Vortex-type BPS solitons in Mass-deformed ABJM model

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contents

1. Introduction

Effective field theory on D-branes $\Rightarrow$ super Yang-Mills (SYM) theory

★ BPS solitons in SYM: nonperturbative object

Recent development on effective field theories on multiple M2's

$\Rightarrow$ ABJM model [Aharony-Bergman-Jafferis-Maldacena]

- U(N)$\times$U(N) or SU(N)$\times$SU(N) gauge symmetry
- $\mathcal{N} = 6$ superconformal Chern-Simons theory
  only $\mathcal{N} = 6$: M2's on $\mathbb{C}^4/\mathbb{Z}_k$ ($\mathcal{N} = 8$ in $k = 1, 2$?)

★ BPS objects in ABJM: some M-brane configration

★ BPS solitons in ABJM model (and BLG model)

1. domain wall (Basu-Harvey equation)
   [Krishnan-Maccaferi, Terashima, Hanaki-Lin, etc.]
   Configration of scalars is important.
2. vortex (point particle) [Hosomichi-Lee-Lee, Kim-Lee, etc.]
   Both of scalars and gauge fields are important.
3. monopole-instanton [Hosomichi-Lee-Lee-Lee-Park-Yi]
   Configration of gauge fields is important.

Here we study BPS vortex-type solitons in ABJM model with SUSY-preserving mass deformation. We find that

- half-BPS: YM-type vortex equation (not CS-type)
- lower-BPS: CS-type vortex equation appears.

2. ABJM model and SUSY-preserving mass deformation

- ABJM model [ABJM, Benna-Klebanov-Klose-Smedbäck]

$$\mathcal{L}_{\rm ABJM} = {\rm Tr}\Bigg[\frac{k}{4\pi}\epsilon^{\mu\nu\rho}\left(A_\mu\partial_\nu A_\rho + \frac{2}{3}iA_\mu A_\nu A_\rho\right) - \frac{k}{4\pi}\epsilon^{\mu\nu\rho}\left(\hat{A}_\mu\partial_\nu\hat{A}_\rho + \frac{2}{3}i\hat{A}_\mu\hat{A}_\nu\hat{A}_\rho\right) - D^\mu Y_A^\dagger D_\mu Y^A + i\psi^{\dagger A}\gamma^\mu D_\mu\psi_A - V_{\rm ferm} - V_0\Bigg].$$

Y^A, ψ_A: $\mathbf{4}$ and $\bar{\mathbf{4}}$ repr. of SU(4)$_R$ = SO(6)$_R$ $\Rightarrow$ $\mathcal{N}=6$

$(\boldsymbol{N}, \bar{\boldsymbol{N}})$ repr. of U(N)×U(N) or SU(N)×SU(N)

Yukawa coupling (the form of $Y^2\psi^2$)

$$V_{\rm ferm} = \frac{2\pi i}{k}\,{\rm Tr}\Big[Y_A^\dagger Y^A \psi^{\dagger B}\psi_B - Y^A Y_A^\dagger \psi_B \psi^{\dagger B} + 2Y^A Y_B^\dagger \psi_A \psi^{\dagger B} - 2Y_A^\dagger Y^B \psi^{\dagger A}\psi_B - \epsilon^{ABCD} Y_A^\dagger \psi_B Y_C^\dagger \psi_D + \epsilon_{ABCD} Y^A \psi^{\dagger B} Y^C \psi^{\dagger D}\Big],$$

bosonic potential (sextic, Y^6-type)

$$V_0 = -\frac{4\pi^2}{3k^2}\,{\rm Tr}\Big[Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger + Y_A^\dagger Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C + 4Y^A Y_B^\dagger Y^C Y_A^\dagger Y^B Y_C^\dagger - 6Y^A Y_B^\dagger Y^B Y_A^\dagger Y^C Y_C^\dagger\Big].$$

- SU(2)×SU(2) gauge group $\Rightarrow$ BLG model ($\mathcal{N}=8$)

• SUSY-preserving mass deformation

[Hosomichi-Lee^3-Park, Gomis-R.-Gomez-Raamsdonk-Verlinde]

$$\Delta\mathcal{L} = -\mathrm{Tr}\Big[Y^A (M^2)_A{}^B Y_B^\dagger + i\psi^{\dagger A} M_A{}^B \psi_B + \frac{4\pi}{k} Y^A Y_A^\dagger Y^B M_B{}^C Y_C^\dagger - \frac{4\pi}{k} Y_A^\dagger Y^A Y_B^\dagger Y^C M_C{}^B\Big],$$

where the mass matrix $M_A{}^B$ is given by

$$M_A{}^B = \mu\,\mathrm{diag}(++--), \quad \mu(>0):\ \text{mass parameter.}$$

- $\mathcal{N}=6$ supersymmetry is preserved.
- R-symmetry is broken from SU(4) to U(1)×SU(2)×SU(2).
- μ can be regarded as (electric or magnetic) FI parameter.

4. Half-BPS vortex-like solution

- Killing spinor equation

$$\delta\psi_A = M_A{}^B \omega_{BC} Y^C - \gamma^\mu \omega_{AB} D_\mu Y^B + \frac{2\pi}{k}\left[-\omega_{AB}\left(Y^C Y_C^\dagger Y^B - Y^B Y_C^\dagger Y^C\right) + 2\omega_{BC} Y^B Y_A^\dagger Y^C\right].$$

The condition of the SUSY parameter ω_{AB} is

$$(\omega_{AB})^* = \omega^{AB} = -\frac{1}{2}\epsilon^{ABCD}\omega_{CD}, \quad \gamma^0 \omega_{AB} = \pm i \omega_{AB},$$

where the double sign is common among ω_{12}, ω_{13} and ω_{14}.

- half-BPS equation $\beta^{A}{}_{B}{}^{C} = \frac{2\pi}{k}(Y^A Y_B^\dagger Y^C - Y^C Y_B^\dagger Y^A).$

$$(D_1 \pm iD_2)Y^1 = 0, \qquad D_i Y^2 = D_i Y^3 = D_i Y^4 = 0,$$

$$D_0 Y^1 \mp i(\beta^{2}{}_{2}{}^{1} + \mu Y^1) = 0, \qquad D_0 Y^2 \pm i(\beta^{1}{}_{1}{}^{2} + \mu Y^2) = 0,$$

$$D_0 Y^3 \pm i\beta^{1}{}_{1}{}^{3} = 0, \qquad D_0 Y^4 \pm i\beta^{1}{}_{1}{}^{4} = 0,$$

$$\beta^{3}{}_{3}{}^{1} = \beta^{4}{}_{4}{}^{1} = \beta^{2}{}_{2}{}^{1} + \mu Y^1, \qquad \beta^{4}{}_{4}{}^{3} = \mu Y^3, \qquad \beta^{3}{}_{3}{}^{4} = \mu Y^4,$$

$$\beta^{3}{}_{3}{}^{2} = \beta^{4}{}_{4}{}^{2} = \beta^{2}{}_{2}{}^{3} = \beta^{2}{}_{2}{}^{4} = 0,$$

$$\beta^{A}{}_{B}{}^{C} = 0 \qquad (A \neq B \neq C \neq A).$$

- Bogomolnyi bound

$$E \geq \frac{\mu}{3}|Q + 2R_{12}|.$$

Q: U(1) charge(= magnetic flux), R_{12}: SU(2) R-charge

• solving the BPS equation

From ${\beta^4}_4{}^3 = \mu Y^3$ and ${\beta^3}_3{}^4 = \mu Y^4$, we have

$$Y^3,\ Y^4 : \begin{cases} \text{Both are zero.} \Rightarrow \text{nontrivial solution} \\ \text{Both are nonzero.} \Rightarrow \text{trivial solution} \end{cases}$$

Then we obtain the simplified half-BPS equation

$$\begin{aligned} &(D_1 \pm i D_2) Y^1 = 0, && D_0 Y^1 = 0, \\ &D_i Y^2 = 0, && D_0 Y^2 \pm i({\beta^1}_1{}^2 + \mu Y^2) = 0, \\ &{\beta^2}_2{}^1 + \mu Y^1 = 0, && Y^3 = Y^4 = 0, \end{aligned}$$

and Gauss' law

$$B = \frac{2\pi i}{k}(Y^2 D_0 Y_2^\dagger - D_0 Y^2 Y_2^\dagger), \quad \hat{B} = -\frac{2\pi i}{k}(Y_2^\dagger D_0 Y^2 - D_0 Y_2^\dagger Y^2).$$

U(2)×U(2) case (similar to BLG case [Kim-Lee])

From the constraint $\beta^2{}_2{}^1 + \mu Y^1 = 0$, we have

$$Y^1 = \sqrt{\frac{k\mu}{2\pi}} \begin{pmatrix} 0 & f \\ 0 & 0 \end{pmatrix}, \qquad Y^2 = \sqrt{\frac{k\mu}{2\pi}} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},$$

where f is the undetermined function. Gauss' law becomes

$$B = \hat{B} = \pm 2\mu^2 \begin{pmatrix} 0 & 0 \\ 0 & 1 - |f|^2 \end{pmatrix}, \quad \Rightarrow \quad A_i = \hat{A}_i = \begin{pmatrix} 0 & 0 \\ 0 & V_i \end{pmatrix}.$$

Then we obtain ANO vortex equation [Abrikosov, Nielsen-Olsen]

$$(D_1 \pm i D_2) f = 0, \quad B_V = \pm 2\mu^2 (1 - |f|^2),$$

where $D_i f = (\partial_i - i V_i) f$. ($f$ is fundamental.)

U(N)×U(N) case

From the constraint $\beta^2{}_2{}^1 + \mu Y^1 = 0$, we have

$$Y^1 = \sqrt{\frac{k\mu}{2\pi}} \begin{pmatrix} \mathbf{0} & F \\ \mathbf{0} & \mathbf{0} \end{pmatrix}, \qquad Y^2 = \sqrt{\frac{k\mu}{2\pi}} \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I_M \end{pmatrix},$$

where F is the $(N-M) \times M$ matrix. Gauss' law becomes

$$B = \hat{B} = \pm 2\mu^2 \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I_M - F^\dagger F \end{pmatrix}, \quad \Rightarrow \quad A_i = \hat{A}_i = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & V_i \end{pmatrix}.$$

Then we obtain non-Abelian vortex equation [TITech group, etc.]

$$(D_1 \pm i D_2) F = 0, \quad B_V = \pm 2\mu^2 (I_M - F^\dagger F),$$

where $D_i F = \partial_i F - i F V_i$. (U($M$) gauge & U($N-M$) flavor)

• comments on lower-BPS case

$\frac{1}{6}$-BPS equation ($\gamma^0\omega_{12} = \pm i\omega_{12}$, $\omega_{13} = \omega_{14} = 0$) with Y^3, $Y^4 = 0$

$$(D_1 \pm iD_2)Y^a = 0, \quad D_0 Y^a \pm i(\beta^b{}_b{}^a + \mu Y^a) = 0, \quad a, b = 1, 2.$$

For U(2)×U(2) case, we use the ansatz

$$Y^1 = \sqrt{\frac{k\mu}{2\pi}} \begin{pmatrix} 0 & f \\ 0 & 0 \end{pmatrix}, \qquad Y^2 = \sqrt{\frac{k\mu}{2\pi}} \begin{pmatrix} 0 & 0 \\ 0 & f \end{pmatrix}.$$

Finally, the equation for f becomes abelian self-dual Chern-Simons vortex equation. [Hong-Kim-Pac, Jackiw-Weinberg, etc.]

$$(D_1 \pm iD_2)f = 0, \quad B_V = \pm 2\mu^2 |f|^2 (1 - |f|^2).$$

Non-abelian extension is possible for U(N)×U(N) case.

5. Summary and discussion

Summary

1. We consider the vortex-like BPS equation of ABJM model with SUSY-preserving mass deformation. In half-BPS case, we obtain ANO vortex equation and non-abelian vortex equation which has U(M) gauge symmetry and U($N - M$) flavor symmetry.

2. In lower-BPS case, we find that the self-dual Chern-Simons vortex equation is contained as a special case.

discussion

- More general solution, classification and moduli space
  - vacuum moduli: [Nastase-Papageorgakis-Ramgoolam]
  - operator level (massless case): [S.-Jabbari-Simón]
- Brane configuration corresponding to BPS solitons
  - half-BPS vortices = D0-branes? [Auzzi-Kumar]
  - polarized brane configuration:

    [Bena, Hanaki-Lin, Arai-Montonen-Sasaki]
- BPS solitons in nonrelativistic ABJM

  [Nakayama-Sakaguchi-Yoshida, Lee-Lee-Lee]
  - abelian vortices (half-BPS): [Kawai-Sasaki]