Higher-Derivative Corrections to the Asymptotic Virasoro Symmetry of 4d Extremal Black Holes

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July 7, 2009
YITP Workshop “Field Theory and String Theory”

Based on arXiv:0903.4176

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Plan to Talk

1. Overview: The Kerr/CFT Correspondence & Our result
2. Asymptotic symmetry & Kerr/CFT (review)
3. Generalization to higher-derivative gravities
4. Summary & Discussion
Overview: The Kerr/CFT Correspondence & Our result

4D External (Kerr) black hole

Near Horizon \[ \rightarrow \] \( U(1) \) fibrated \( \text{AdS}_2 \) geometry (NHEK)

\[
ds^2 = A(\theta)^2 \left( -r^2 dt^2 + \frac{dr^2}{r^2} \right) + d\theta^2 + B(\theta)^2 (d\phi + kr dt)^2
\]

We can find a dual chiral \( \text{CFT}_2 \) through asymptotic symmetry & compute the central charge \( c \) (\( = \text{d.o.f. of the theory} \)).

\[ \Downarrow \]

Reproduce the Bekenstein-Hawking entropy correctly!

We successfully extended this result to arbitrary higher-derivative action & reproduced the Iyer-Wald entropy !!
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Symmetry Correspondences in \( \text{AdS}_3/\text{CFT}_2 \) & \( \text{AdS}_2/\text{CFT}_2 \)

- cf) \( \text{AdS}_{d+1}/\text{CFT}_d \) correspondence (\( d \geq 3 \))
  - \( d \)-dim conformal symmetry = \( \text{SO}(2, d) \)
  
  \( \cong \)
  
  \( \text{AdS}_{d+1} \) isometry = \( \text{SO}(2, d) \)

- \( \text{AdS}_3/\text{CFT}_2 \) correspondence
  - 2-dim conformal symmetry = \( \text{Virasoro}_L \times \text{Virasoro}_R \)
    (enhanced from \( \text{SO}(2, 2) \cong \text{SL}(2, \mathbb{R})_L \times \text{SL}(2, \mathbb{R})_R \))
    
    \( \cong \)
    
    \( \text{something} = \text{Virasoro}_L \times \text{Virasoro}_R \)
    (enhanced from \( \text{AdS}_3 \) isometry \( \text{SL}(2, \mathbb{R})_L \times \text{SL}(2, \mathbb{R})_R \))

- \( \text{AdS}_2/\text{CFT}_2 \) correspondence
  - 2-dim chiral conformal symmetry = \( \text{Virasoro}_L \) (from \( \text{SL}(2, \mathbb{R})_L \))
    
    \( \cong \)
    
    \( \text{something} = \text{Virasoro}_L \)
    (enhanced from \( \text{AdS}_2 \) isometry \( \text{SL}(2, \mathbb{R})_L \ ??) \)
What is the symmetry enhancement on the gravity side?

- **Consider asymptotic** $(r \to \infty)$ **symmetry**, not exact isometry.

- **Asymptotic symmetry**
  
  \[ g_{ab} = \bar{g}_{ab} + h_{ab}, \quad h_{ab} \sim O(r^{k_{ab}}) \quad (r \to \infty) \]

  \[ (\bar{g}_{ab}: \text{background metric}) \]

In fact,

- **Asymptotic symmetry group (ASG) of AdS$_3$**
  
  \[ = \text{Virasoro}_L \times \text{Virasoro}_R \]

  (under an appropriate boundary condition)
Asymptotic symmetry of NHEK & entropy reproduction

- asymptotic symmetry = Virasoro \((\leftarrow \text{appropriate b.c. \\& constraint})\)
  \[\xi_n = -e^{-in\phi}(inr\partial_r + \partial_\phi), \quad i[\xi_m, \xi_n]_{Lie} = (m - n)\xi_{m+n}\]

  (enhancement from \(U(1)_\phi\) of \(S^1\), not \(SL(2, \mathbb{R})\) of AdS\(_2\))

- locally no center \(\Rightarrow\) it appears from topology (\& boundary)
  - “asymptotic Noether charge” \(Q_n\) corresponding to \(\xi_n\)
  - construct a symplectic structure on the phase space
  - compute the Dirac bracket
  \[i\{Q_m, Q_n\} = (m - n)Q_{m+n} + \frac{c}{12}m(m^2 + a)\delta_{m+n}\]

  \[c = \frac{3k}{2\pi} \int_{\text{horizon}} dA, \quad T_{FT} = \frac{1}{2\pi k}\]

  for dual CFT,
  \[\left[ L_m, L_n \right] = (m - n)L_{m+n} + \frac{c}{12}m(m^2 + 1)\delta_{m+n}\]

  by Cardy formula,
  \[S_{CFT} = \frac{\pi^2}{3}cT_{FT} = \frac{1}{4} \int_{\text{horizon}} dA = S_{BH}\]
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Kerr/CFT in higher-derivative gravities ?

Motivation:

- LEET of string theory
- universality of gauge/gravity dualities in more general gravity theories?

Setup:

- 4D general higher-derivative pure gravity Lagrangian:
  \[ L(g_{ab}, R_{abcd}, \nabla e_1 R_{abcd}, \nabla (e_1 \nabla e_2) R_{abcd}, \ldots, \nabla (e_1 \ldots \nabla e_k) R_{abcd}) \]

- Near horizon extremal BH geometry:
  \[ ds^2 = A(\theta)^2 \left( -r^2 dt^2 + \frac{dr^2}{r^2} \right) + d\theta^2 + B(\theta)^2 (d\phi + kr dt)^2 \]

Goal:

- reproduce Iyer-Wald formula for BH entropy
  \[ S_{IW} = -2\pi \int_{\text{horizon}} dA \frac{\delta L}{\delta R_{abcd}} \epsilon_{ab} \epsilon_{cd} \]
What is necessary?

Action-independent properties:

- b.c. & asymptotic symmetry generators \((\xi_n)\)
- \(T_{FT} = \frac{1}{2\pi k}\)

Therefore our main task is:

- find the appropriate definition of the asymptotic charges and symplectic form (= Poisson bracket) on phase space
- compute the central charge \(c\)
Computation of central charge and entropy reproduction

Essence

- definition of symplectic form and charges:
  
  \[ c = -12k \int_{\text{horizon}} dA \, \frac{\delta L}{\delta R_{abcd}} \epsilon_{ab} \epsilon_{cd}, \quad T_{FT} = \frac{1}{2\pi k} \]

  \[ \therefore S_{CFT} = S_{IW}! \]

  perfectly reproduce Iyer-Wald entropy!!
Main result:

- We generalized the Kerr/CFT correspondence to arbitrary higher-derivative gravities.
- It always reproduces Iyer-Wald entropy.

Some possible suggestions:

1. gauge/gravity dualities exist for arbitrary gravity theories, without any relations to string theory or something?

2. extremal BH entropy is a too trivial quantity? (some underlying mechanism?)

3. or something?
Thank you.