CFT Duals for Extreme Black Holes

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based on

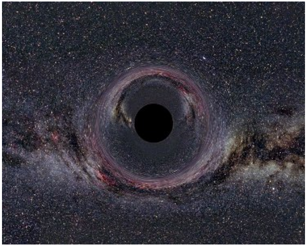
- arXiv:0811.4393 with T. Hartman, K. Murata and A. Strominger
- arXiv:0902.1001 with G. Compere and K. Murata

July 7, 2009 @ YITP

Introduction: How to interpret black hole entropy?

Black hole has an entropy (Bekenstein '73, Hawking '74)

$$S_{BH} = \frac{\text{Area(Horizon)}}{4}$$



We don't know inside the black hole
$\Rightarrow$ Is there fundamental degrees of freedom inside it?

- it is mysterious that the entropy is proportional to the area of the black hole, not its volume
- its (microscopic) origin remains to be fully understood

Various approach

For specific case, there are several explanations

- Counting BPS states (SUSY BH) (Strominger-Vafa '96)
- Attractor mechanism (Extremal) (Ferrrara-Kallosh-Strominger '95, Sen '05 Goldstein-Iizuka-Jena-Trivedi '05)
- AdS_3/CFT_2 (BTZ) (Strominger '97)
- Near horizon conformal symmetry (Carlip '98 '99)
- OSV conjecture $Z_{BH} = |Z_{top}|^2$ (Ooguri-Strominger-Vafa '04)
- Entanglement entropy (Extremal) (Azeyanagi-TN-Takayanagi '07)

$$\vdots$$

Remarkably, the extremality plays an important role even though the approaches are quite different

The Kerr/CFT correspondence

Kerr/CFT correspondence (Guica-Hartman-Song-Strominger '08)

Extreme Kerr black hole in 4D $\quad \Leftrightarrow \quad$ 2D (chiral) CFT

Key points

1. Extract "Virasoro algebra" from diffeomorphism
2. Evaluate the central charge c of this Virasoro algebra
3. Define the dual temperature T_L

Roughly speaking

$$\text{Kerr BH with ang. mom. } J \quad \Leftrightarrow \quad \text{state } |J\rangle \text{ in CFT}$$

Purpose

The statistical entropy in CFT agrees with the black hole entropy

$$S_{CFT} = \frac{\pi^2}{3} c T_L = S_{BH}$$

We can obtain the (in a sence) microscopic interpretation of the black hole entropy

Can we apply this strategy to more general black holes ?

$\Rightarrow$ yes

we can construct the dual CFT thanks to the extremality

Generalization to Kerr-Newman-(A)dS black hole

To illustrate the detail, let us consider the Kerr-Newman-(A)dS black hole

This is the most general solution in

4D Einstein-Maxwell theory

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R + \frac{6}{\ell^2} - \frac{1}{4}F^2\right)$$

Notice that

- it becomes setting the charges to zero the Kerr black hole
- Also it becomes the Reissner-Nordstrom black hole setting the angular momentum to zero

Kerr-Newman-(A)dS black hole

The metric is given by (Caldarelli-Cognola-Klemm '99)

$$ds^2 = -\frac{\Delta_r}{\rho^2}\left(d\hat{t} - \frac{a}{\Xi}\sin^2\theta d\hat{\phi}\right)^2 + \frac{\rho^2}{\Delta_r}d\hat{r}^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta}{\rho^2}\sin^2\theta\left(a d\hat{t} - \frac{\hat{r}^2+a^2}{\Xi}d\hat{\phi}\right)^2$$

with

$$\Delta_r = (\hat{r}^2+a^2)\left(1+\frac{\hat{r}^2}{\ell^2}\right) - 2M\hat{r} + q^2 , \qquad \Delta_\theta = 1 - \frac{a^2}{\ell^2}\cos^2\theta ,$$

$$\rho^2 = \hat{r}^2 + a^2\cos^2\theta , \qquad \Xi = 1 - \frac{a^2}{\ell^2} , \qquad q^2 = q_e^2 + q_m^2$$

Extreme limit

In the extreme limit ($T_H \to 0$), the inner and outer horizons degenerate to a single horizon at r_+

The extremality condition

$$a^2 = \frac{r_+^2(1 + 3r_+^2/\ell^2) - q^2}{1 - r_+^2/\ell^2}$$

$$M = \frac{r_+[(1 + r_+^2/\ell^2)^2 - q^2/\ell^2]}{1 - r_+^2/\ell^2}$$

Entropy at extremality

$$S(T_H = 0) = \frac{\pi(2r_+^4/\ell^2 + 2r_+^2 - q^2)}{1 - 2r_+^2/\ell^2 - 3r_+^4/\ell^4 + q^2/\ell^2}$$

Near horizon limit

To take the near horizon limit, we introduce new coordinates (Bardeen-Horowitz '99)

$$\hat{r} = r_+ + \epsilon r_0 r \ , \qquad \hat{t} = t r_0/\epsilon \ , \qquad \hat{\phi} = \phi + \Omega_H \frac{t r_0}{\epsilon}$$

In the limit of $\epsilon \to 0$

Near horizon metric

$$ds^2 = \Gamma(\theta)\left[-r^2 dt^2 + \frac{dr^2}{r^2} + \alpha(\theta) d\theta^2\right] + \gamma(\theta)(d\phi + k r dt)^2$$

where

$$\Gamma(\theta) = \frac{\rho_+^2 r_0^2}{r_+^2 + a^2} \ , \quad \alpha(\theta) = \frac{r_+^2 + a^2}{\Delta_\theta r_0^2} \ , \quad \gamma(\theta) = \frac{\Delta_\theta (r_+^2 + a^2)^2 \sin^2\theta}{\rho_+^2 \Xi^2}$$

and we have defined

$$\rho_+^2 = r_+^2 + a^2 \cos^2\theta \ , \quad r_0^2 = \frac{(r_+^2 + a^2)(1 - r_+^2/\ell^2)}{1 + 6r_+^2/\ell^2 - 3r_+^4/\ell^4 - q^2/\ell^2} \ , \quad k = \frac{2 a r_+ \Xi r_0^2}{(r_+^2 + a^2)^2}$$

Isometry

- The isometry is $U(1) \times SL(2, R)$ ($U(1)$: ϕ, $SL(2, R)$: AdS$_2$ part)
- We will calculate the central charge for this general form for simplicity
- Similar form appears as the near horizon limit of the extreme black hole in the fairly general gravity theory

Asymptotic Symmetry Group and Boundary Conditions

Under the diffeomorphism ξ

$$g_{\mu\nu} \quad \rightarrow g_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

This changes the form of the metric at spatial infinity

The asymptotic symmetry group (ASG) of a spacetime is

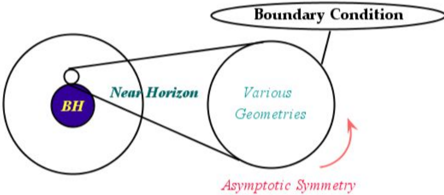
- A symmetry which preserves the boundary conditions in the diffeomorphism
- A part of the diffeomorphism becomes ASG

Sketch of ASG

The boundary condition determines the family of the geometries

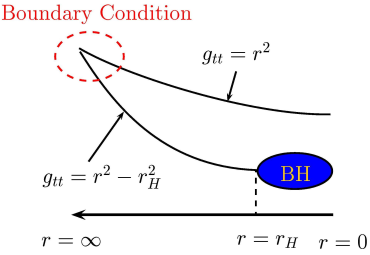
$$\mathcal{L}_\xi g \sim 0 \quad \text{up to B.C.}$$

ξ : Asymptotic Symmetry Group



We require

- ASG includes the Virasoro algebra (not too strong)
- Conserved
  charge is finite (not too week)



How to choose boundary conditions?

For the general form

$$\begin{aligned} ds^2 &= \Gamma(\theta)\left[-r^2dt^2 + \frac{dr^2}{r^2} + \alpha(\theta)d\theta^2\right] + \gamma(\theta)(d\phi + krdt)^2 \\ A &= f(\theta)(d\phi + krdt) \end{aligned}$$

we choose the boundary condition such that

- the ASG includes the Virasoro algebra
- the charges is finite

like the Brown-Henneaux's case

The appropriate boundary conditions determine the family of the geometries in which the charges are finite

Boundary condition and ASG

Such a boundary condition is (in the basis (t, ϕ, θ, r))

$$h_{\mu\nu} \sim \mathcal{O}\begin{pmatrix} r^2 & 1 & 1/r & 1/r^2 \\ & 1 & 1/r & 1/r \\ & & 1/r & 1/r^2 \\ & & & 1/r^3 \end{pmatrix}$$

ASG

$$\zeta[\epsilon] = \epsilon(\phi)\partial_\phi - r\epsilon'(\phi)\partial_r$$

Virasoro algebra

$$i[\zeta_n, \zeta_m]_{L.B.} = (n-m)\zeta_{n+m}$$
$$\zeta_n \equiv \zeta[\epsilon = -e^{-in\phi}]$$

Central charge

- The Noether charge Q_m associated with the ASG L_m could have the central charge

$$i\{Q_m, Q_n\}_{DB} = (m-n)Q_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}$$

- The central charge has contributions from gravitational part and gauge field

$$c = c_{grav} + c_{gauge}$$

Central charge

Results

$$c_{grav} = 3k \int_0^{\pi} d\theta \sqrt{\Gamma(\theta)\alpha(\theta)\gamma(\theta)}$$

$$c_{gauge} = 0$$

For the Kerr-Newman-(A)dS black hole

$$c = \frac{12 r_+ \sqrt{(3r_+^4/\ell^2 + r_+^2 - q^2)(1 - r_+^2/\ell^2)}}{1 + 6r_+^2/\ell^2 - 3r_+^4/\ell^4 - q^2/\ell^2}$$

Temperature

The extremality constraint requires

$$0 = T_H dS = dM_{ADM} - (\Omega_H dJ + \Phi_e dQ_e + \Phi_m dQ_m)$$

For such constrained variations we may write

$$-dI_{gr} = dS = \frac{dJ}{T_L} + \frac{dQ_e}{T_e} + \frac{dQ_m}{T_m}$$

Like GKP-W relation, we assume

$$\rho_{gravity} \equiv \rho_{CFT}$$

$$\rho_{gravity} = \exp\left(-I_{gr}\right), \qquad \rho_{CFT} = \exp\left(-\frac{L_0}{T_L} - \frac{\hat{q}_e}{T_e} - \frac{\hat{q}_m}{T_m}\right)$$

Then we obtain the temperature T_L of dual CFT

Entropy

For Kerr-Newman-(A)dS case

$$T_L = \frac{(1+6r_+^2/\ell^2 - 3r_+^4/\ell^4 - q^2/\ell^2)[2r_+^2(1+r_+^2/\ell^2) - q^2]}{4\pi r_+[(1+r_+^2/\ell^2)(1-3r_+^2/\ell^2)+q^2/\ell^2]\sqrt{(1-r_+^2/\ell^2)(3r_+^4/\ell^2+r_+^2-q^2)}}$$

Then

$$S_{CFT} = \frac{\pi^2}{3}cT_L = \frac{\pi(2r_+^4/\ell^2 + 2r_+^2 - q^2)}{1-2r_+^2/\ell^2 - 3r_+^4/\ell^4 + q^2/\ell^2}$$

This agrees in precise with the Bekenstein-Hawking entropy of the Kerr-Newman-(A)dS black hole!

Notice that the temperature is rewritten as the surprisingly simple form

$$\boxed{T_L = \frac{1}{2\pi k}}$$

The Extreme Black Hole/CFT correspondence

We treated the KNAdS black hole as the following general form

$$ds^2 = \Gamma(\theta)\left[-r^2 dt^2 + \frac{dr^2}{r^2} + \alpha(\theta)d\theta^2\right] + \gamma(\theta)(d\phi + krdt)^2$$

It was shown that the above form is obtained as the near horizon geometry of the extremal black hole constructed in (Kunduri-Lucietti-Reall '07)

General action

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\Big(R - \frac{1}{2}f_{AB}(\chi)\partial_\mu\chi^A\partial^\mu\chi^B - V(\chi) - \frac{1}{4}g_{IJ}(\chi)F^I_{\mu\nu}F^{J\mu\nu}\Big) + \frac{1}{2}\int h_{IJ}(\chi)F^I \wedge F^J$$

The near horizon scalar fields and gauge fields have the form

$$\chi^A = \chi^A(\theta)\ , \quad A^I = f^I(\theta)(d\phi + krdt)$$

Construct dual CFT

The Bekenstein-Hawking entropy of such a black hole is

$$S_{BH} = \frac{\pi}{2}\int_0^\pi d\theta\sqrt{\Gamma(\theta)\alpha(\theta)\gamma(\theta)}$$

- We must take the contribution of the non-gravitational part such as the scalar fields into account
- Remarkably, even in the presence of the non-gravitational fields, the central charge is always given by

Central charges (Compere-Murata-TN '09)

$$c = c_{grav} = 3k\int_0^\pi d\theta\sqrt{\Gamma(\theta)\alpha(\theta)\gamma(\theta)}$$

$$c_{other} = 0$$

Entropy

General temperature (assumption)

$$T_L = \frac{1}{2\pi k}$$

We naively apply this formula to the general cases[1]

$$S_{CFT} = \frac{\pi^2}{3} c_{grav} T_L$$
$$= \frac{\pi}{2} \int_0^\pi d\theta \sqrt{\Gamma(\theta)\alpha(\theta)\gamma(\theta)} = \frac{\text{Area(horizon)}}{4}$$

in agreement with the Bekenstein-Hawking entropy!

[1] Recently, this conjecture has been checked under some assumption (Chow-Cvetic-Lu-Pope '08)

Summary

- The entropy of the Kerr-Newman-(A)dS black hole is reproduced as the statistical entropy of dual CFT
- If we assume the formula for the temperature of CFT, we can apply this idea to the fairly general four-dimensional extremal black holes
- The Reissner-Nordstrome black hole also can be treated, but there is a dual description by embedding it into 5D space