格子シミュレーションによる共形場の理論の探索

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arXiv : 0902.3768 (hep-lat) and Work in progress

QFT2009 @ YITP2009/7/10

Collaborators

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Numerical simulation was carried out on the vector supercomputer NEC SX-8 in YITP, Kyoto University and RCNP, Osaka University

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Plan to talk

- Introduction
- Step scaling
- Wilson loop scheme
  Twisted Polyakov loop scheme
- simulation (quenched QCD)
- Nf=12 case (TPL scheme)

Introduction

Introduction

- Fixed point・・・universality, symmetry
- Examples of fixed point

scalar theory (Renormalization group, large-N)

2-dim. NLSM : Gaussian fixed point

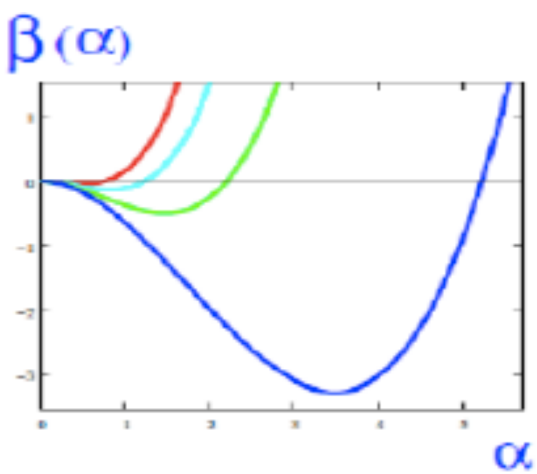
minimal series (non-trivial)

3-dim. LSM : IR (Wilson-Fischer) fixed point

gauge theory

4-dim. QCD : Gaussian fixed point

4-dim. large flavor SU(N) theory : Non-trivial IR fixed point？？



The existence of conformal window (Caswell 1974)

perturbative 2-loop

$$\mu \frac{d}{d\mu} \alpha(\mu) = \beta(\alpha) = -b\,\alpha^2(\mu) - c\,\alpha^3(\mu)$$

The phase structure (Banks-Zaks 1982)

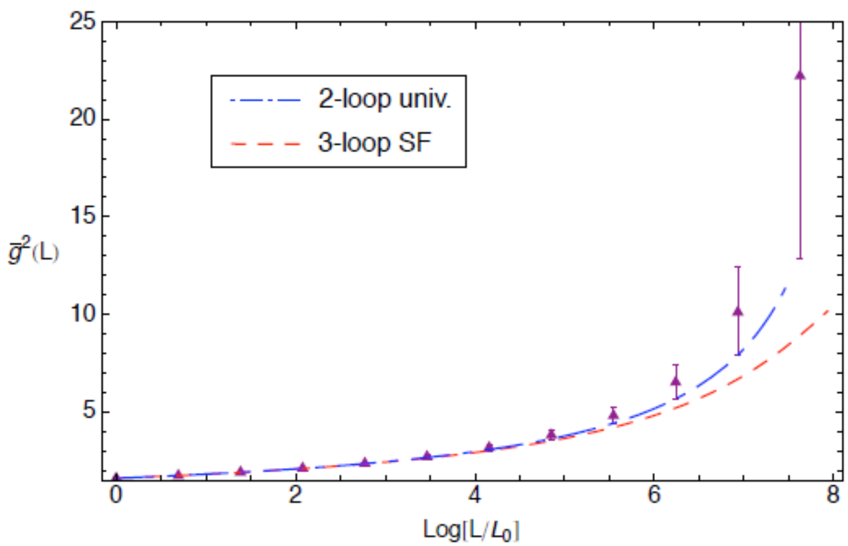
$N_f^{af} - N_f$ expansion

Previous studies in lattice QCD

- Damgaard, Heller, Krasnitz, Olesen: Phys.Lett.B400:169
- Iwasaki et al: Phys.Rev.D76:034504
- Appelquist, Fleming and Neil: Phys.Rev.Lett.100:171607
  arXiv:0901:3766

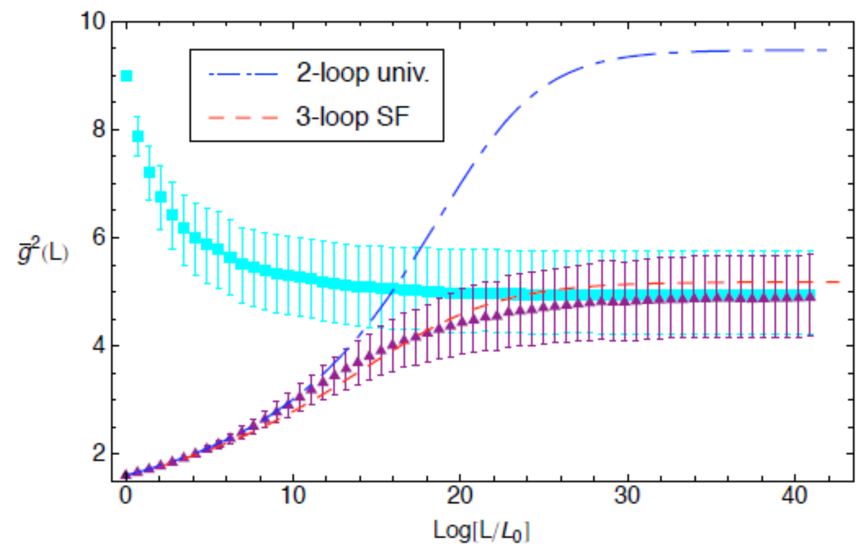
The running coupling constant in Schrodinger functional scheme.

Nf=8



There is no edivence of fixed point.

Nf=12



There is a flat region in low energy scale.

Practical conditions of the renormalized coupling on the lattice:

Luscher et.al.
Nucl.Phys.Proc.Suppl.30:139-148,1993.

1. $g^2(L)$ is non-perturbatively defined
2. $g^2(L)$ can be computed through numerical simulation
3. A perturbative computation is possible. (In principle)

Examples of scheme:

- Schrodinger functional scheme
- Wilson loop scheme : arXiv:0902.3768 (hep-lat)
- Twiseted Polyakov Loop scheme :

(Wilson loop scheme and Twiseted Polyakov Loop scheme:) no O(a/L) error scheme

The renormalized coupling constant on the lattice and the running coupling constant

renormalized coupling on the lattice:

$$g^2\left(L_0, \frac{a}{L_0}, \beta\right) = Z_R(L_0, \frac{a}{L_0}, \beta) g_0^2$$

- Lattice size (L_0)
- Lattice spacing (a)
- bare coupling constant $\beta = 2N/g_0^2$

How to take the continuum limit

$$g_R^2\left(\frac{1}{L_0}\right) \equiv \lim_{a \to 0} Z_R\left(\frac{a}{L_0}, g_0^2\right)\Big|_{L_0} g_0^2$$

To take the continuum limit, we have to set the scale "a".
It corresponds to tuning g_0^2 to keep a certain input physical parameter constant.

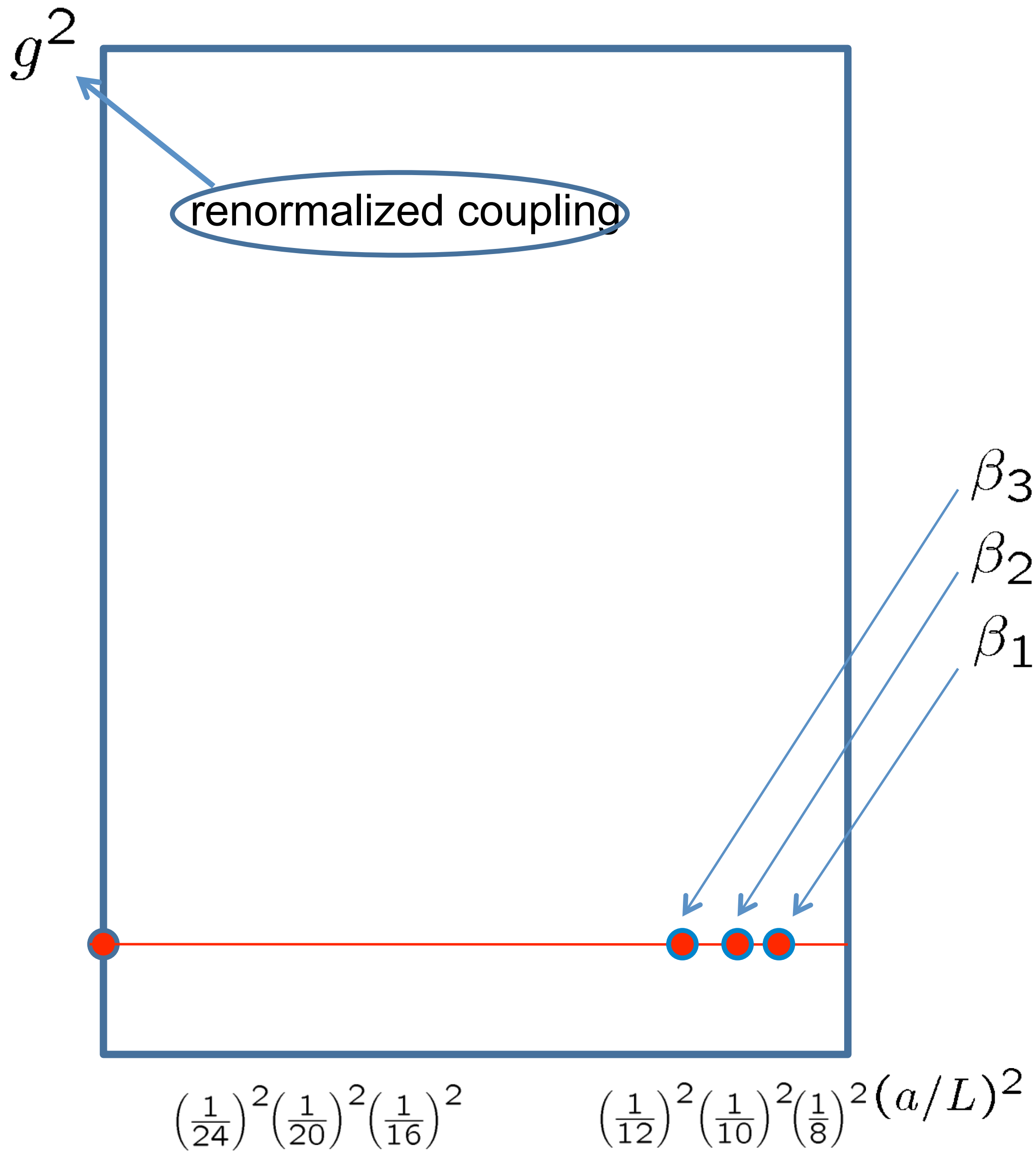
Examples of input parameters: Λ_{QCD}, mass, Sommer scale….

To measure the running coupling

input $g_R(1/L_0)$ ⇒ output $g_R(s/L_0)$

s : scaling parameter

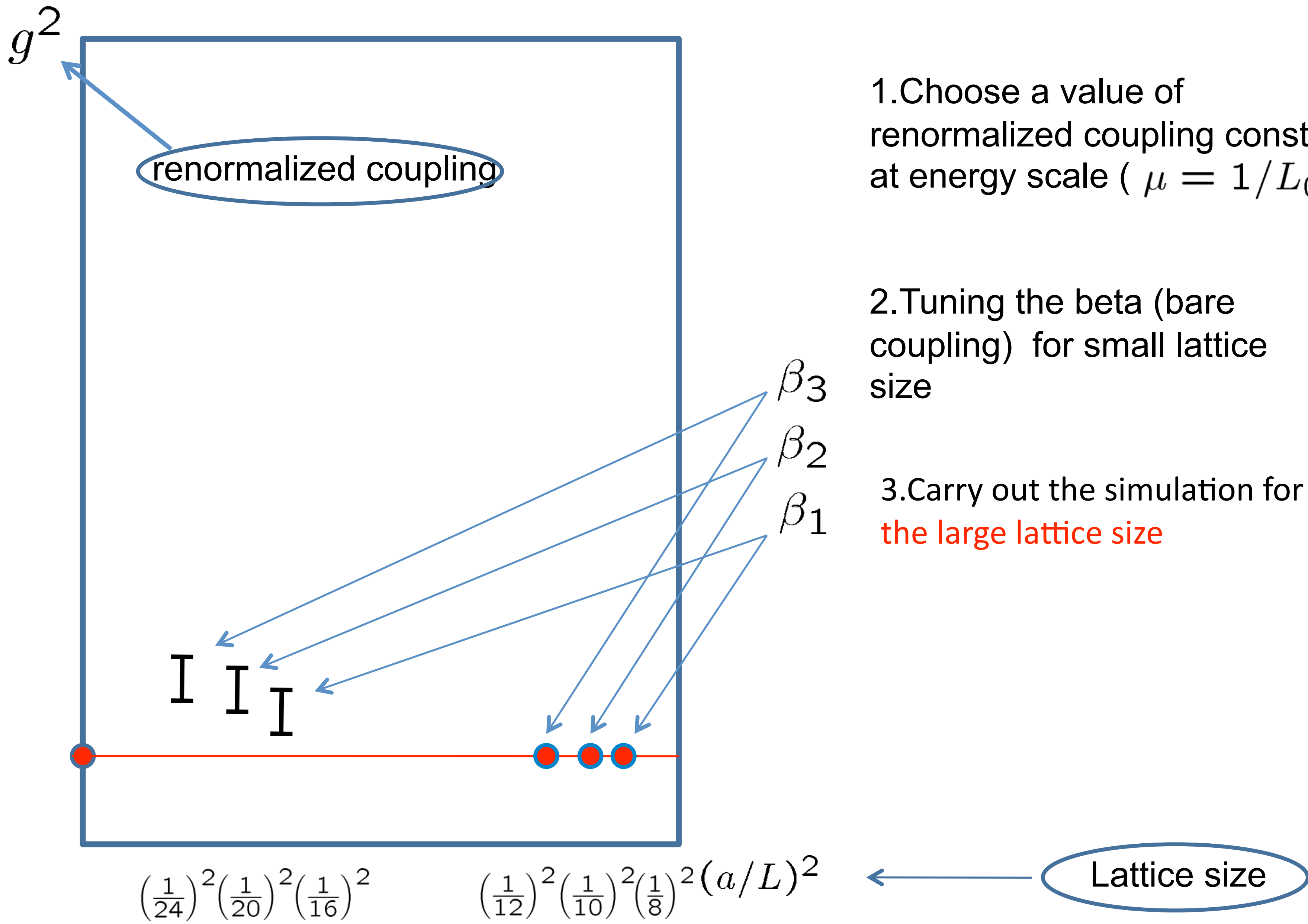
Step scaling



1.Choose a value of renormalized coupling constant at energy scale ($\mu = 1/L_0$)

2.Tuning the beta (bare coupling) for small lattice size

Step scaling

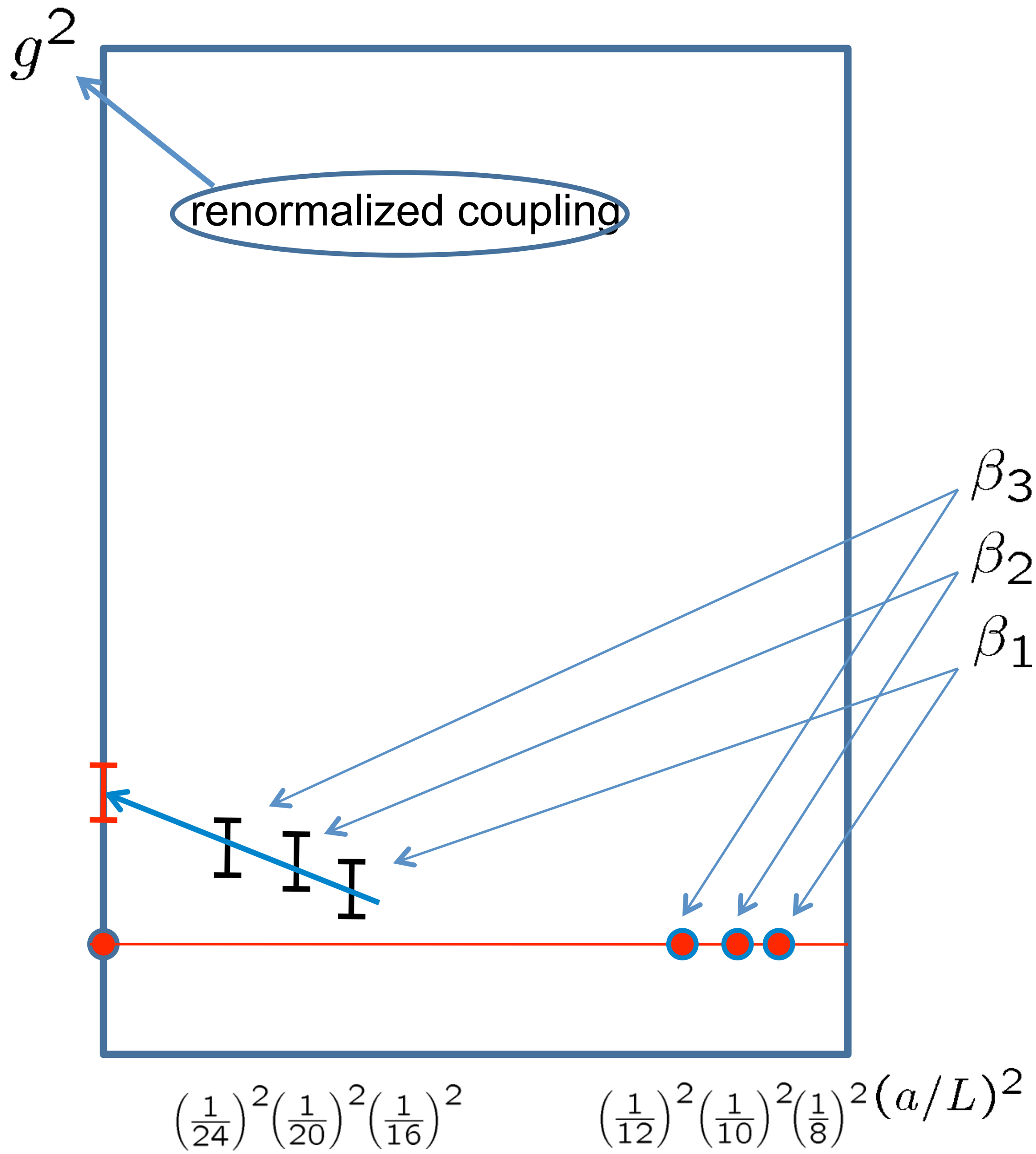


1.Choose a value of renormalized coupling constant at energy scale ($\mu = 1/L_0$)

2.Tuning the beta (bare coupling) for small lattice size

3.Carry out the simulation for the large lattice size

Step scaling



1.Choose a value of renormalized coupling constant at energy scale ($\mu = 1/L_0$)

2.Tuning the beta (bare coupling) for small lattice size

3.Carry out the simulation for the large lattice size

4.Take the continuum limit (energy scale $\mu = 1/2L_0$)

Wilson loop scheme
Twisted Polyakov loop scheme

Twisted Polyakov loop scheme

Previous works of SU(2) gauge theory: de Divitiis et. al. NPB422:382
Twisted boundary condition: 't Hooft NPB153:131

$$U_\mu(x+\hat{\nu}L/a) = \Omega_\nu U_\mu(x)\Omega_\nu^\dagger \quad (\nu = 1,2)$$

Ω : twist matrix

$$\Omega_1\Omega_2 = e^{i2\pi/3}\Omega_2\Omega_1, \quad \Omega_\mu\Omega_\mu^\dagger = 1, \quad (\Omega_\mu)^3 = 1, \quad \mathrm{Tr}[\Omega_\mu] = 0$$

Polyakov loop in Twisted direction

$$P_1(y,z,t) = \mathrm{Tr}\left([\Pi_j U_1(x=j,y,z,t)]\underline{\Omega_1}\,\underline{e^{i2\pi y/3L}}\right)$$

To satisfy the gauge inv. and translation inv.

Non-perturbative definition of renormalized coupling in TPL scheme

$$g_{TP}^2 = \frac{1}{k}\frac{\langle\sum_{y,z} P_1(y,z,L/2a)P_1(0,0,0)^*\rangle}{\langle\sum_{x,y} P_3(x,y,L/2a)P_3(0,0,0)^*\rangle}$$

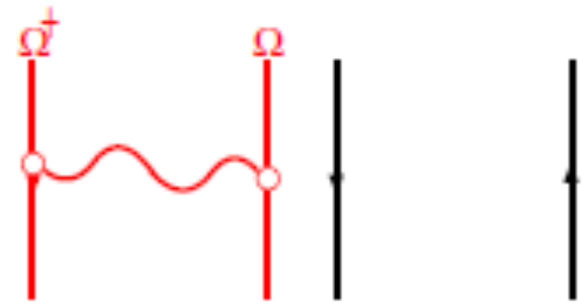
Non-perturbative definition of renormalized coupling in TPL scheme

$$g^2_{TP} = \frac{1}{k}\frac{\langle \sum_{y,z} P_1(y,z,L/2a)P_1(0,0,0)^* \rangle}{\langle \sum_{x,y} P_3(x,y,L/2a)P_3(0,0,0)^* \rangle}$$

At tree level, g^2_{TP} is proportional to bare coupling.

$$g^2_{TP}|_{tree} = g_0^2$$

$$\begin{aligned} k &= \frac{1}{24\pi^2}\Sigma\frac{(-1)^n}{n^2+(1/3)^2} \\ &= 0.03184\cdots \end{aligned}$$



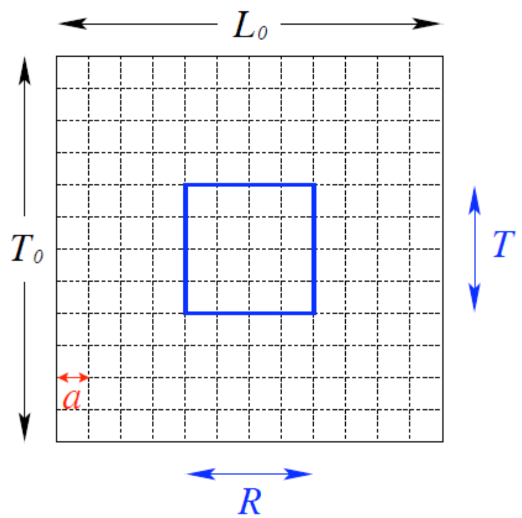
In $L \to \infty$ the ratio becomes unity

$$g^2_{\rm TP}(\mu = 1/L \to 0) \to \frac{1}{k} \sim 32$$

In our works, we have to check the g^2_{TP} at IR fixed point which should be smaller than this value

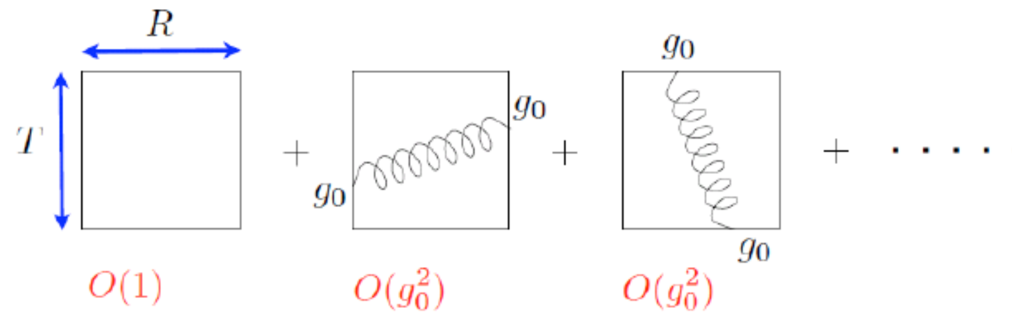
Wilson loop scheme

arXiv : 0902.3768 (hep-lat)



We take $L_0 = T_0, R = T$

$$W(L_0, R, T_0, T, a, g_0) \; =$$



Renormalized coupling in "Wilson loop scheme"

$$g_W^2 = -R^2 \frac{\partial^2}{\partial R \partial T} \ln \langle W(R,T) \rangle \Big|_{T=R} \Big/ k$$

One the lattice simulation, Creutz ratio:

$$CR(\hat{R}, \hat{T}) = -\ln \left(\frac{W(\hat{R},\hat{T}) W(\hat{R}-1,\hat{T}-1)}{W(\hat{R},\hat{T}-1) W(\hat{R}-1,\hat{T})} \right)$$

The coupling constant of this scheme also has no O(a) error.

simulation (quenched QCD)

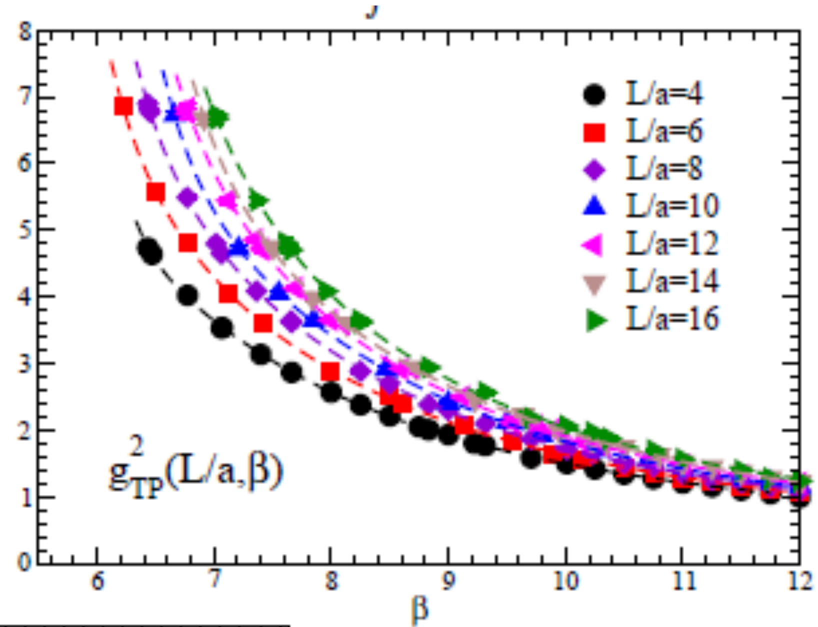
Quenched QCD test (TPL)

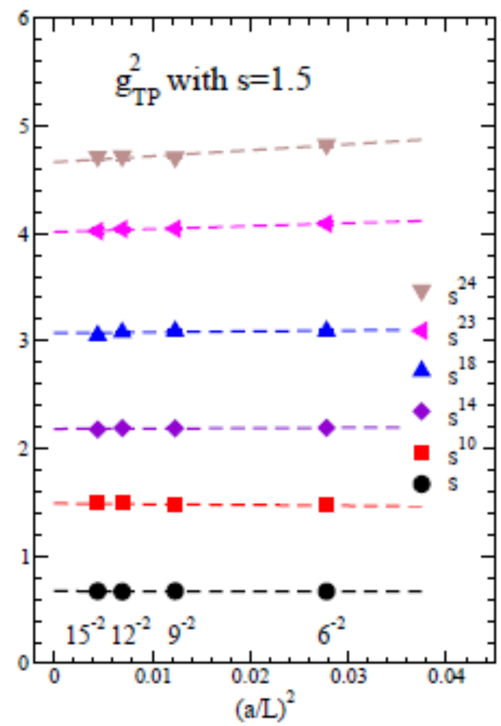
Lattice size

$$s = 1 : L = 4, 6, 8, 10$$
$$s = 1.5 : L = 6, 9, 12, 15$$

parameter $6.2 < \beta < 16$

- measure Ploop at every Monte Carlo sweep
  (200,000-400,000 sweeps)
- interpolation for L=9,15
- step scaling (s=1.5)
- extrapolation



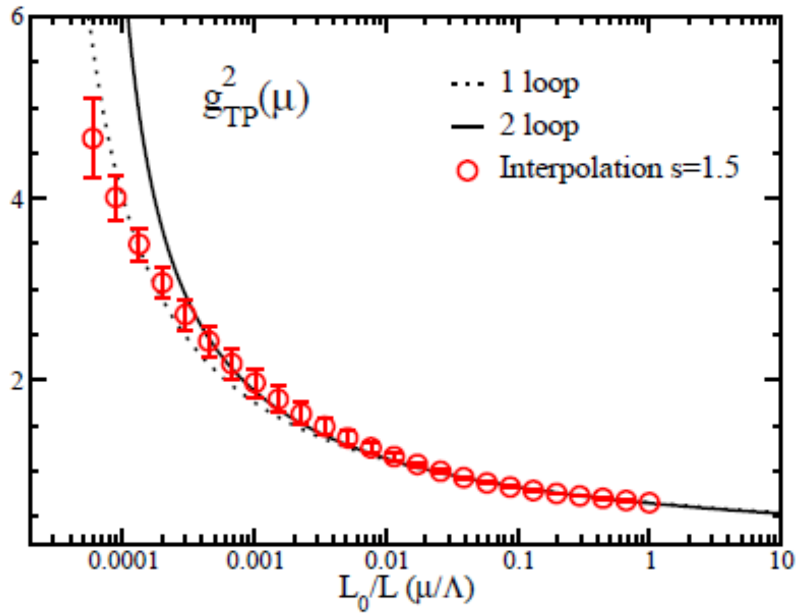


linear fn. of a^2 ,no O(a/L)

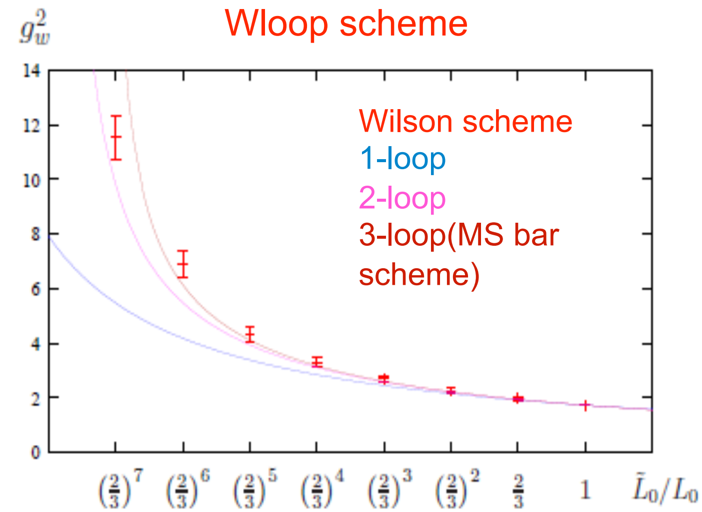
small statistical error for all L/a
24 steps

Running of the renormalized coupling constant in Quenched QCD

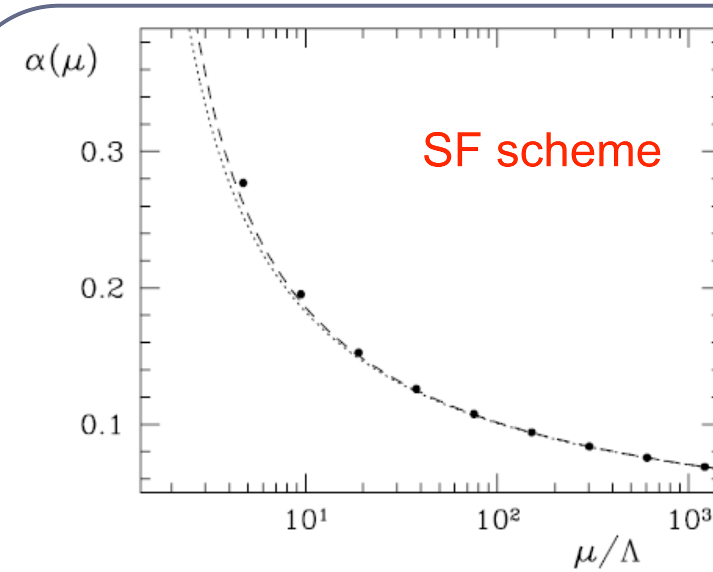
TPL scheme



Wloop scheme



arXiv : 0902.3768 (hep-lat)



Capitani et.al:NPB B544 : 669-698

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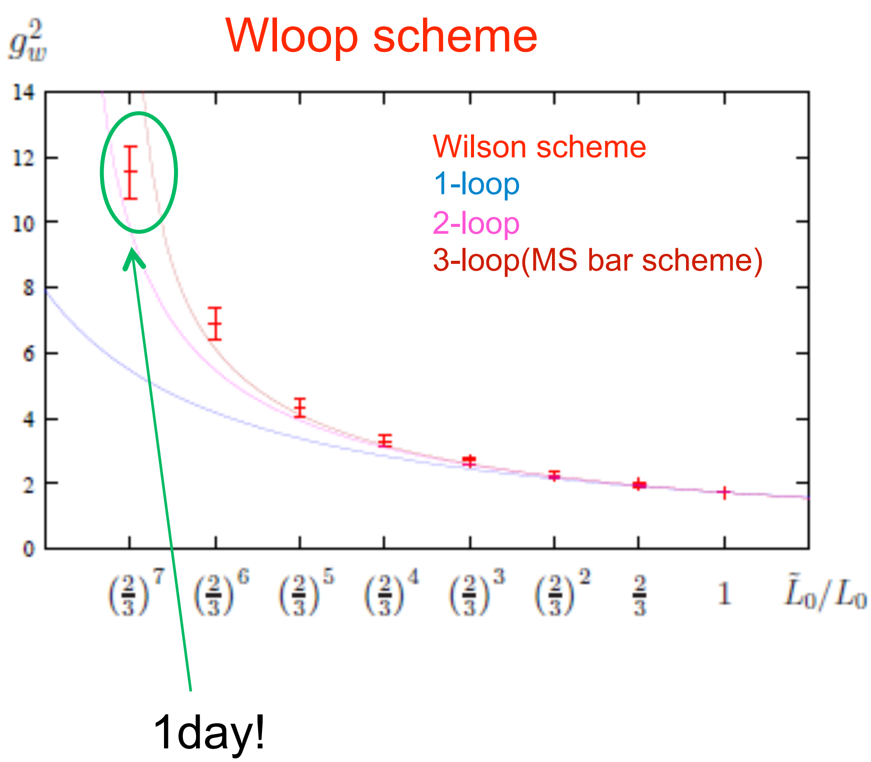
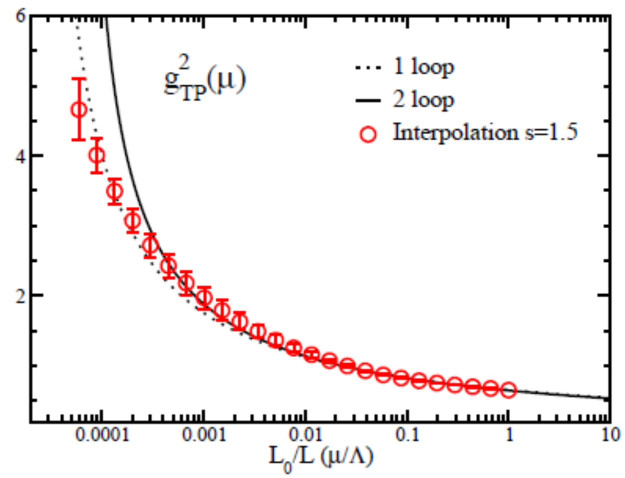
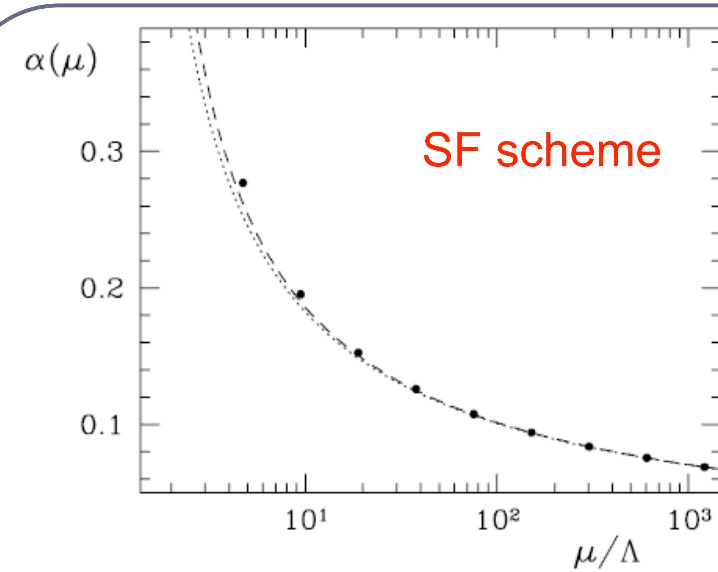


TPL scheme





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Summary of the quenched QCD test for Wilson and Polyakov loop scheme

- TPL scheme and WL scheme have nice scaling.
- Small systematic error (no O(a/L) error)
- Smearing drastically reduces the statistical error in WL scheme.
- These methods are promising. We will investigate the large flavor QCD using these renormalization scheme.

Nf=12 case (TPL scheme)

Massless large flavor QCD with twisted boundary condition

Problem of twisted boundary condition of fermions

$$
\begin{aligned}
U_\mu(x+\hat{\nu}L/a) &= \Omega_\nu U_\mu(x)\Omega_\nu^\dagger \quad (\nu=1,2) \\
\psi(x+\hat{\nu}L/a) &= \Omega_\nu \psi(x)
\end{aligned}
$$

$$
\begin{aligned}
\psi(x+\hat{\nu}L/a+\hat{\rho}L/a) &= \Omega_\rho\Omega_\nu\psi(x) = e^{i2\pi/3}\Omega_\nu\Omega_\rho\psi(x) \\
&= \Omega_\nu\Omega_\rho\psi(x) \text{ inconsistent}
\end{aligned}
$$

Introduce "smell" $\rightarrow \psi^a_\alpha(x)$: $N_c \times N_s(=N_c)$ matrix

Parisi, 1983(Unpublished)

$$
\psi^a_\alpha(x+\hat{\nu}L/a) = e^{i\pi/3}\Omega_\nu^{ab}\psi^b_\beta(x)(\Omega_\nu^\dagger)_{\beta\alpha} \quad (\nu=1,2)
$$

Apart from boundary condition, smell likes extra flavor.

$$
N_f = N_s \times n \geq N_c
$$

Staggered fermion requires $N_f = 4 \times n \Rightarrow N_f = 4 \times N_s \geq 12$ simulation

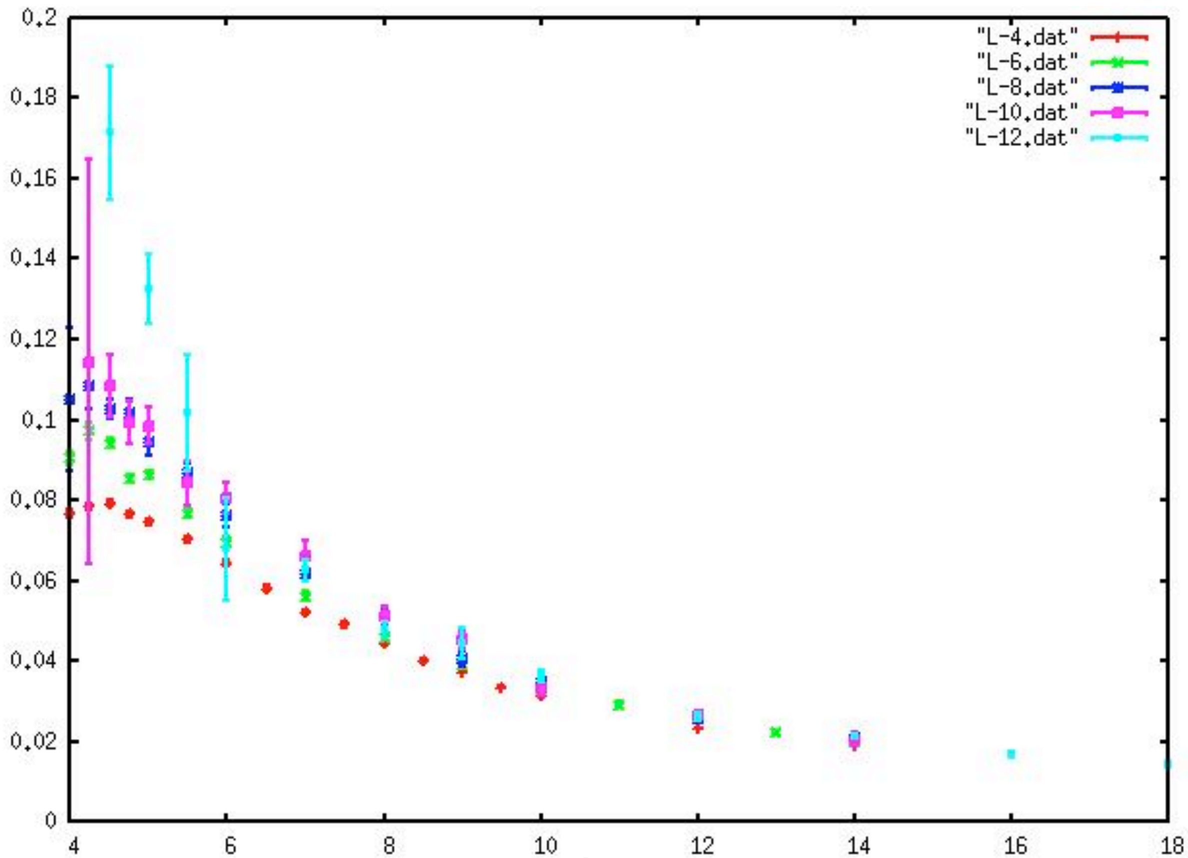
same as Appelquist et al.

- Hybrid Monte Carlo algorithm (exact algorithm)
- staggered fermion (massless 4N flavor)
- Polyakov loop scheme

Lattice size
$s=1 : L=6,8,10$
$s=1.5 : L=9,12,15$
parameter $4.0 < \beta < 18$

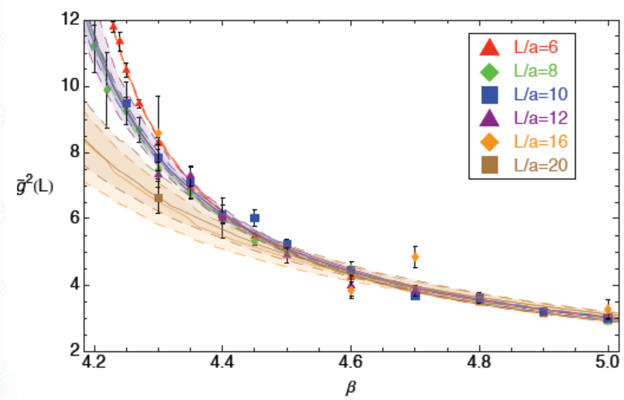
Our data (Preliminary)

ratio of Ploop



beta

cf: Appelquist et.al.



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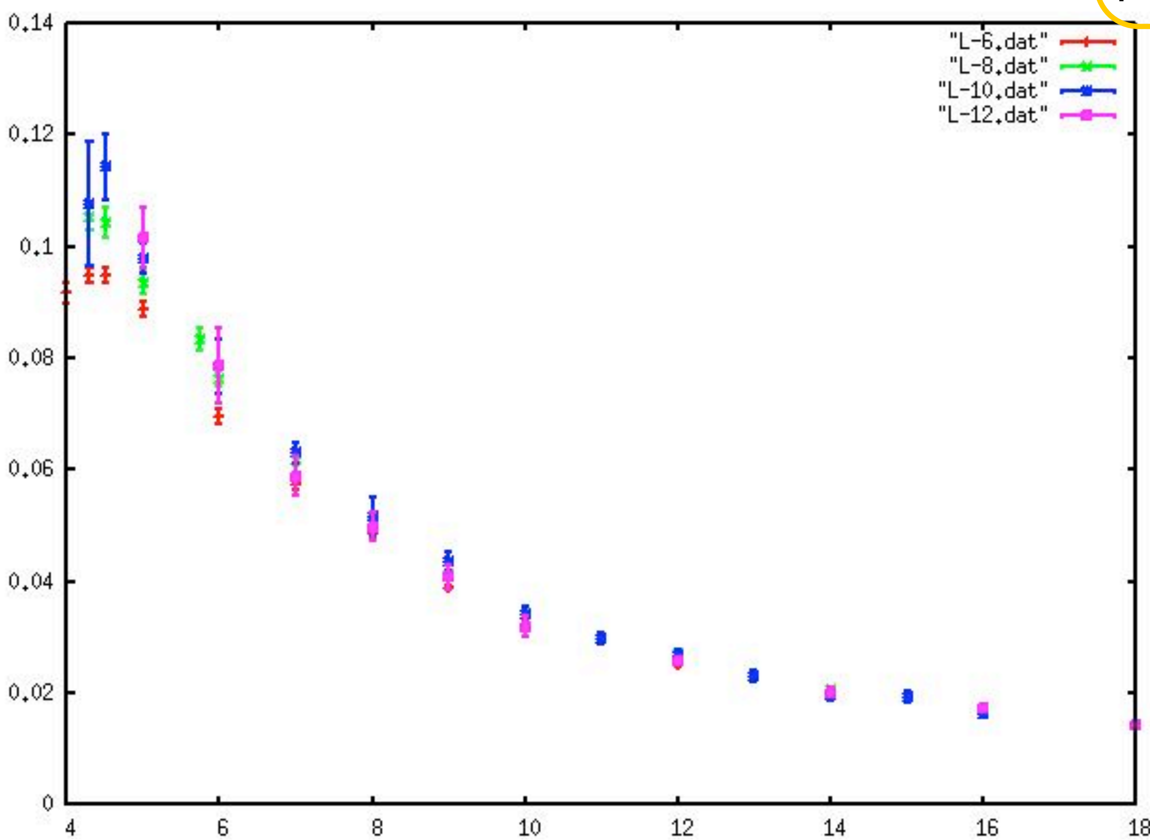

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