99) M.I and K. Ghoroku ('08))
Then, the embedded configuration of the D5 brane is

$$S = T_5 \Omega_4 R^4 \int dt d\theta \sin^4\theta (-\sqrt{e^{\Phi}(r^2 + r'^2 + (r/R)x'^2) - F_{\theta t}^2} + 4A_t)$$

We eliminate the gauge filed in favor of D.

$$U_{D5} = \frac{N}{3\pi^2\alpha'} \int d\theta \, e^{\Phi/2} f \sqrt{r^2 + r'^2 + (r/R)^4 x'^2} \sqrt{V_\nu(\theta)}$$
$$V_\nu(\theta) = D(\nu, \theta)^2 + \sin^8\theta$$

where we used $T_5 \Omega_4 R^4 = N/(3\pi^2\alpha')$.

Then, we change variable in U_{D5} action from θ to x as

$$U_{D5} = \frac{N}{3\pi^2\alpha'} \int dx\, e^{\Phi/2} \sqrt{r^2\dot{\theta}^2 + \dot{r}^2 + (r/R)^4}\, \sqrt{V_\nu(\theta)} \equiv \frac{N}{3\pi^2\alpha'} \int dx L$$

The coordinate x is not included in the action explicitly, then we can introduce an integral constant h

$$h = \dot{r} p_r + \dot{\theta} p_\theta - L$$

$$p_r = \frac{\partial L}{\partial \dot{r}} = \dot{r} \left(\frac{R}{r}\right)^2 Q\,, \quad p_\theta = \frac{\partial L}{\partial \dot{\theta}} = r^2 \dot{\theta} \left(\frac{R}{r}\right)^2 Q\,,$$

and

$$Q = \sqrt{e^\Phi V_\nu - \left(\frac{p_\theta^2}{r^2} + p_r^2\right)}$$

Then h is written in terms of the momentum as

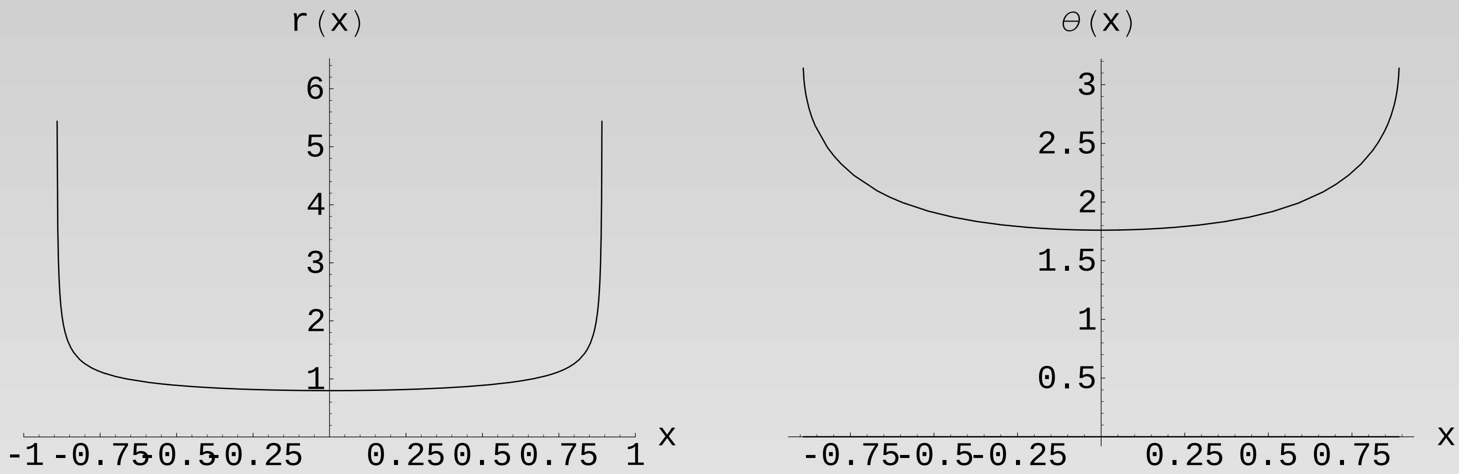
$$h = -\left(\frac{r}{R}\right)^2 \sqrt{e^{\Phi} V_\nu - \left(\frac{p_\theta^2}{r^2} + p_r^2\right)}\,,$$

and the equations of motion are obtained as x

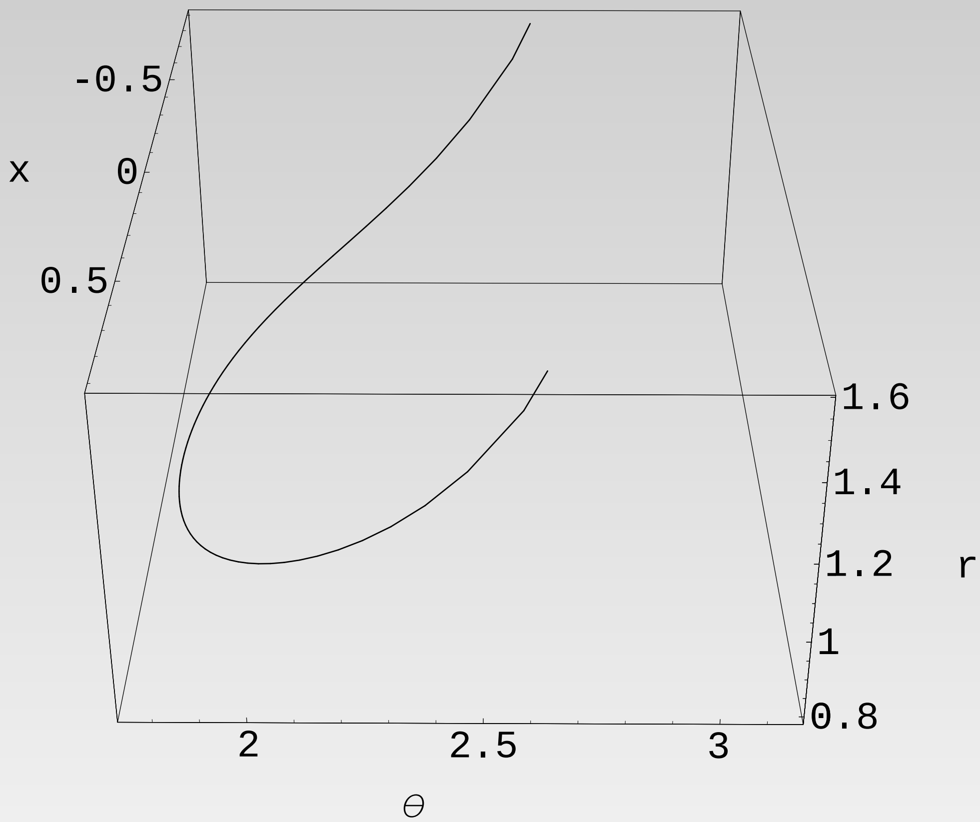
$$\dot{r} = \left(\frac{r}{R}\right)^2 \frac{p_r}{Q}\,, \quad \dot{\theta} = \left(\frac{r}{R}\right)^2 \frac{p_\theta}{r^2 Q}\,,$$

$$\dot{p}_r = -\frac{\partial h}{\partial r}\,, \quad \dot{p}_\theta = -\frac{\partial h}{\partial \theta}$$

The equations are solved numerically.



The typical $D5/\overline{D5}$ solution for $\nu = 0.5$ at $h = -1$.


-0.5
x
0
0.5
1.6
1.4
1.2
r
1
0.8
2
2.5
3
θ

There are two cusps at $\theta = \pi$. The flux numbers at cusps are given as

$$\frac{2N}{3\pi}|D(\nu,\pi)| = \left|\frac{\delta S}{\delta F_{t\theta}}\right||_{\theta=\pi} = N - k \quad (k \equiv N\nu)$$

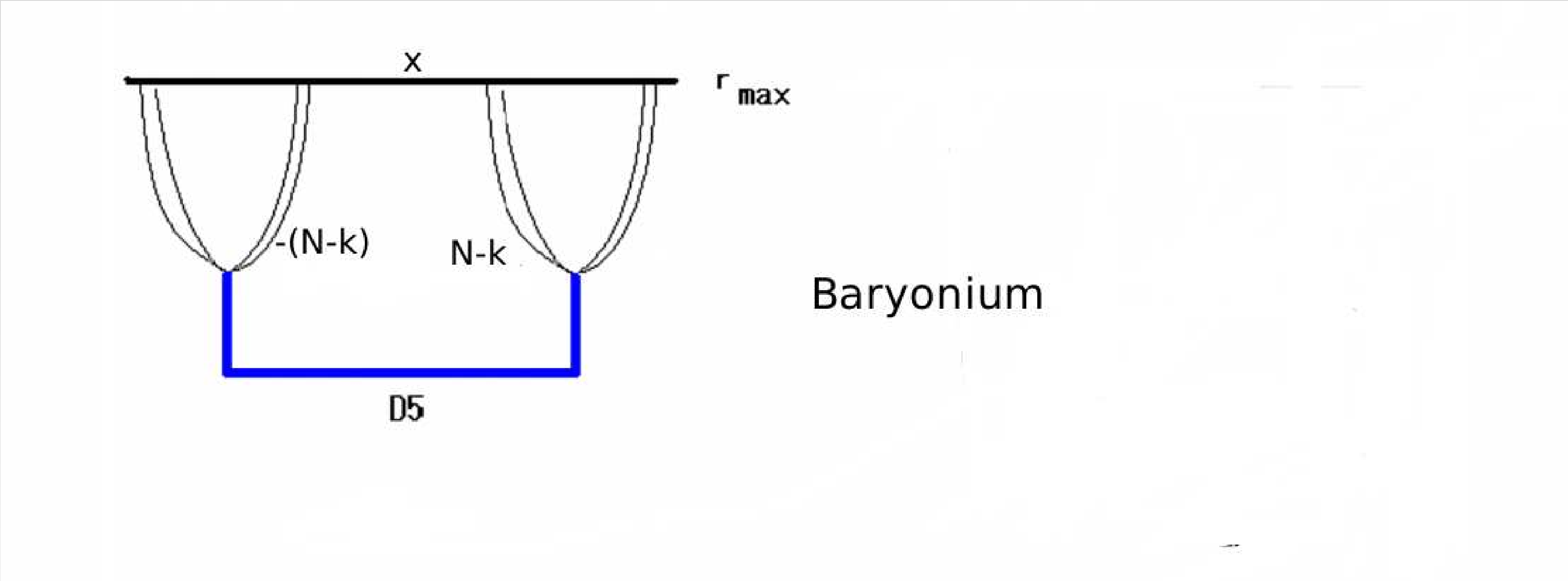
We cannot decide the sign of D because U_{D5} contains D in the squared form D^2.
However, in order to preserve the flux, D should have opposite sign at the two cusps as

$$\frac{2N}{3\pi}D(\nu,\pi,x^{\pm}) = \pm\frac{\delta S}{\delta F_{t\theta}}|_{\theta=\pi} = \pm(N-k)$$

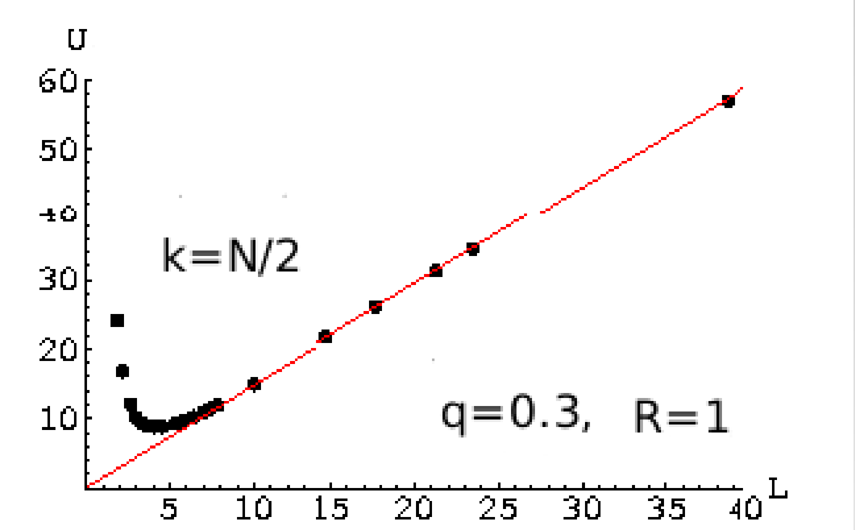
where, $x^{\pm} = \pm x(\theta = \pi)$

Then, this solution can be interpreted as the $D5/\bar{D}5$ configuration.

$(N-k)$ F-strings must be attached at both cusps but with opposite orientation to cancel out the singularity at the two cusps.
Such a state forms the $(N-k)$ quarks and $(N-k)$ anti-quarks bound state.



the $U_{D5} - L$ relation for h=-1 $(L \equiv x^{+} - x^{-})$



The energy of vertex part of a baryonium (U_{D5}) has a minimum at the finite L.

Summary and future work

- We find a baryonium solution by solving the equation of motion for the D5-brane action.
- Since the equation of D5 action contains the displacement flux operator D in the squared form D^2, the $D5/\bar{D}5$ bound state is obtained from the D5-brane action.
- There is a minimum of U_{D5} at finite L. Then, $D5$ and $\bar{D}5$ are separated enough not to be destabilized by the tachyon. Then, the baryonium state found here would be stable.
- As for the total mass of the baryonium, we must add F-strings at the cusps of D5-brane. The detail of the analyses will given in the future.