

Meson-Nucleon Coupling from AdS/QCD

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Plan

1: Introduction

2: Meson sector (review)

3: Meson-Nucleon coupling
(review & our results)

4: Summary

Introduction

AdS/CFT correspondence has opened a new avenue to study strongly coupled gauge theories

Application to QCD and hadron physics

Top down: String theory \rightarrow QCD

Sakai & Sugimoto (2005)

Bottom up: QCD \rightarrow 5D holographic model

Erlich, Katz, Son & Stephanov (2005)

Da Lord & Pomarol (2005)

Introduction

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\Rightarrow 5D model of Meson sector
(vector, axial-vector meson masses,
decay consts, $g_{\rho \pi \pi}, \dots$)

As for baryons, two approaches are known

1: Skyrmion

Hashimoto, Sakai & Sugimoto (2008)
and many papers

2: Bulk fermion

Hong, Inami & Yee (2007)[spin 1/2]
Ahn, Hong, Park & Siwach (2009)[spin 3/2]

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 π NN coupling

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Our work

By extending the model of Hong, Inami & Yee,
we calculated **(Axial-)Vector Meson-Nucleon
couplings** using the 5D Holographic QCD

Meson sector

"QCD and a Holographic Model of Hadrons"
Erlich, Katz, Son and Stephanov
PRL95 261602 (2005)

"Chiral Symmetry Breaking from Five Dimensional Spaces"
Da Lord and Pomarol
NPB721 79 (2005)

AdS/CFT dictionary

4D

5D

Global symmetry \Leftrightarrow Gauge symmetry

Operator \mathcal{O} \Leftrightarrow 5D field

Dimension[\mathcal{O}] \Leftrightarrow Bulk mass

$1/N_c$ \Leftrightarrow g_5^2

Resonance \Leftrightarrow KK modes

Following this dictionary,
we guess a holographic model of QCD

5D $SU(N_f)_L \times SU(N_f)_R$ gauge theory on a slice of AdS_5

$$ds^2 = \frac{1}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \quad \varepsilon \leq z \leq z_m (= \Lambda_{QCD}^{-1})$$

(Axial-)Vector meson (spin 1) \Leftrightarrow (Axial-)Vector gauge field

4D: $\mathcal{O}(x)$	5D: $\phi(x, z)$	$SU(N_f)_L \times SU(N_f)_R$	p	Δ	$M_5^2 = (\Delta - p)(\Delta + p - 4)$
$\bar{q}_L \gamma^\mu t^a q_L$	L_μ^a	$(adj, 1)$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$	R_μ^a	$(1, adj)$	1	3	0
$\bar{q}_R^\alpha q_L^\beta$	$z^{-1} X^{\alpha\beta}$	(\bar{N}_f, N_f)	0	3	-3

$$S_{\text{meson}} = \int d^5x \sqrt{-g} \text{Tr} \left[-\frac{1}{2g_5^2} (L_{MN}^2 + R_{MN}^2) + |D_M X|^2 - M_5^2 |X|^2 \right]$$

Bulk scalar field "X" Source of chiral symmetry breaking

Classical solution

$$X(z) = Mz + \Sigma z^3$$

"explicit breaking"

Quark Mass

UV B.C.

$$\left. \frac{X(z)}{z} \right|_{z=\varepsilon} = M$$

"spontaneous breaking"

Chiral condensate

$$\begin{aligned} \langle \bar{q}q \rangle &= \frac{\partial}{\partial M} \left\langle \exp \left[i \int d^4x M \bar{q}q \right] \right\rangle \Big|_{M=0} \\ &= \frac{\delta}{\delta M} S_5[X] \Big|_{X=X_{cl}, M=0, z=\varepsilon} \sim \Sigma \end{aligned}$$

Defining $V_M \equiv \frac{1}{2}(L_M + R_M)$, $A_M \equiv \frac{1}{2}(L_M - R_M)$ and adding

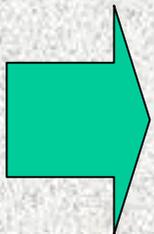
$$\mathcal{L}_{gf} = -\frac{1}{2\xi_V g_5^2 z} \left[\partial_\mu V^\mu - \xi_V z \partial_z \left(\frac{V_z}{z} \right) \right]^2 - \frac{1}{2\xi_A g_5^2 z} \left[\partial_\mu A^\mu - \xi_A z \partial_z \left(\frac{A_z}{z} \right) + 2\sqrt{2} g_5^2 \frac{\xi_A}{z^2} \langle X \rangle^2 P \right]^2$$

leads to the quadratic Lagrangian in the unitary gauge

$$\mathcal{L}_{quadratic} = -\frac{1}{4g_5^2 z} V_\mu^a \left[-\eta^{\mu\nu} \partial^2 + \partial^\mu \partial^\nu + \eta^{\mu\nu} z \partial_z \left(\frac{1}{z} \right) \partial_z \right] V_\nu^a \quad X = \langle X \rangle e^{iP}$$

$$-\frac{1}{4g_5^2 z} A_\mu^a \left[-\eta^{\mu\nu} \partial^2 + \partial^\mu \partial^\nu + \eta^{\mu\nu} z \partial_z \left(\frac{1}{z} \right) \partial_z \right] A_\nu^a + \frac{\langle X \rangle^2}{2z^3} (\partial_\mu P^a - A_\mu^a)^2$$

Mode equations and boundary conditions



$$0 = \left[m_n^2 + z \partial_z \left(\frac{1}{z} \right) \partial_z \right] f_n^V, \quad f_n^V(\varepsilon) = \partial_z f_n^V(z_m) = 0$$

$$0 = \left[m_n^2 + z \partial_z \left(\frac{1}{z} \right) \partial_z - \frac{2g_5^2 \langle X \rangle^2}{z^2} \right] f_n^A, \quad f_n^A(\varepsilon) = \partial_z f_n^A(z_m) = 0$$

Meson-Nucleon coupling

"Baryons in AdS/QCD"

Hong, Inami and Yee, PLB646 165 (2007)

"Meson-Nucleon Coupling from AdS/QCD"

N.M. and Motoi Tachibana

arXiv:0904.3816(accepted in EPJC)

Spin $\frac{1}{2}$ Baryon \Leftrightarrow Spin $\frac{1}{2}$ Dirac fermion

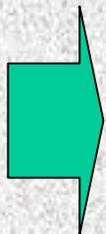
Hong, Inami & Yee (2007)

$$S_{Baryon} = \int d^5x \sqrt{-g} \left[i\bar{N}_1 e_A^M \Gamma^A D_M N_1 + i\bar{N}_2 e_A^M \Gamma^A D_M N_2 - \frac{5}{2} \bar{N}_1 N_1 + \frac{5}{2} \bar{N}_2 N_2 \right]$$

$$D_\mu = \partial_\mu + \frac{1}{2z} \Gamma_z \Gamma_\mu - iL_\mu, \quad D_z = \partial_\mu - iL_z, \quad m_5^2 = (\Delta - 2)^2 = \left(\frac{9}{2} - 2 \right)^2$$

To incorporate chiral symmetry breaking, the following Yukawa coupling is introduced

$$S_{Yukawa} = \int d^5x \sqrt{-g} \left[-g_Y \bar{N}_1 X N_1 - g_Y \bar{N}_2 X^\dagger N_2 \right]$$



$$\begin{pmatrix} \partial_z - \frac{\Delta}{z} & -\frac{g_Y \langle X \rangle}{z} \\ -\frac{g_Y \langle X \rangle^\dagger}{z} & \partial_z - \frac{4-\Delta}{z} \end{pmatrix} \begin{pmatrix} f_{1L}^n \\ f_{2L}^n \end{pmatrix} = -m_n \begin{pmatrix} f_{1R}^n \\ f_{2R}^n \end{pmatrix}, \quad \begin{pmatrix} \partial_z - \frac{4-\Delta}{z} & \frac{g_Y \langle X \rangle}{z} \\ \frac{g_Y \langle X \rangle^\dagger}{z} & \partial_z - \frac{\Delta}{z} \end{pmatrix} \begin{pmatrix} f_{1R}^n \\ f_{2R}^n \end{pmatrix} = m_n \begin{pmatrix} f_{1L}^n \\ f_{2L}^n \end{pmatrix}$$

$f_{1R}^n(z_m) = f_{2L}^n(z_m) = f_{1L}^n(\varepsilon) = f_{2R}^n(\varepsilon) = 0$, Other B.C. from EOM

π NN coupling

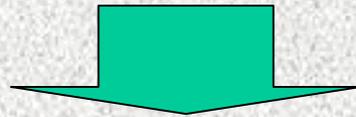
π NN coupling is generated from

1: Gauge coupling (5th component)

2: Yukawa coupling

3: **Pauli term** (through Goldberger-Treiman relation)

$$\mathcal{L}_{\pi NN} = \int_0^{z_m} dz \sqrt{-g} \left[\frac{i}{2} \bar{N}_1 \Gamma^z (-iA_z^L) N_1 - \frac{i}{2} (iA_z^L \bar{N}_1) \Gamma^z N_1 + (L \leftrightarrow R, 1 \leftrightarrow 2) \right] \\ + \int_0^{z_m} dz \sqrt{-g} \left[-g_Y \bar{N}_1 X N_1 - g_Y \bar{N}_2 X^\dagger N_2 \right]$$



$$g_{\pi N^l N^l} = \int_0^{z_m} dz \frac{1}{z^4} \left[f_\pi \left(f_{1L}^{l*} f_{1R}^l - f_{2L}^{l*} f_{2R}^l \right) - \frac{g_Y}{2v(z) z g_5^2} \partial_z \left(\frac{f_\pi}{z} \right) \left(f_{1L}^{l*} f_{2R}^l - f_{2L}^{l*} f_{1R}^l \right) \right]$$

Vector & Axial-vector Meson-Nucleon coupling

Vector & Axial-vector meson-nucleon couplings come from

1: Gauge coupling

2: Pauli term

$$\begin{aligned}\mathcal{L}_{\text{gauge}} &= \int_0^{z_m} dz \sqrt{-g} \left[\frac{i}{2} \bar{N}_1 e_A^M \Gamma^A (-iA_M^L) N_1 - \frac{i}{2} (iA_M^L \bar{N}_1) e_A^M \Gamma^A N_1 \right. \\ &\quad \left. + \frac{i}{2} \bar{N}_2 e_A^M \Gamma^A (-iA_M^R) N_2 - \frac{i}{2} (iA_M^R \bar{N}_2) e_A^M \Gamma^A N_2 \right] \\ &\supset \int_0^{z_m} dz \frac{1}{z^4} \left[\bar{N}_1 \gamma^\mu V_\mu N_1 + \bar{N}_2 \gamma^\mu V_\mu N_2 + \bar{N}_1 \gamma^\mu A_\mu N_1 - \bar{N}_2 \gamma^\mu A_\mu N_2 \right]\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{\text{Pauli}} &= c \int_0^{z_m} dz \sqrt{-g} \left[i\bar{N}_1 \Gamma^{MN} F_{MN}^L N_1 - i\bar{N}_2 \Gamma^{MN} F_{MN}^R N_2 \right] \\
&\rightarrow -c \int_0^{z_m} dz \frac{1}{z^3} \left[\bar{N}_{1L} \gamma^\mu \gamma^5 (\partial_\mu V_z - \partial_z V_\mu) N_{1L} + \bar{N}_{1R} \gamma^\mu \gamma^5 (\partial_\mu V_z - \partial_z V_\mu) N_{1R} - (1 \leftrightarrow 2) \right] \\
&\quad -c \int_0^{z_m} dz \frac{1}{z^3} \left[\bar{N}_{1L} \gamma^\mu \gamma^5 (\partial_\mu A_z - \partial_z A_\mu) N_{1L} + \bar{N}_{1R} \gamma^\mu \gamma^5 (\partial_\mu A_z - \partial_z A_\mu) N_{1R} + (1 \leftrightarrow 2) \right]
\end{aligned}$$



$$\begin{aligned}
g_{v^n N^l N^l} &\equiv \int_0^{z_m} dz \frac{1}{z^4} \left[f_n^V + cz \partial_z f_n^V \right] \left[|f_{1L}^l|^2 + |f_{1R}^l|^2 \right] \\
g_{a^n N^l N^l} &\equiv \int_0^{z_m} dz \frac{1}{z^4} \left[f_n^A + cz \partial_z f_n^A \right] \left[|f_{1L}^l|^2 + |f_{1R}^l|^2 \right] \\
g_{\partial\pi N^l N^l} &\equiv -c \int_0^{z_m} dz \frac{1}{z^3} f_\pi \left[|f_{1L}^l|^2 + |f_{1R}^l|^2 \right] \Rightarrow g_{\partial\pi NN} = \frac{m_\pi}{2m_N} g_{\pi NN} \text{ (GT)}
\end{aligned}$$

Numerical Results

Data: $g_{\rho NN} = 4.2 \sim 6.5$, $g_{\pi NN} \sim 13.6$

$z_m^{-1} (GeV)$	g_Y	$g_{\rho NN}$	$g_{a_1 NN}$	$g_{\pi NN}$
0.6	26.5 ~ 26.9	-4.3 ~ -6.2	-8.2 ~ -10.5	-20.1 ~ -22.0
0.7	33.6 ~ 34.0	-5.1 ~ -6.2	-10.1 ~ -11.4	-19.8 ~ -20.7
0.8	38.6 ~ 40.2	-4.2 ~ -6.4	-10.0 ~ -13.1	-18.8 ~ -20.5
0.9	42.5 ~ 44.1	-5.1 ~ -6.5	-13.0 ~ -15.1	-19.8 ~ -20.9
1.0	39.0 ~ 43.8	-4.2 ~ -6.5	-13.7 ~ -17.6	-19.9 ~ -21.7

fitted

prediction

50% dev.

Parameters: z_m, g_Y (free parameters), $m_N = 0.94 GeV$

$M = 2.34 MeV$, $\Sigma = (311 MeV)^3 \leftarrow m_\pi, f_\pi$

$g_5 = 2\pi \leftarrow$ Matching to pQCD Erlich et al (2005)

$c \leftarrow$ Nucleon g_2 Hong, Kim, Siwach & Yee (2007)

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fitted

prediction

50% dev.

Comparison to Skyrme approach for baryons

Similar results: $g_{\rho NN} = 5.8$, $g_{\pi NN} \sim 7.46$

Hashimoto, Sakai & Sugimoto (2008)

Summary

- We have formulated meson-baryon couplings in 5D holographic QCD
- Spin $\frac{1}{2}$ baryon \Leftrightarrow 5D Dirac fermion
- πNN , ρNN , $a_1 NN$ couplings were computed
- In particular, $a_1 NN$ coupling is a prediction, our model can be tested by measuring this coupling

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- We have formulated meson-baryon couplings in 5D holographic QCD
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Ways out to improve the results in our simplified model

- Deformation of the metric
- Anomalous dimension \rightarrow Bulk mass correction
- Quantum gravity, Stringy effects

Backup Slides

ρ meson mass & its mode function

$$m_\rho \sim \frac{3}{4} \pi z_m^{-1}, f^\rho(z) \sim \frac{z J_1(m_\rho z)}{\sqrt{\int_0^{z_m} dz z [J_1(m_\rho z)]^2}}$$

a_1 meson mass & its mode function

Mode equation cannot be solved analytically
due to a z -dependent mass

Approximation: brane localized mass @ IR brane

$$\tan\left(m_{a_1} z_m - \frac{3}{4} \pi\right) \sim \frac{g_5^2 \sigma^2}{2m_{a_1}} z_m^4, f^{a_1}(z) \sim \frac{z J_1(m_{a_1} z)}{\sqrt{\int_0^{z_m} dz z [J_1(m_{a_1} z)]^2}}$$

Pion mode function

Note that in the unitary gauge, if $z \partial_z \left(\frac{A_z}{z} \right) - 2\sqrt{2} \frac{v^2 g_5^2}{z^2} P = 0$

its orthogonal combination of A_z & P are massless \Rightarrow Pion

$$\mathcal{L}(A_z, P) = \frac{z^4}{2g_5^2} (\partial_\mu A_z)^2 + \frac{z^8}{8g_5^2 v^2} \left[\partial_z \left(\frac{\partial_\mu A_z}{z} \right) \right]^2 - \frac{v^2 z^2}{8g_5^4} \left[\partial_z \left(\frac{z^3}{v^2} \partial_z \left(\frac{A_z}{z} \right) \right) - 4g_5^2 A_z \right]^2$$

Pion mode function is defined from $A_z = f_\pi(z) \pi(x)$ and obtained by

$$\partial_z \left(\frac{z^3}{v^2} \partial_z \left(\frac{f_\pi}{z} \right) \right) - 4g_5^2 f_\pi = 0, \quad 1 = \int_0^{z_m} dz \left[\frac{1}{2g_5^2 z} f_\pi^2 + \frac{z^3}{8v^2 g_5^4} \left(\partial_z \left(\frac{f_\pi}{z} \right) \right)^2 \right]$$