Black Holes in the Dilatonic Einstein-Gauss-Bonnet Theory in Various Dimensions
– Case with negative cosmological term –

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Based on collaboration with Z.-K. Guo and T. Torii
reference: Prog. Theor. Phys. 121 (2009) 253 [arXiv:0811.3068 [gr-qc]].

See also
Prog. Theor. Phys. 120 (2008) 581 [arXiv:0806.2481 [gr-qc]];
Prog. Theor. Phys. 121 (2009) 959 [arXiv:0902.4072 [hep-th]].

1 Introduction

- **Check of the predictions of superstring theories**

The situation where the effects of quantum gravity become important

$\Rightarrow$ **Black holes (singularity)**
Early universe (singularity)

It is urgent to see whether and how these problems are resolved and if superstrings can give realistic models of particles and their interaction including gravity

Here we consider black holes.

- **We need dilaton!!**

Many studies of black holes have been performed by using low-energy effective theories inspired by string theories, which typically involve not only the metric but also the dilaton field (as well as several gauge fields).

There are studies of such solutions in Einstein theories with dilaton.

- **What about higher order corrections?**

It is known that there are correction terms of higher orders in the curvature to the lowest effective supergravity action coming from superstrings. The simplest correction is the Gauss-Bonnet (GB) term coupled to the dilaton field.

However, black holes in Einstein-GB theories have been studied much but WITHOUT DILATON!

In order to understand properties of black holes in string theories, we should include dilaton!

- **Another motivation:**

Many people consider the application to the calculation of shear viscosity in strongly coupled gauge theories using black hole solutions in five-dimensional Einstein-GB theory via AdS/CFT correspondence, but without dilaton. In order to see this in the context of superstrings, we should again include dilaton, and also search for asymptotically AdS solutions.

Last year we presented asymptotically flat solution for spherically symmetric space (curvature of the space $k = +1$). When we examine $k = 0$, it turns out that there is no solution without c.c. There are several sources of (negative) cosmological constant in superstrings. e.g. RR 10-form.

2 Dilatonic Einstein-GB theory

2.1 Basic equations

The action:

$$S = \frac{1}{2\kappa_D^2}\int d^D x\sqrt{-g}\left[R - \frac{1}{2}(\partial_\mu\phi)^2 + \alpha_2 e^{-\gamma\phi} R_{\rm GB}^2 - \Lambda e^{\lambda\phi}\right],$$

R**: the scalar curvature,** ϕ**: a dilaton field,**
$R_{\rm GB}^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$**: the GB combination,**
$\kappa_D^2 = 8\pi G_D$**: a** D**-dimensional gravitational constant,**
$\alpha_2 = \alpha'/8$**:** α **is the Regge slope parameter** α'**,** $\gamma = 1/2$**,** Λ**: (negative) cosmological "constant."**

Line element in D**-dimensional static spacetime**

$$ds_D^2 = -Be^{-2\delta}dt^2 + B^{-1}dr^2 + r^2 h_{ij}dx^i dx^j,$$

where $h_{ij}dx^i dx^j$ **represents the line element of a** $(D-2)$**-dimensional hypersurface with constant curvature of signature** k **and volume** Σ_k **for** $k = \pm 1, 0$**.**

Master equations:

$$
\begin{aligned}
&\left[(k-B)\tilde r^{D-3}\right]'\frac{D-2}{\tilde r^{D-4}}h-\frac{1}{2}B\tilde r^2\phi'^2-(D-1)_4\, e^{-\gamma\phi}\frac{(k-B)^2}{\tilde r^2}\\
&\quad+4(D-2)_3\,\gamma e^{-\gamma\phi}B(k-B)(\phi''-\gamma\phi'^2)\\
&\quad+2(D-2)_3\,\gamma e^{-\gamma\phi}\phi'\frac{(k-B)[(D-3)k-(D-1)B]}{\tilde r}-\tilde r^2\tilde\Lambda e^{\lambda\phi}=0\,,\\
&\delta'(D-2)\tilde r h+\frac{1}{2}\tilde r^2\phi'^2-2(D-2)_3\,\gamma e^{-\gamma\phi}(k-B)(\phi''-\gamma\phi'^2)=0\,,\\
&(e^{-\delta}\tilde r^{D-2}B\phi')'=\gamma(D-2)_3 e^{-\gamma\phi-\delta}\tilde r^{D-4}\Big[(D-4)_5\frac{(k-B)^2}{\tilde r^2}+2(B'-2\delta' B)B'\\
&\quad-4(k-B)BU(r)-4\frac{D-4}{\tilde r}(B'-\delta' B)(k-B)\Big]+e^{-\delta}\tilde r^{D-2}\lambda\tilde\Lambda e^{\lambda\phi},
\end{aligned}
$$

where we have defined

$$
\begin{aligned}
\tilde r &\equiv \frac{r}{\sqrt{\alpha_2}},\quad \tilde m\equiv\frac{Gm}{\alpha_2^{(D-3)/2}},\qquad (D-m)_n\equiv(D-m)(D-m-1)(D-m-2)\cdots(D-n),\\
h &\equiv 1+2(D-3)e^{-\gamma\phi}\Big[(D-4)\frac{k-B}{\tilde r^2}+\gamma\phi'\frac{3B-k}{\tilde r}\Big],\\
\tilde h &\equiv 1+2(D-3)e^{-\gamma\phi}\Big[(D-4)\frac{k-B}{\tilde r^2}+\gamma\phi'\frac{2B}{\tilde r}\Big],
\end{aligned}
$$

$$
\begin{aligned}
U(r) &\equiv \frac{1}{2\tilde{h}}\Bigg[(D-3)_4\frac{k-B}{\tilde{r}^2 B} - 2\frac{D-3}{\tilde{r}}\left(\frac{B'}{B}-\delta'\right) - \frac{1}{2}\phi'^2 \\
&\quad +(D-3)e^{-\gamma\phi}\Bigg[(D-4)_6\frac{(k-B)^2}{\tilde{r}^4 B} - 4(D-4)_5\frac{k-B}{\tilde{r}^3}\left(\frac{B'}{B}-\delta'-\gamma\phi'\right) \\
&\quad -4(D-4)\gamma\frac{k-B}{\tilde{r}^2}\left(\gamma\phi'^2+\frac{D-2}{\tilde{r}}\phi'-\Phi\right) + 8\frac{\gamma\phi'}{\tilde{r}}\Bigg\{\left(\frac{B'}{2}-\delta' B\right)\left(\gamma\phi'-\delta'+\frac{2}{\tilde{r}}\right) \\
&\qquad -\frac{D-4}{2\tilde{r}}B'\Bigg\} + 4(D-4)\left(\frac{B'}{2B}-\delta'\right)\frac{B'}{\tilde{r}^2} - 4\frac{\gamma}{\tilde{r}}\Phi(B'-2\delta' B)\Bigg]\Bigg], \\
\Phi &\equiv \phi'' + \left(\frac{B'}{B}-\delta'+\frac{D-2}{\tilde{r}}\right)\phi'.
\end{aligned}
$$

Symmetries

1. Scaling transf. $B \to a^2 B, \quad \tilde{r} \to a\tilde{r}$, **(**$a$**: an arbitrary constant).**

$\Rightarrow$ generate solutions with different horizon radii $\tilde{r}_H$ but the same $\tilde{\Lambda}$.

$\Rightarrow$ The mass scales like

$$\tilde{M}_0 \propto \tilde{r}_H^{D-1}, \qquad \tilde{\Lambda}\textbf{: fixed}$$

2. Scaling of c.c.:

$$\phi \to \phi - \phi_*, \quad \tilde{\Lambda} \to e^{(\lambda-\gamma)\phi_*}\tilde{\Lambda}, \quad B \to e^{-\gamma\phi_*}B\,,$$

$\Rightarrow$ generate solutions for different cosmological constants $\tilde{\Lambda}$ but with

the same horizon radius $\tilde{r}_H$. $\Rightarrow$ The mass scales as

$$\tilde{M}_0 \propto |\tilde{\Lambda}|^{\gamma/(\gamma-\lambda)}, \qquad \tilde{r}_H\text{: fixed}$$

3. Another shift symmetry

$$\delta \to \delta - \delta_\infty, \quad t \to e^{-\delta_\infty} t,$$

$\Rightarrow$ the asymptotic value of $\delta = 0$.

2.2 Boundary conditions

1. The existence of a regular horizon $\tilde{r}_H$:

$$B(\tilde{r}_H) = 0, \quad |\delta_H| < \infty, \quad |\phi_H| < \infty\,.$$

2. The nonexistence of singularities outside the event horizon ($\tilde{r} > \tilde{r}_H$):

$$B(\tilde{r}) > 0, \quad |\delta| < \infty, \quad |\phi| < \infty\,.$$

3. "AdS asymptotic behavior" ($\tilde{r} \to \infty$):

$$B \sim \tilde{b}_2 \tilde{r}^2 - \frac{2\tilde{M}}{\tilde{r}^\mu}, \qquad \delta(r) \sim \delta_0 + \frac{\delta_1}{\tilde{r}^\sigma}, \quad \phi \sim \phi_0 + \frac{\phi_1}{\tilde{r}^\nu}\,,$$

with finite constants $\tilde{b}_2 > 0$, $\tilde{M}$, δ_0, δ_1, ϕ_0, ϕ_1 and positive constant μ, σ, ν.

Given the b.c. at the horizon, ϕ'_H is determined:

$$
\begin{aligned}
&B_H = 0, \quad h_H = \tilde{h}_H = 1, \\
&B'_H = -\frac{\tilde{\Lambda}}{D-2}\tilde{r}_H e^{\lambda\phi_H}, \\
&\phi'_H = -\frac{1}{\tilde{r}_H}\Big[2\gamma(D-3)\tilde{\Lambda}e^{(\lambda-\gamma)\phi_H} + (D-2)\lambda\Big], \\
&\delta'_H = -\frac{1}{2(D-2)}\tilde{r}_H(\phi'_H)^2. \quad \Rightarrow \textbf{ no solution without c.c.}
\end{aligned}
$$

Effective potential picture

The dilaton field equation

$$
\Box\phi - \frac{d\tilde{V}_{\rm eff}}{d\phi} = 0,
$$

with the "effective potential"

$$
\tilde{V}_{\rm eff} = -e^{-\gamma\phi}\tilde{R}^2_{\rm GB} + \tilde{\Lambda}e^{\lambda\phi}.
$$

For the asymptotic AdS behavior for B, this gives

$$
\tilde{V}_{\rm eff} = -(D)_3\,\tilde{b}_2^2\,e^{-\gamma\phi} + \tilde{\Lambda}e^{\lambda\phi}.
$$

When $\lambda > 0$, the effective potential has a maximum (Fig. 1 (a)), and the dilaton field would approach a finite constant ϕ_0 at $r = \infty$. Otherwise, the dilaton diverges, and we consider only the case of $\lambda > 0$.

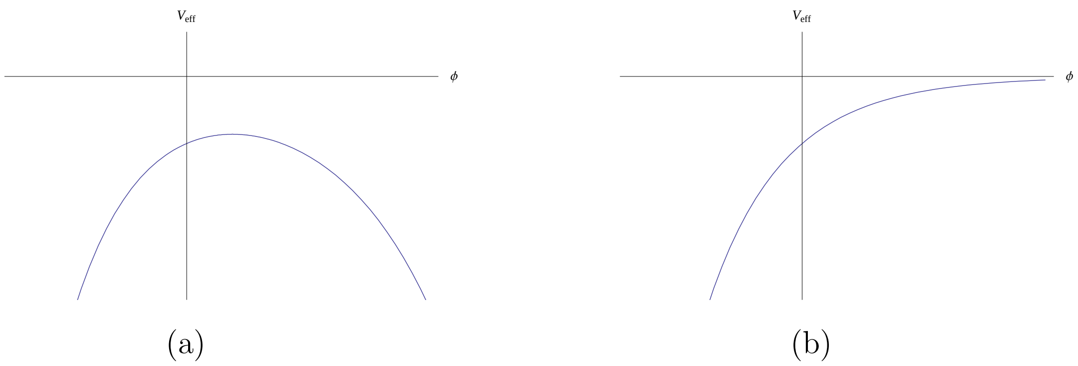


Figure 1: The effective potentials of the dilaton field in the Liouville potential case with (a) $\lambda > 0$ and (b) $\lambda < 0$.

The asymptotic forms of the fields

$$\phi \sim \phi_0 + \frac{\phi_+}{\tilde r^{\nu_+}} + \cdots , \quad B \sim \tilde b_2 \tilde r^2 - \frac{2\tilde M_+}{\tilde r^{\nu_+ - 2}} - \frac{2\tilde M_0}{\tilde r^{D-3}} + \cdots , \quad \delta \sim \delta_0 + \frac{\delta_+}{\tilde r^{\nu_+}} + \cdots .$$

where

$$\nu_\pm = \frac{D-1}{2}\left[1 \pm \sqrt{1 - \frac{\tilde m^2}{\tilde m_{BF}^2}}\right], \quad \tilde m_{BF}^2 = -\frac{(D-1)^2}{4\tilde\ell_{AdS}^2} = -\frac{(D-1)^2}{4}\tilde b_2,$$

There is a term $1/r^{\nu_-}$ (non-normalizable); we tune the boundary condition such that this term disappears.

We choose the following parameters in our numerical analysis:

$$\gamma = \frac{1}{2}, \quad \lambda = \frac{1}{3}, \quad \tilde\Lambda < 0, \quad \phi_- = 0, \quad \delta_0 = 0,$$

3 $D = 4$ black hole solutions

For the horizon radius $\tilde{r}_H = 1$ and $\tilde{\Lambda} = -3/2$ ($\tilde{\ell} = 2$) with the additional boundary conditions, we find $\phi_H = 2.33422$ in order to obtain $\phi_- = 0$, and $\delta_H = -0.02893$, $\phi_0 = 2.43279$ and $\tilde{M}_0 = 0.28014$. (See next figure.)

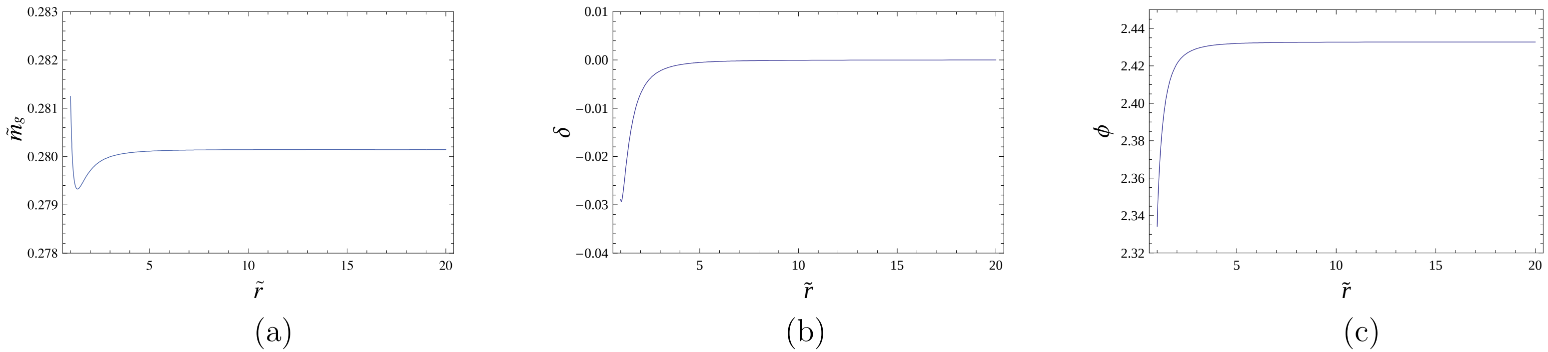


Figure 2: The configurations of the field functions (a) $\tilde{m}_g$, (b) δ and (c) ϕ in four dimensions for $\tilde{r}_H = 1$ and $\tilde{\Lambda} = -3/2$.

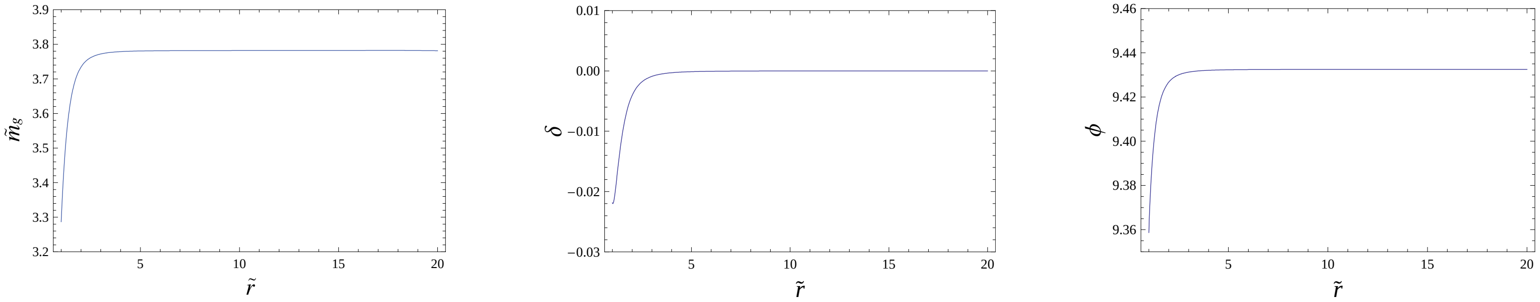
By using the symmetry, we can generate solutions for other $\tilde{\Lambda}$ and $\tilde{r}_H$ and the gravitational mass $\tilde{M}_0$:

$$\tilde{M}_0 = 0.28014 \left(\frac{2|\tilde{\Lambda}|}{3} \right)^3 \tilde{r}_H^{\,3} .$$

4 $D = 5$ solutions

For the horizon radius $\tilde{r}_H = 1$ and $\tilde{\Lambda} = -3$ ($\tilde{\ell} = 2$) with the additional boundary conditions at the horizon, we find $\phi_H = 9.35869$, $\delta_H =$

-0.02188, $\phi_0 = 9.43249$ **and** $\tilde{M}_0 = 3.78189$**.**



(a) (b) (c)

Figure 3: The configurations of the field functions (a) $\tilde{m}_g$, (b) δ and (c) ϕ in five dimensions for $\tilde{r}_H = 1$ and $\tilde{\Lambda} = -3$.

The gravitational mass $\tilde{M}_0$ for this case is given by

$$\tilde{M}_0 = 3.7819 \left(\frac{|\tilde{\Lambda}|}{3} \right)^3 \tilde{r}_H^4 .$$

5 $D = 6$ solutions

For the horizon radius $\tilde{r}_H = 1$ and $\tilde{\Lambda} = -5$ ($\tilde{\ell} = 2$), we find $\phi_H = 13.8108$, $\delta_H = -0.01621$, $\phi_0 = 13.86530$ and $\tilde{M}_0 = 19.93321$.

The regular black hole solutions exist for all $\tilde{r}_H > 0$.

The gravitational mass $\tilde{M}_0$ is given as

$$\tilde{M}_0 = 19.933 \left(\frac{|\tilde{\Lambda}|}{5} \right)^3 \tilde{r}_H^5 .$$

The mass of the black hole approaches 0 as $\tilde{r}_H \to 0$.

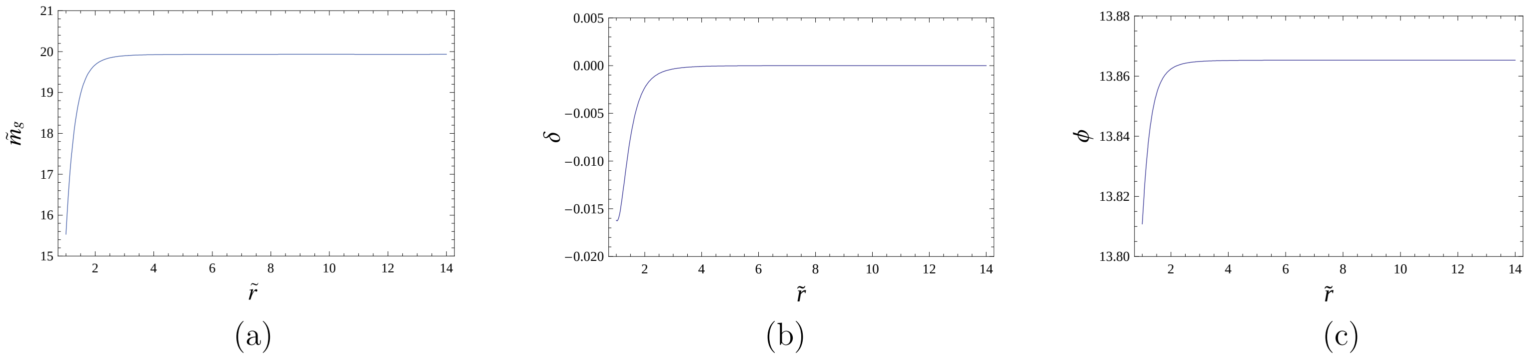


Figure 4: The configurations of the field functions (a) $\tilde{m}$, (b) δ and (c) ϕ in six dimensions for $\tilde{r}_H = 1$ and $\tilde{\Lambda} = -5$.

6 $D = 10$ solutions

For the horizon radius $\tilde{r}_H = 1$ and $\tilde{\Lambda} = -18$ ($\tilde{\ell} = 2$), we find $\phi_H = 23.6338$, $\delta_H = -0.0024575$, $\phi_0 = 23.64366$ and $\tilde{M}_0 = 771.67622$.

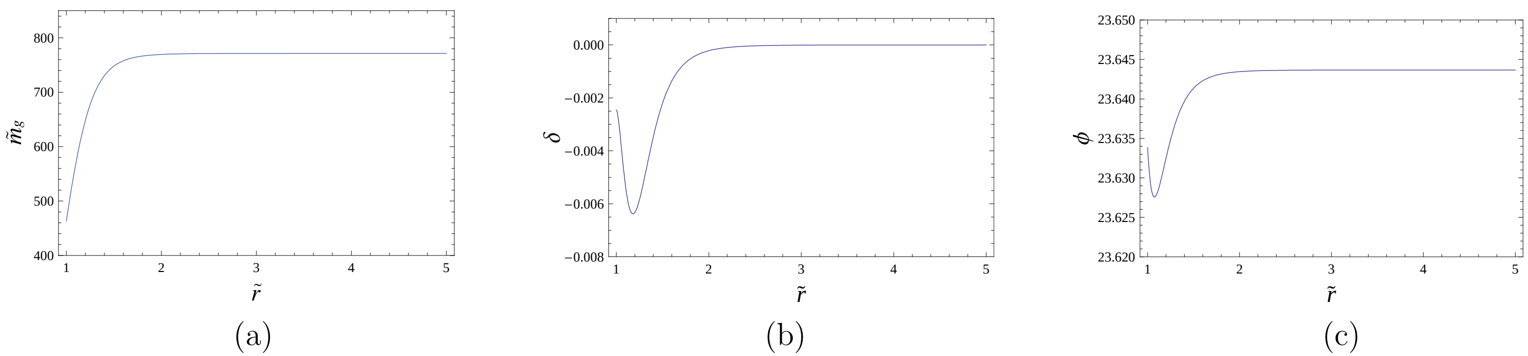


Figure 5: The configurations of the field functions (a) $\tilde{m}$, (b) δ and (c) ϕ in ten dimensions for $\tilde{r}_H = 1$ and $\tilde{\Lambda} = -18$.

The gravitational mass $\tilde{M}_0$ is given by

$$\tilde{M}_0 = 771.68 \left(\frac{|\tilde{\Lambda}|}{18} \right)^3 \tilde{r}_H^{\,9} \,.$$

7 Discussions

Properties:

Regular black hole solutions exist for all $\tilde{r}_H > 0$.

The mass of the dilatonic black holes approaches zero as $\tilde{r}_H \to 0$.

The dilaton field ϕ monotonically increases for large black holes.

These are in agreement with the non-dilatonic case,

$$B(\tilde{r}) = \frac{1}{2(D-3)_4}\left(1 \mp \sqrt{1 - \frac{4(D-3)_4}{\tilde{\ell}^2} + \frac{8(D-3)_4\bar{M}}{\tilde{r}^{D-1}}}\right)\tilde{r}^2,$$

$$\delta(\tilde{r}) \equiv 0$$

Mass is given by

$$\bar{M} = \frac{1}{2\tilde{\ell}^2}\tilde{r}_H^{\,D-1} \quad \Leftrightarrow \quad \tilde{M}_0 \propto |\tilde{\Lambda}|^{\gamma/(\gamma-\lambda)}\tilde{r}_H^{\,D-1}\,.$$ **dilatonic case**

Hawking temperature: $\tilde{T}_H = \dfrac{e^{-\delta_H}}{4\pi}B'_H = -\dfrac{e^{-\delta_H}}{4(D-2)\pi}\tilde{\Lambda}\tilde{r}_H e^{\lambda\phi_H},$

Not much qualitative difference between the dilatonic and the non-dilatonic cases.

This solution has been used to study higher order corrections to shear viscosity to entropy density and the naive lower bound $1/4\pi$ as well as the new bound $4/25\pi$ from GB correction (without dilaton) may be violated.

R. G. Cai, Z. Y. Nie, N. Ohta and Y. W. Sun, Phys. Rev. D 79 (2009) 066004 [arXiv:0901.1421 [hep-th]].

	$k=1$	$k=0$	$k=-1$
$\Lambda=0$	**I**	**II (No)**	**IV (No)**
$\Lambda=1$	**IV (No)**	**IV (No)**	**IV**
$\Lambda=-1$	**III**	**II**	**III**

IV: to appear soon, No: no solution

Remaining problems:

1. **The global structures:**

   Our numerical analysis was limited to outer spacetime of the event horizon.

   The global structures of the solutions such as the existence of the inner horizon and (central or branch) singularity have not been clarified. This may be done by integrating field equations inward numerically.

2. **The ambiguity of the frames:** **We have studied the solution in the Einstein frame.**

There is, however, a possibility that the properties of solutions changes drastically by transforming to the string frame. In particular, the conformal transformation may become singular.

3. **Charged solution:**

   It would be also interesting to extend our analysis to dilatonic black holes (large and small) with charges.

4. **Stability:**

   The stability of our solutions is another important subject to study.