Meson-Nucleon Coupling from AdS/QCD

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Plan

1: Introduction

2: Meson sector (review)

3: Meson-Nucleon coupling
(review & our results)

4: Summary

Introduction

AdS/CFT correspondence has opened a new avenue to study strongly coupled gauge theories

Application to QCD and hadron physics

Top down: String theory → QCD

Sakai & Sugimoto (2005)

Bottom up: QCD → 5D holographic model

Erlich, Katz, Son & Stephanov (2005)
Da Lord & Pomarol (2005)

Introduction

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Da Lord & Pomarol (2005)

⇒ 5D model of Meson sector (vector, axial-vector meson masses, decay consts, $g_{\rho\pi\pi}$,...)

As for baryons, two approaches are known

1: Skyrmion

Hashimoto, Sakai & Sugimoto (2008)
and many papers

2: Bulk fermion

Hong, Inami & Yee (2007)[spin 1/2]
Ahn, Hong, Park & Siwach (2009)[spin3/2]

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Our work

By extending the model of Hong, Inami & Yee, we calculated (Axial-)Vector Meson-Nucleon couplings using the 5D Holographic QCD

Meson sector

"QCD and a Holographic Model of Hadrons"
Erlich, Katz, Son and Stephanov
PRL95 261602 (2005)

"Chiral Symmetry Breaking from Five Dimensional Spaces"
Da Lord and Pomarol
NPB721 79 (2005)

AdS/CFT dictionary

4D		5D
Global symmetry	$\Leftrightarrow$	Gauge symmetry
Operator $\mathcal{O}$	$\Leftrightarrow$	5D field
Dimension[$\mathcal{O}$]	$\Leftrightarrow$	Bulk mass
1/Nc	$\Leftrightarrow$	g_5^2
Resonance	$\Leftrightarrow$	KK modes

Following this dictionary,
we guess a holographic model of QCD

5D SU(Nf)L x SU(Nf)R gauge theory on a slice of AdS5

$$ds^2 = \frac{1}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right), \quad \varepsilon \le z \le z_m \left(= \Lambda_{QCD}^{-1}\right)$$

(Axial-)Vector meson (spin 1) ⇔ (Axial-)Vector gauge field

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	$SU(N_f)_L \times SU(N_f)_R$	p	Δ	$M_5^2 = (\Delta - p)(\Delta + p - 4)$
$\bar{q}_L \gamma^\mu t^a q_L$	L_μ^a	$(adj, 1)$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$	R_μ^a	$(1, adj)$	1	3	0
$\bar{q}_R^\alpha q_L^\beta$	$z^{-1} X^{\alpha\beta}$	$(\bar{N}_f, N_f)$	0	3	-3

$$S_{\text{meson}} = \int d^5x \sqrt{-g}\, \mathrm{Tr}\left[-\frac{1}{2g_5^2}\left(L_{MN}^2 + R_{MN}^2\right) + \left|D_M X\right|^2 - M_5^2 \left|X\right|^2 \right]$$

Bulk scalar field “X”
Source of chiral symmetry breaking

Classical solution

$$X(z) = Mz + \Sigma z^3$$

“explicit breaking”
Quark Mass

“spontaneous breaking”
Chiral condensate

UV B.C.

$$\left.\frac{X(z)}{z}\right|_{z=\varepsilon} = M$$

$$\langle \bar{q}q \rangle = \frac{\partial}{\partial M}\left\langle \exp\left[i\int d^4x M\bar{q}q\right]\right\rangle\Big|_{M=0}$$

$$= \frac{\delta}{\delta M} S_5[X]\Big|_{X=X_{cl},M=0,z=\varepsilon} \sim \Sigma$$

Defining $V_M \equiv \frac{1}{2}\left(L_M + R_M\right),\ A_M \equiv \frac{1}{2}\left(L_M - R_M\right)$ and adding

$$\mathcal{L}_{gf} = -\frac{1}{2\xi_V g_5^2 z}\left[\partial_\mu V^\mu - \xi_V z \partial_z \left(\frac{V_z}{z}\right)\right]^2 - \frac{1}{2\xi_A g_5^2 z}\left[\partial_\mu A^\mu - \xi_A z \partial_z \left(\frac{A_z}{z}\right) + 2\sqrt{2} g_5^2 \frac{\xi_A}{z^2}\left\langle X \right\rangle^2 P\right]^2$$

leads to the quadratic Lagrangian in the unitary gauge

$$\mathcal{L}_{quadratic} = -\frac{1}{4g_5^2 z} V_\mu^a \left[-\eta^{\mu\nu}\partial^2 + \partial^\mu \partial^\nu + \eta^{\mu\nu} z \partial_z \left(\frac{1}{z}\right)\partial_z\right] V_\nu^a \qquad X = \left\langle X \right\rangle e^{iP}$$

$$-\frac{1}{4g_5^2 z} A_\mu^a \left[-\eta^{\mu\nu}\partial^2 + \partial^\mu \partial^\nu + \eta^{\mu\nu} z \partial_z \left(\frac{1}{z}\right)\partial_z\right] A_\nu^a + \frac{\left\langle X \right\rangle^2}{2z^3}\left(\partial_\mu P^a - A_\mu^a\right)^2$$

Mode equations and boundary conditions

$$0 = \left[m_n^2 + z\partial_z\left(\frac{1}{z}\right)\partial_z\right] f_n^V,\quad f_n^V(\varepsilon) = \partial_z f_n^V(z_m) = 0$$

$$0 = \left[m_n^2 + z\partial_z\left(\frac{1}{z}\right)\partial_z - \frac{2g_5^2\left\langle X \right\rangle^2}{z^2}\right] f_n^A,\quad f_n^A(\varepsilon) = \partial_z f_n^A(z_m) = 0$$

Meson-Nucleon coupling

"Baryons in AdS/QCD"
Hong, Inami and Yee, PLB646 165 (2007)

"Meson-Nucleon Coupling from AdS/QCD"
N.M. and Motoi Tachibana
arXiv:0904.3816(accepted in EPJC)

Spin ½ Baryon ⇔ Spin ½ Dirac fermion

Hong, Inami & Yee (2007)

$$S_{Baryon}=\int d^5x\sqrt{-g}\left[i\bar{N}_1 e_A^M\Gamma^A D_M N_1+i\bar{N}_2 e_A^M\Gamma^A D_M N_2-\frac{5}{2}\bar{N}_1N_1+\frac{5}{2}\bar{N}_2N_2\right]$$

$$D_\mu=\partial_\mu+\frac{1}{2z}\Gamma_z\Gamma_\mu-iL_\mu,\ D_z=\partial_\mu-iL_z,\ m_5^2=(\Delta-2)^2=\left(\frac{9}{2}-2\right)^2$$

To incorporate chiral symmetry breaking, the following Yukawa coupling is introduced

$$S_{Yukawa}=\int d^5x\sqrt{-g}\left[-g_Y\bar{N}_1XN_1-g_Y\bar{N}_2X^\dagger N_2\right]$$

$$\Rightarrow\quad \begin{pmatrix}\partial_z-\frac{\Delta}{z} & -\frac{g_Y\langle X\rangle}{z}\\ -\frac{g_Y\langle X\rangle^\dagger}{z} & \partial_z-\frac{4-\Delta}{z}\end{pmatrix}\begin{pmatrix}f_{1L}^n\\ f_{2L}^n\end{pmatrix}=-m_n\begin{pmatrix}f_{1R}^n\\ f_{2R}^n\end{pmatrix},\ \begin{pmatrix}\partial_z-\frac{4-\Delta}{z} & \frac{g_Y\langle X\rangle}{z}\\ \frac{g_Y\langle X\rangle^\dagger}{z} & \partial_z-\frac{\Delta}{z}\end{pmatrix}\begin{pmatrix}f_{1R}^n\\ f_{2R}^n\end{pmatrix}=m_n\begin{pmatrix}f_{1L}^n\\ f_{2L}^n\end{pmatrix}$$

$f_{1R}^n(z_m)=f_{2L}^n(z_m)=f_{1L}^n(\varepsilon)=f_{2R}^n(\varepsilon)=0$, Other B.C. from EOM

π NN coupling

π NN coupling is generated from

1: Gauge coupling (5th component)
2: Yukawa coupling
3: Pauli term (through Goldberger-Treiman relation)

$$\mathcal{L}_{\pi NN} = \int_0^{z_m} dz\sqrt{-g}\left[\frac{i}{2}\bar{N}_1\Gamma^z\left(-iA_z^L\right)N_1 - \frac{i}{2}\left(iA_z^L\bar{N}_1\right)\Gamma^z N_1 + \left(L \leftrightarrow R, 1 \leftrightarrow 2\right)\right]$$

$$+\int_0^{z_m} dz\sqrt{-g}\left[-g_Y\bar{N}_1 X N_1 - g_Y\bar{N}_2 X^\dagger N_2\right]$$

$$g_{\pi N^l N^l} = \int_0^{z_m} dz\frac{1}{z^4}\left[f_\pi\left(f_{1L}^{l*}f_{1R}^l - f_{2L}^{l*}f_{2R}^l\right) - \frac{g_Y}{2v(z)zg_5^2}\partial_z\left(\frac{f_\pi}{z}\right)\left(f_{1L}^{l*}f_{2R}^l - f_{2L}^{l*}f_{1R}^l\right)\right]$$

Vector & Axial-vector Meson-Nucleon coupling

Vector & Axial-vector meson-nucleon couplings come from

1: Gauge coupling

2: Pauli term

$$\mathcal{L}_{\text{gauge}} = \int_0^{z_m} dz \sqrt{-g} \left[\frac{i}{2} \bar{N}_1 e_A^M \Gamma^A \left(-iA_M^L \right) N_1 - \frac{i}{2} \left(iA_M^L \bar{N}_1 \right) e_A^M \Gamma^A N_1 \right.$$

$$\left. + \frac{i}{2} \bar{N}_2 e_A^M \Gamma^A \left(-iA_M^R \right) N_2 - \frac{i}{2} \left(iA_M^R \bar{N}_2 \right) e_A^M \Gamma^A N_2 \right]$$

$$\supset \int_0^{z_m} dz \frac{1}{z^4} \left[\bar{N}_1 \gamma^\mu V_\mu N_1 + \bar{N}_2 \gamma^\mu V_\mu N_2 + \bar{N}_1 \gamma^\mu A_\mu N_1 - \bar{N}_2 \gamma^\mu A_\mu N_2 \right]$$

$$\mathcal{L}_{\text{Pauli}} = c\int_0^{z_m} dz\sqrt{-g}\left[i\bar{N}_1\Gamma^{MN}F^L_{MN}N_1 - i\bar{N}_2\Gamma^{MN}F^R_{MN}N_2 \right]$$

$$\to -c\int_0^{z_m} dz\,\frac{1}{z^3}\left[\bar{N}_{1L}\gamma^\mu\gamma^5\left(\partial_\mu V_z - \partial_z V_\mu\right)N_{1L} + \bar{N}_{1R}\gamma^\mu\gamma^5\left(\partial_\mu V_z - \partial_z V_\mu\right)N_{1R} - (1\leftrightarrow 2)\right]$$

$$-c\int_0^{z_m} dz\,\frac{1}{z^3}\left[\bar{N}_{1L}\gamma^\mu\gamma^5\left(\partial_\mu A_z - \partial_z A_\mu\right)N_{1L} + \bar{N}_{1R}\gamma^\mu\gamma^5\left(\partial_\mu A_z - \partial_z A_\mu\right)N_{1R} + (1\leftrightarrow 2)\right]$$

$$\Downarrow$$

$$g_{v^n N^l N^l} \equiv \int_0^{z_m} dz\,\frac{1}{z^4}\left[f_n^V + cz\partial_z f_n^V \right]\left[\left|f_{1L}^l\right|^2 + \left|f_{1R}^l\right|^2 \right]$$

$$g_{a^n N^l N^l} \equiv \int_0^{z_m} dz\,\frac{1}{z^4}\left[f_n^A + cz\partial_z f_n^A \right]\left[\left|f_{1L}^l\right|^2 + \left|f_{1R}^l\right|^2 \right]$$

$$g_{\partial\pi N^l N^l} \equiv -c\int_0^{z_m} dz\,\frac{1}{z^3}\,f_\pi\left[\left|f_{1L}^l\right|^2 + \left|f_{1R}^l\right|^2 \right] \Rightarrow g_{\partial\pi NN} = \frac{m_\pi}{2m_N}\,g_{\pi NN}\,(\text{GT})$$

Numerical Results

Data: $g_{\rho NN}$ = 4.2~6.5, $g_{\pi NN}$ ~ 13.6

z_m^{-1} (GeV)	g_Y	$g_{\rho NN}$	$g_{a_1 NN}$	$g_{\pi NN}$
0.6	26.5 ~ 26.9	−4.3 ~ −6.2	−8.2 ~ −10.5	−20.1 ~ −22.0
0.7	33.6 ~ 34.0	−5.1 ~ −6.2	−10.1 ~ −11.4	−19.8 ~ −20.7
0.8	38.6 ~ 40.2	−4.2 ~ −6.4	−10.0 ~ −13.1	−18.8 ~ −20.5
0.9	42.5 ~ 44.1	−5.1 ~ −6.5	−13.0 ~ −15.1	−19.8 ~ −20.9
1.0	39.0 ~ 43.8	−4.2 ~ −6.5	−13.7 ~ −17.6	−19.9 ~ −21.7

fitted prediction 50% dev.

Parameters: z_m, g_Y (free parameters), m_N=0.94GeV
M=2.34MeV, Σ=(311MeV)^3 ← m_π, f_π
$g_5=2\pi$ ← Matching to pQCD Erlich et al (2005)
c ← Nucleon g-2 Hong, Kim, Siwach & Yee (2007)

Numerical Results

Data: $g_{\rho NN} = 4.2 \sim 6.5$, $g_{\pi NN} \sim 13.6$

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fitted　　prediction　　50% dev.

Comparison to Skyrmion approach for baryons

Similar results: $g_{\rho NN} = 5.8$, $g_{\pi NN} \sim 7.46$

Hashimoto, Sakai & Sugimoto (2008)

Summary

- We have formulated meson-baryon couplings in 5D holographic QCD
- Spin $\frac{1}{2}$ baryon ⇔ 5D Dirac fermion
- πNN, ρNN, a_1NN couplings were computed
- In particular, a_1NN coupling is a prediction, our model can be tested by measuring this coupling

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Ways out to improve the results in our simplified model

- Deformation of the metric
- Anomalous dimension $\rightarrow$ Bulk mass correction
- Quantum gravity, Stringy effects

Backup Slides

ρ meson mass & its mode function

$$m_\rho \sim \frac{3}{4}\pi z_m^{-1},\ f^\rho(z) \sim \frac{zJ_1(m_\rho z)}{\sqrt{\int_0^{z_m} dzz\left[J_1(m_\rho z)\right]^2}}$$

a_1 meson mass & its mode function

Mode equation cannot be solved analytically due to a z-dependent mass

Approximation: brane localized mass@ IR brane

$$\tan\left(m_{a_1} z_m - \frac{3}{4}\pi\right) \sim \frac{g_5^2\sigma^2}{2m_{a_1}} z_m^4,\ f^{a_1}(z) \sim \frac{zJ_1(m_{a_1} z)}{\sqrt{\int_0^{z_m} dzz\left[J_1(m_{a_1} z)\right]^2}}$$

Pion mode function

Note that in the unitary gauge, if $z\partial_z\left(\frac{A_z}{z}\right)-2\sqrt{2}\frac{v^2g_5^2}{z^2}P=0$

its orthogonal combination of A_z & P are massless ⇒ Pion

$$\mathcal{L}(A_z,P)=\frac{z^4}{2g_5^2}\left(\partial_\mu A_z\right)^2+\frac{z^8}{8g_5^2v^2}\left[\partial_z\left(\frac{\partial_\mu A_z}{z}\right)\right]^2-\frac{v^2z^2}{8g_5^4}\left[\partial_z\left(\frac{z^3}{v^2}\partial_z\left(\frac{A_z}{z}\right)\right)-4g_5^2A_z\right]^2$$

Pion mode function is defined from $A_z = f_\pi(z)\pi(x)$ and obtained by

$$\partial_z\left(\frac{z^3}{v^2}\partial_z\left(\frac{f_\pi}{z}\right)\right)-4g_5^2f_\pi=0,\ \ 1=\int_0^{z_m}dz\left[\frac{1}{2g_5^2z}f_\pi^2+\frac{z^3}{8v^2g_5^4}\left(\partial_z\left(\frac{f_\pi}{z}\right)\right)^2\right]$$