Quantum Field Theory at a Lifshitz Point

Makoto Sakamoto (Kobe Univ.)

The purpose of my talk is to explain

ideas and physics

behind the Hořava-Lifshitz gravity.

P. Hořava, P.R.D.79 (2009) 084008, arXiv:0901.3775;
JHEP 0903(2009) 020, arXiv:0812.4287.

The purp o explain

ide a d ph cs

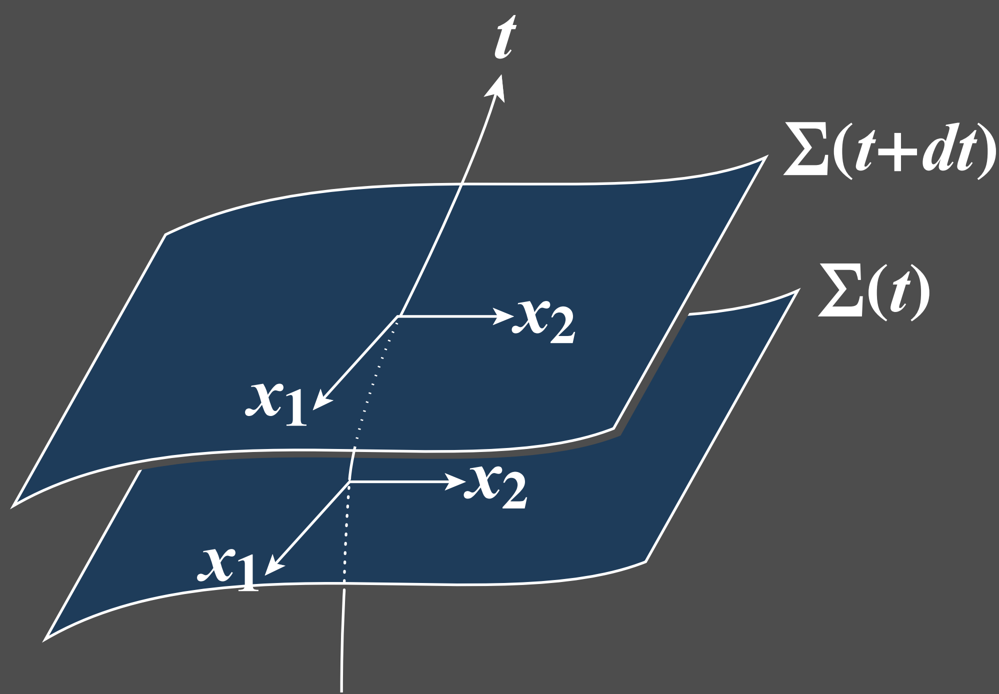
behind th av ifsh gravity.

P. Hořava, P. 9 (20 0 008 v:0901.3775;

JH 003(09) 020; 0812.4287.

ADM Decomposition of Metric

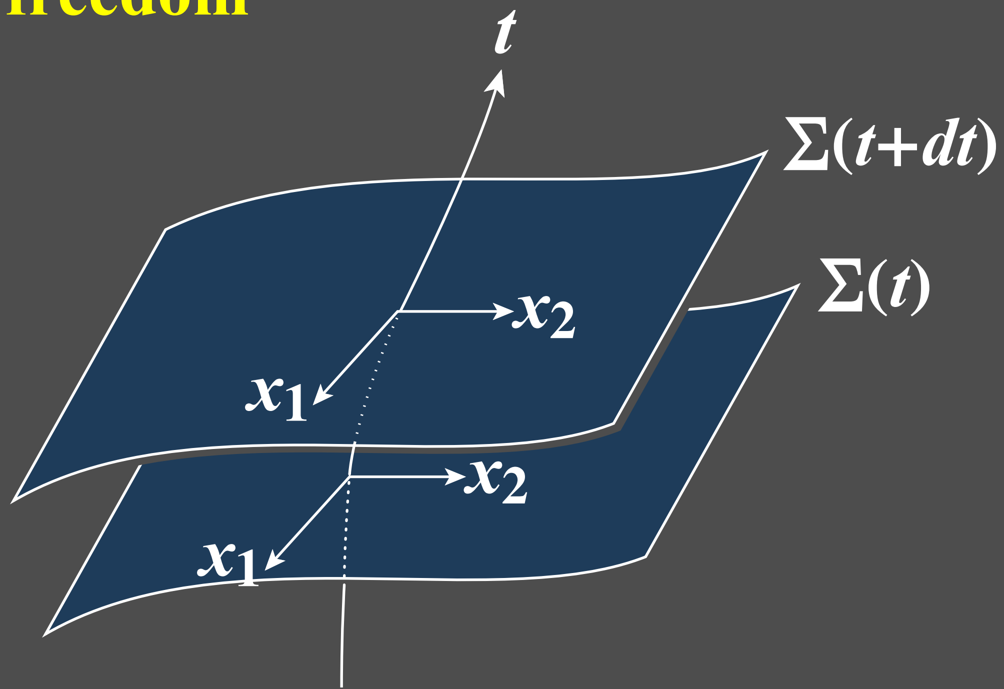
$$ds^2 = -N^2 c^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$



ADM Decomposition of Metric

$$ds^2 = -\underbrace{N^2}_{\text{lapse}} c^2 dt^2 + g_{ij}(dx^i + \underbrace{N^i}_{\text{shift}} dt)(dx^j + N^j dt)$$

dynamical degrees of freedom: N, g_{ij}, N^i



ADM Decomposition of Metric

lapse

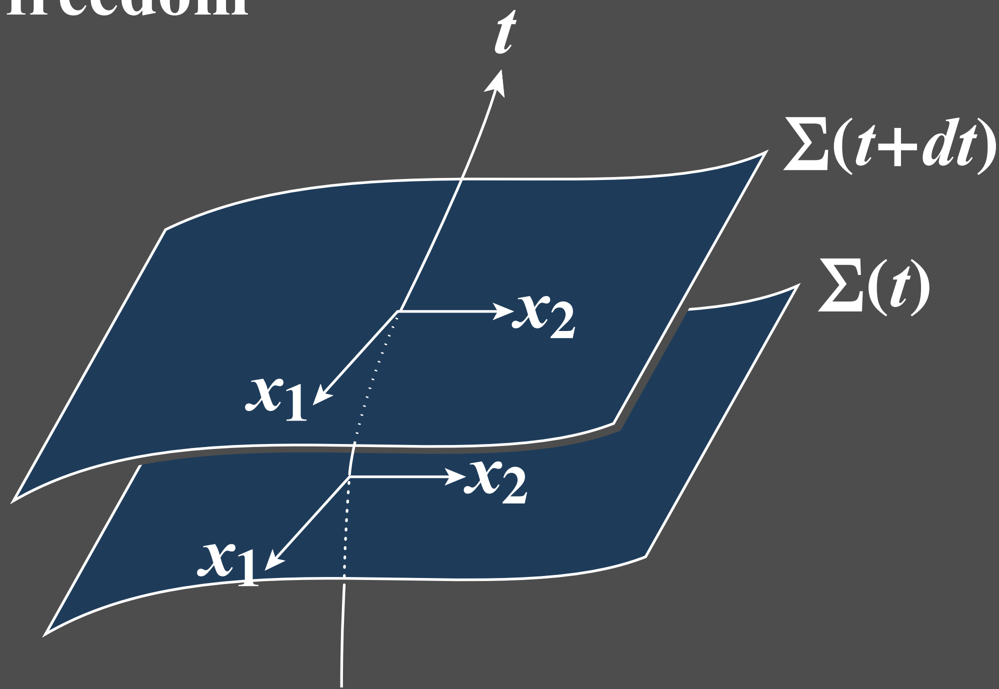
shift

$$ds^2 = -N^2 c^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

dynamical degrees of freedom

4d scalar curvature

$$\boxed{{}^{(4)}\mathcal{R} = K_{ij}\,\mathcal{G}^{ijkl}K_{kl} + R}$$



ADM Decomposition of Metric

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$$ds^2 = -N^2c^2dt^2 + g_{ij}(dx^i+N^idt)(dx^j+N^jdt)$$

dynamical degrees of freedom

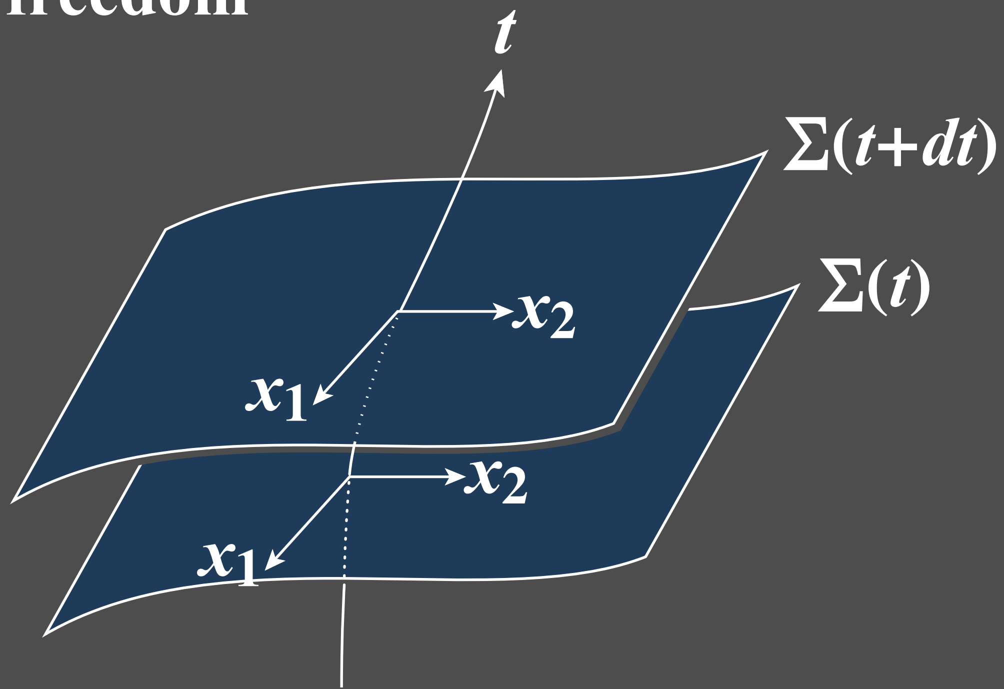
4d scalar curvature

extrinsic curvature

DeWitt metric

$$^{(4)}\mathcal{R} = K_{ij}\,\mathcal{G}^{ijkl}K_{kl} + R$$

3d scalar curvature constructed from spatial metric g_{ij}



z=3 Hořava-Lifshitz Gravity in 3+1dim

$$\mathcal{L}_{\mathrm{HL}} = \frac{2}{\kappa^2} K_{ij} \mathcal{G}^{ijkl} K_{kl} - \frac{\kappa^2}{2w^4} C^{ij} \mathcal{G}_{ijkl} C^{kl}$$

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► **Extrinsic curvature**

$$K_{ij} \equiv \frac{1}{2N}(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

z=3 Hořava-Lifshitz Gravity in 3+1dim ③

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► "DeWitt metric"

parameter

$$\mathcal{G}^{ijkl} \equiv \frac{1}{2}(g^{ik} g^{jl} + g^{il} g^{jk}) - \lambda g^{ij} g^{kl}$$

z=3 Hořava-Lifshitz Gravity in 3+1dim 3

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$$\mathcal{G}^{ijkl} \equiv \frac{1}{2}(g^{ik} g^{jl} + g^{il} g^{jk}) - \underbrace{\lambda}_{\text{parameter}} g^{ij} g^{kl}$$

- Cotton tensor

$$C^{ij} \equiv \epsilon^{ikl} \nabla_k (R^j{}_l - \frac{1}{4} R \delta^j{}_l)$$

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relevant deformation

$$- \frac{\mu w^2}{2} (R^{ij} - \frac{1}{2} R g^{ij} + \Lambda_W g^{ij})$$

Hořava-Lifshitz vs. Einstein

$$\mathcal{L}_{\mathrm{HL}} = \frac{2}{\kappa^2} K_{ij} \mathcal{G}^{ijkl} K_{kl} - \frac{\kappa^2}{2w^4} C^{ij} \mathcal{G}_{ijkl} C^{kl}$$

$$\mathcal{L}_{\mathrm{E}} = \frac{2}{\kappa^2} K_{ij} \mathcal{G}^{ijkl} K_{kl} + \frac{2}{\kappa^2} (R - 2\Lambda)$$

Hořava-Lifshitz vs. Einstein

kinetic term

$$\mathcal{L}_{\mathrm{HL}} = \frac{2}{\kappa^2} K_{ij} \mathcal{G}^{ijkl} K_{kl} - \frac{\kappa^2}{2w^4} C^{ij} \mathcal{G}_{ijkl} C^{kl}$$

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λ is a coupling constant.

$$\mathcal{G}^{ijkl} \equiv \frac{1}{2}(g^{ik} g^{jl} + g^{il} g^{jk}) - \lambda g^{ij} g^{kl}$$

λ=1

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"potential" term

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$$C^{ij} \equiv \epsilon^{ikl} \nabla_k \left(R^j{}_l - \frac{1}{4} R \delta^j{}_l \right)$$

$(\partial^3 g_{ij})^2 + \cdots$

6th power of ∂

2nd power of ∂

$(\partial g_{ij})^2 + \cdots$

$$\mathcal{L}_{\mathrm{E}} = \frac{2}{\kappa^2} K_{ij} \mathcal{G}^{ijkl} K_{kl} + \frac{2}{\kappa^2} (\overbrace{R - 2}\Lambda)$$

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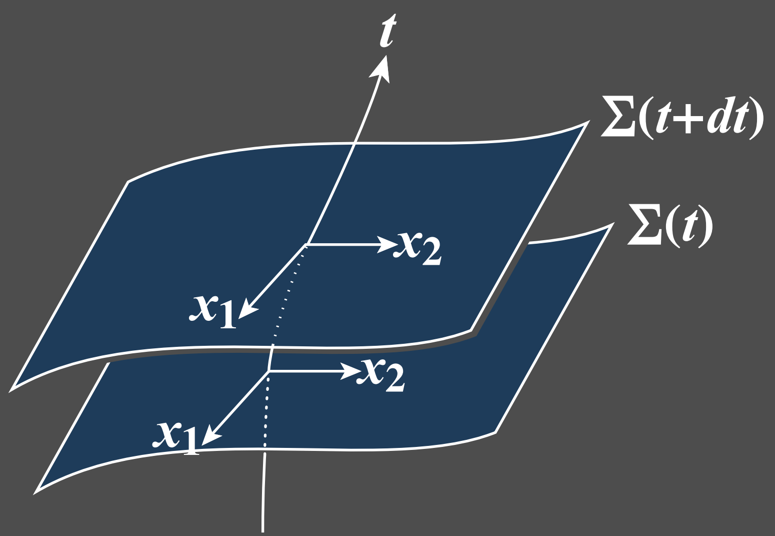

$$\updownarrow$$

Local Invariance

$$\mathcal{L}_{\mathrm{E}} = \frac{2}{\kappa^2} K_{ij} \mathcal{G}^{ijkl} K_{kl} + \frac{2}{\kappa^2}(R - 2\Lambda)$$

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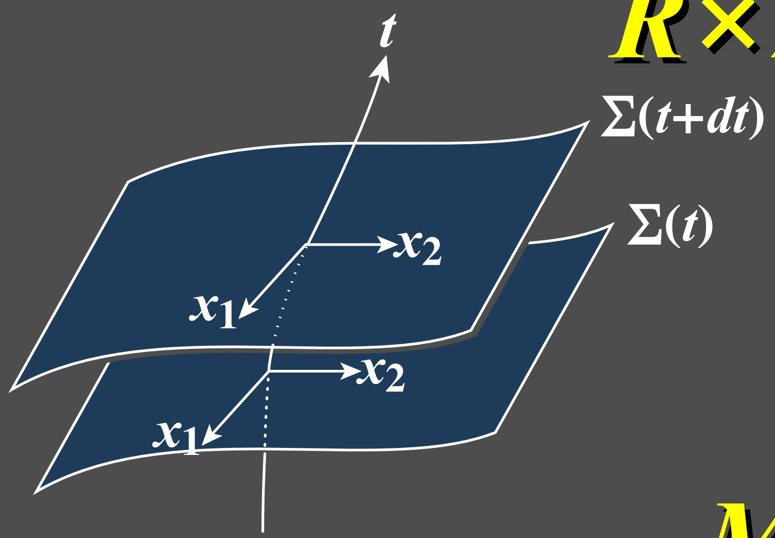

$R \times \Sigma$ $\boxed{\delta t = f(t) \quad , \quad \delta x^i = \zeta^i(t,x)}$

$(\lambda \neq 1/3)$

$\updownarrow$ **Local Invariance**

M^4 $\boxed{\delta t = f(t,x) \quad , \quad \delta x^i = \zeta^i(t,x)}$

$$\mathcal{L}_{\mathrm{E}} = \frac{2}{\kappa^2} K_{ij} \mathcal{G}^{ijkl} K_{kl} + \frac{2}{\kappa^2}(R - 2\Lambda)$$

Novel Properties

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► **anisotropy between space and time**

$$z = 3$$

nonrelativistic at UV

Novel Properties

► anisotropy between space and time

$z=3$ nonrelativistic at UV $\longrightarrow$ $z=1$ relativistic at IR

Novel Properties

- ▶ anisotropy between space and time

$$\underset{\text{nonrelativistic at UV}}{z=3} \longrightarrow \underset{\text{relativistic at IR}}{z=1}$$

- ▶ **Lorentz symmetry as an emergent sym. at IR**

The causal structure drastically changes at UV!

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- ►power-counting renormalizable in 3+1 dim.

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- ► closely related to 3d topological massive gravity

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The causal structure drastically changes at UV!

- power-counting renormalizable in 3+1 dim.
- closely related to 3d topological massive gravity
- spectral dimension

$$d_s = 1+\frac{D}{z} = \begin{cases} 2 & \text{at UV} \ (z=D=3) \\ 4 & \text{at IR} \ (z=1,\ D=3) \end{cases}$$

Gravity/Temperature fluctuations feel

Novel Properties

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d_s: Gravity/Temperature fluctuations feel

This is consistent with CDT approach.
Ambjorn et al, 2005,
Phys.Rev.Lett.95(2005)171301.

Ideas & Physics behind HL Gravity

Tests of Lorentz Invariance

Nonrenormalizability in Quantum Gravity

Lifshitz Point

Detailed Balance

Tests of Lorentz Invariance

Lorentz Violation could be induced by Quantum Gravity but suppressed by $M_{\mathrm{Pl}} \approx 10^{19}\,\mathrm{GeV}$.

Mattingly 2005, Living Rev. Relativity 8(2005)5;
Jacobson, Liberati and Mattingly 2006, Ann. Phys. 321(2006)150.

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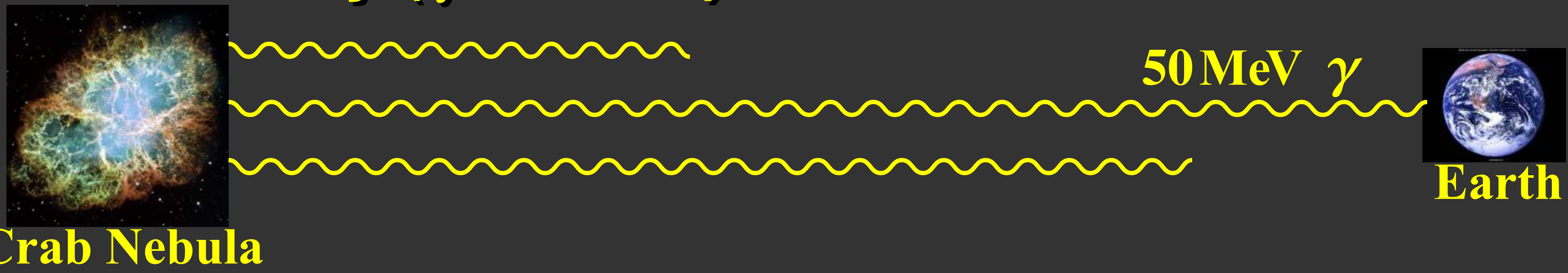
Photon decay ($\gamma \to e^+ e^-$)



$$\text{photon:} \quad \omega^2 = k^2$$

$$\text{electron:} \quad E^2 = m^2 + k^2$$

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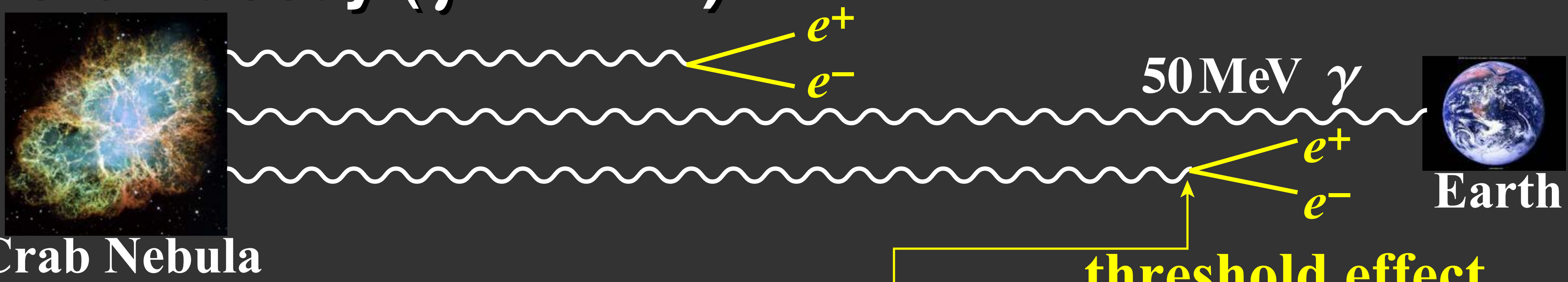

threshold effect
(otherwise forbidden!)

photon: $\omega^2 = k^2\left(1 + \boxed{\xi \frac{k}{M_{\mathrm{Pl}}}}\right)$

electron: $E^2 = m^2 + k^2\left(1 + \boxed{\eta \frac{k}{M_{\mathrm{Pl}}}}\right)$

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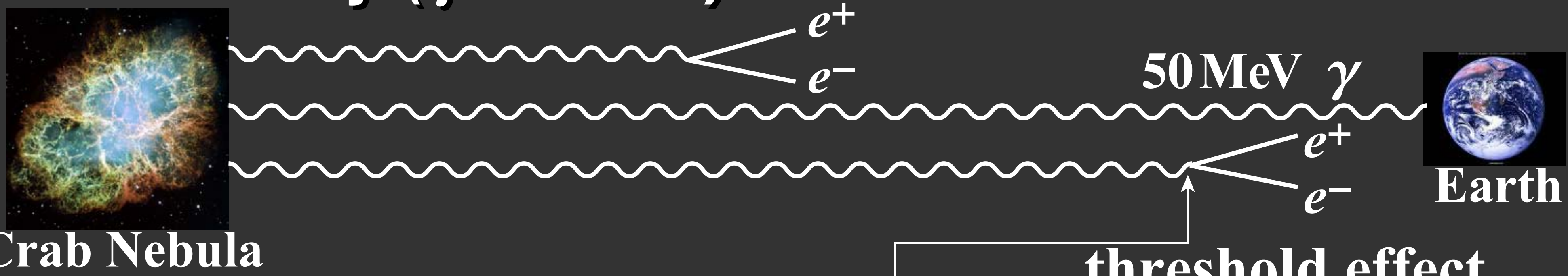

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$$\xi \lesssim 10^{-4}$$
$$\eta \lesssim 0.2$$

birefringence

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Photon time of flight

GRBs/AGN

low energy γ

high energy γ

Earth

$$\frac{c(E)}{c} = 1 + \xi \frac{E}{M_{\rm Pl}}$$

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$$\boxed{|\xi| \lesssim O(10)}$$

MAGIC, AGN Markarian 501,
Phys. Lett. B668(2008)253,
FERMI, GRB 080916C,
SCIENCE 323(2009)1688,
H.E.S.S., AGN PKS 2155-304,
arXiv:0904.3184.

Nonrenormalizability in Quantum Gravity

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► **gravitational constant κ^2 in D+1 dim.**

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$$1 + \kappa^2 \underbrace{\Lambda}_{\text{cutoff}}{}^{D-1} + \kappa^4 \Lambda^{2(D-1)} + \kappa^6 \Lambda^{3(D-1)} + \underbrace{\cdots}_{\textit{UV divergences are uncontrollable!}}$$

Nonrenormalizability in Quantum Gravity

- ► gravitational constant κ^2 in D+1 dim.

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- ► **power-counting renormalizable**

$$[\kappa^2] \geq 0$$

Nonrenormalizability in Quantum Gravity

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- ► power-counting renormalizable

$$[\kappa^2] \geq 0 \longrightarrow \underbrace{D}_{\text{space dimensions}} \leq 1$$

Nonrenormalizability in Quantum Gravity

► **higher-derivative gravity**

dimensionless parameter

$$\mathcal{L}_{\mathrm{E}} = \frac{2}{\kappa^2}\,{}^{(4)}\mathcal{R} + \alpha\,{}^{(4)}\mathcal{R}^2$$

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► **higher-derivative gravity**

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$$\mathcal{L}_{\mathrm{E}} = \frac{2}{\kappa^2}\,{}^{(4)}\mathcal{R} + \alpha\,{}^{(4)}\mathcal{R}^2$$

propagator

$$\frac{1}{k^2} \longrightarrow \frac{1}{k^2 - k^4/M^2}$$

cures the UV catastrophe!

Nonrenormalizability in Quantum Gravity

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good news

- **renormalizable**
- **asymptotically-free**

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- violation of unitarity

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▶ higher-derivative gravity

dimensionless parameter

$$\mathcal{L}_{\mathrm{E}} = \frac{2}{\kappa^2}\,{}^{(4)}\mathcal{R} + \alpha\,{}^{(4)}\mathcal{R}^2$$

massless graviton

massive ghost

propagator

$$\frac{1}{k^2} \longrightarrow \frac{1}{k^2 - k^4/M^2} = \frac{1}{k^2} - \frac{1}{k^2 - M^2}$$

cures the UV catastrophe!

good news

- renormalizable
- asymptotically-free

bad news

- violation of unitarity

Nonrenormalizability in Quantum Gravity

► Anisotropic scaling between space & time

$$\left.\begin{array}{l} x \rightarrow bx \\ t \rightarrow b^z t \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} [x] = -1 \\ [t] = -z \end{array}\right.$$

Lorentz inv.
$\Rightarrow z = 1$

Nonrenormalizability in Quantum Gravity

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$$S = \int dt d^3x \sqrt{g} N \left\{ \frac{2}{\kappa^2} (\dot{g}_{ij})^2 + \cdots \right\}$$

Nonrenormalizability in Quantum Gravity

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Lorentz inv.
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$$[g_{ij}] = [N] = 0$$

$$\underbrace{S}_{0} = \int \underbrace{dt}_{-z} \underbrace{d^3x}_{-3} \underbrace{\sqrt{g}N}_{+0} \left\{ \underbrace{\frac{2}{\kappa^2}}_{-[\kappa^2]} \underbrace{(\dot{g}_{ij})^2}_{+2z} + \cdots \right\}$$

$$0 = -z - 3 + 0 - [\kappa^2] + 2z$$

Nonrenormalizability in Quantum Gravity

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$$[g_{ij}] = [N] = 0$$

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$$\Rightarrow \ [\kappa^2] = z - 3$$

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$$[g_{ij}] = [N] = 0$$

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$$0 = -z - 3 + 0 - [\kappa^2] + 2z$$

$$\Rightarrow [\kappa^2] = z - 3 \quad \Rightarrow [\kappa^2] \geq 0 \quad \text{if} \quad \boxed{z \geq 3}$$

power-counting renormalizable

Nonrenormalizability in Quantum Gravity

▶Nonrelativistic quantum gravity

$$\mathcal{L}_{\mathrm{HL}} = \frac{2}{\kappa^2} K_{ij} \mathcal{G}^{ijkl} K_{kl} - \frac{\kappa^2}{2w^4} C^{ij} \mathcal{G}_{ijkl} C^{kl}$$

Nonrenormalizability in Quantum Gravity

► Nonrelativistic quantum gravity

$$\mathcal{L}_{\mathrm{HL}} = \frac{2}{\kappa^2}\underbrace{K_{ij}\,\mathcal{G}^{ijkl}K_{kl}}_{\approx\,(\dot{g}_{ij})^2} - \frac{\kappa^2}{2w^4}\underbrace{C^{ij}\,\mathcal{G}_{ijkl}\,C^{kl}}_{\approx\,(\partial^3 g_{ij})^2}$$

$z = 3$ **anisotropy!**

Nonrenormalizability in Quantum Gravity

►Nonrelativistic quantum gravity

$$\mathcal{L}_{\mathrm{HL}} = \frac{2}{\kappa^2}\underbrace{K_{ij}\mathcal{G}^{ijkl}K_{kl}}_{\approx\ (\dot{g}_{ij})^2} - \frac{\kappa^2}{2w^4}\underbrace{C^{ij}\mathcal{G}_{ijkl}C^{kl}}_{\approx\ (\partial^3 g_{ij})^2}$$

$z=3$ anisotropy!

propagator

$$\frac{1}{\omega^2-\gamma \mathrm{k}^6}$$

well controls UV divergences!

no ghost!

Nonrenormalizability in Quantum Gravity

▸Where is Einstein's General Relativity?

Nonrenormalizability in Quantum Gravity

► **Where is Einstein's General Relativity?**

Don't worry!

Nonrenormalizability in Quantum Gravity

▶ **Where is Einstein's General Relativity? Don't worry!**

$$S_{\mathrm{HL}} = \int dt d^3x \sqrt{g} N \left\{ \frac{2}{\kappa^2} K_{ij} \mathcal{G}^{ijkl} K_{kl} - \frac{\kappa^2}{2w^4} C^{ij} \mathcal{G}_{ijkl} C^{kl} \right\}$$

Nonrenormalizability in Quantum Gravity

- ►**Where is Einstein's General Relativity?**

Don't worry!

$$S_{\mathrm{HL}} = \int dt d^3x \sqrt{g} N \left\{ \frac{2}{\kappa^2} K_{ij} \mathcal{G}^{ijkl} K_{kl} - \frac{\kappa^2}{2w^4} C^{ij} \mathcal{G}_{ijkl} C^{kl} \right\}$$

$$C^{ij} \equiv \epsilon^{ikl} \nabla_k \left(R^j{}_l - \frac{1}{4} R \delta^j{}_l \right) + \frac{\mu w^2}{2} \left(R^{ij} - \frac{1}{2} R g^{ij} - \Lambda_W g^{ij} \right)$$

Nonrenormalizability in Quantum Gravity

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$$= \int dt d^3x \sqrt{g} N \left\{ \frac{2}{\kappa^2} K_{ij} \mathcal{G}^{ijkl} K_{kl} - \frac{\kappa^2}{2w^4} \left(\nabla_i R_{jk} \nabla^i R^{jk} + \cdots \right) \right.$$

$$\left. + \cdots + \frac{\kappa^2 \mu^2}{8(1-3\lambda)} \left(\Lambda_W R - 3 \Lambda_W{}^2 \right) \right\}$$

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Einstein Gravity naturally appears at IR!

Nonrenormalizability in Quantum Gravity

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Einstein Gravity naturally appears at IR!

$$S_{\mathrm{E}} = \int c dt d^3x \sqrt{g} N \frac{1}{16\pi G_{\mathrm{N}}} \left\{ \overbrace{\frac{1}{c^2} K_{ij} \mathcal{G}^{ijkl} K_{kl}} + \left(\overbrace{R} - \overbrace{2\Lambda} \right) \right\}$$

Nonrenormalizability in Quantum Gravity

► Emergent parameters at IR

light velocity $$c \equiv \frac{\kappa^2 \mu}{4}\sqrt{\frac{\Lambda_W}{1-3\lambda}}$$

Newton's constant $$G_N \equiv \frac{\kappa^2}{32\pi c}$$

cosmological constant $$\Lambda \equiv \frac{3}{2}\Lambda_W$$

parameters at UV

parameters at IR

Nonrenormalizability in Quantum Gravity

▶ **Where is Einstein's General Relativity?**

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$$S_{\mathrm{HL}} = \int dt d^3x \sqrt{g} N \left\{ \frac{2}{\kappa^2} K_{ij} \mathcal{G}^{ijkl} K_{kl} - \frac{\kappa^2}{2w^4} C^{ij} \mathcal{G}_{ijkl} C^{kl} \right\}$$

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$$\left. \cdots + \frac{\kappa^2 \mu^2}{8(1-3\lambda)} \left(\Lambda_W R - 3\Lambda_W{}^2 \right) \right\}$$

?

$\lambda \to 1$ at IR

…ity naturally appears at IR!

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Lifshitz Point

Lifshitz Point

Ginzburg-Landau theory

order parameter

$$F = a\Phi^2 + b\Phi^4 + c\Phi^6 + \cdots$$
$$+ \alpha(\nabla\Phi)^2 + \beta(\nabla^2\Phi)^2 + \cdots$$

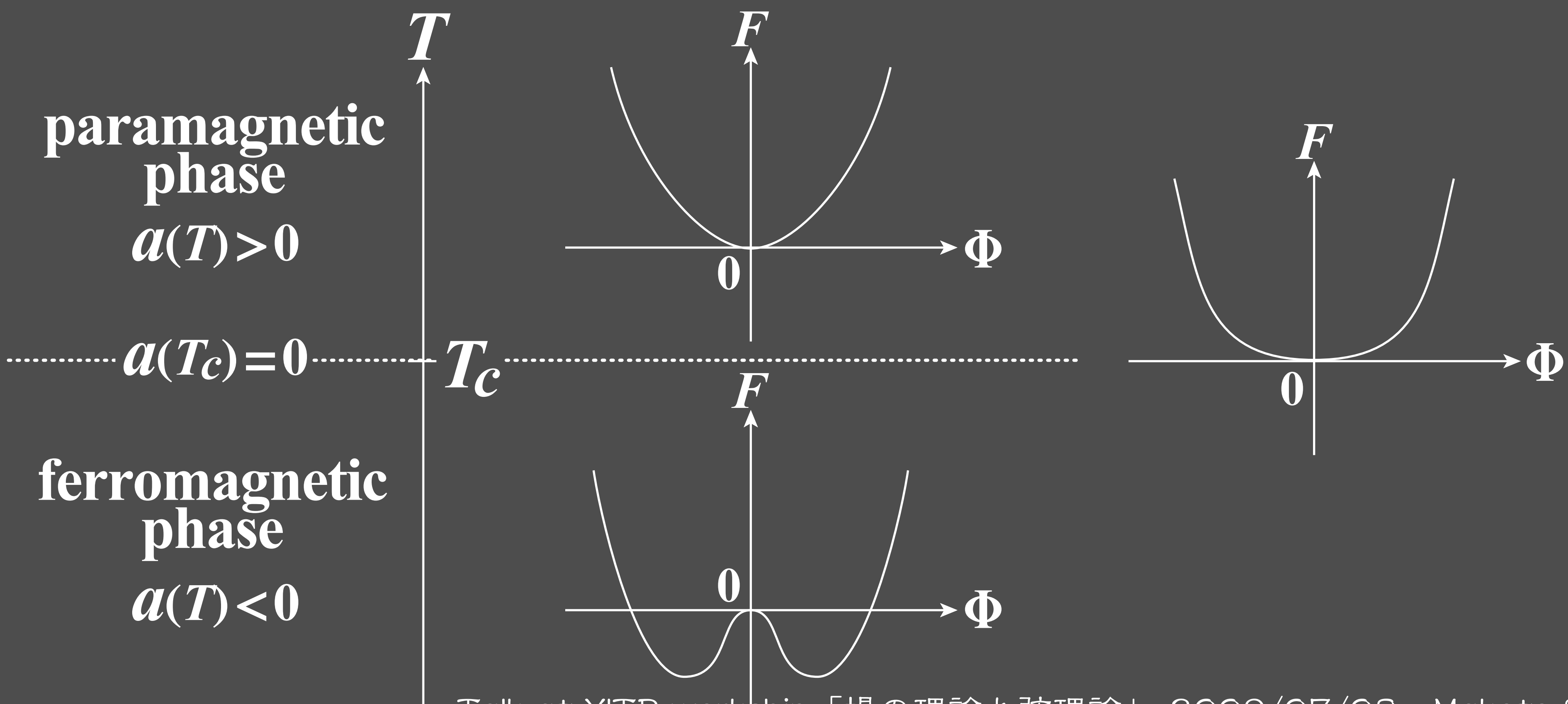
can qualitatively describe the dynamics of phase transitions/critical phenomena.

Lifshitz Point

2nd order phase transition

$$F = a\Phi^2 + b\Phi^4 + c\Phi^6 + \cdots$$
$$+ \alpha(\nabla\Phi)^2 + \beta(\nabla^2\Phi)^2 + \cdots$$

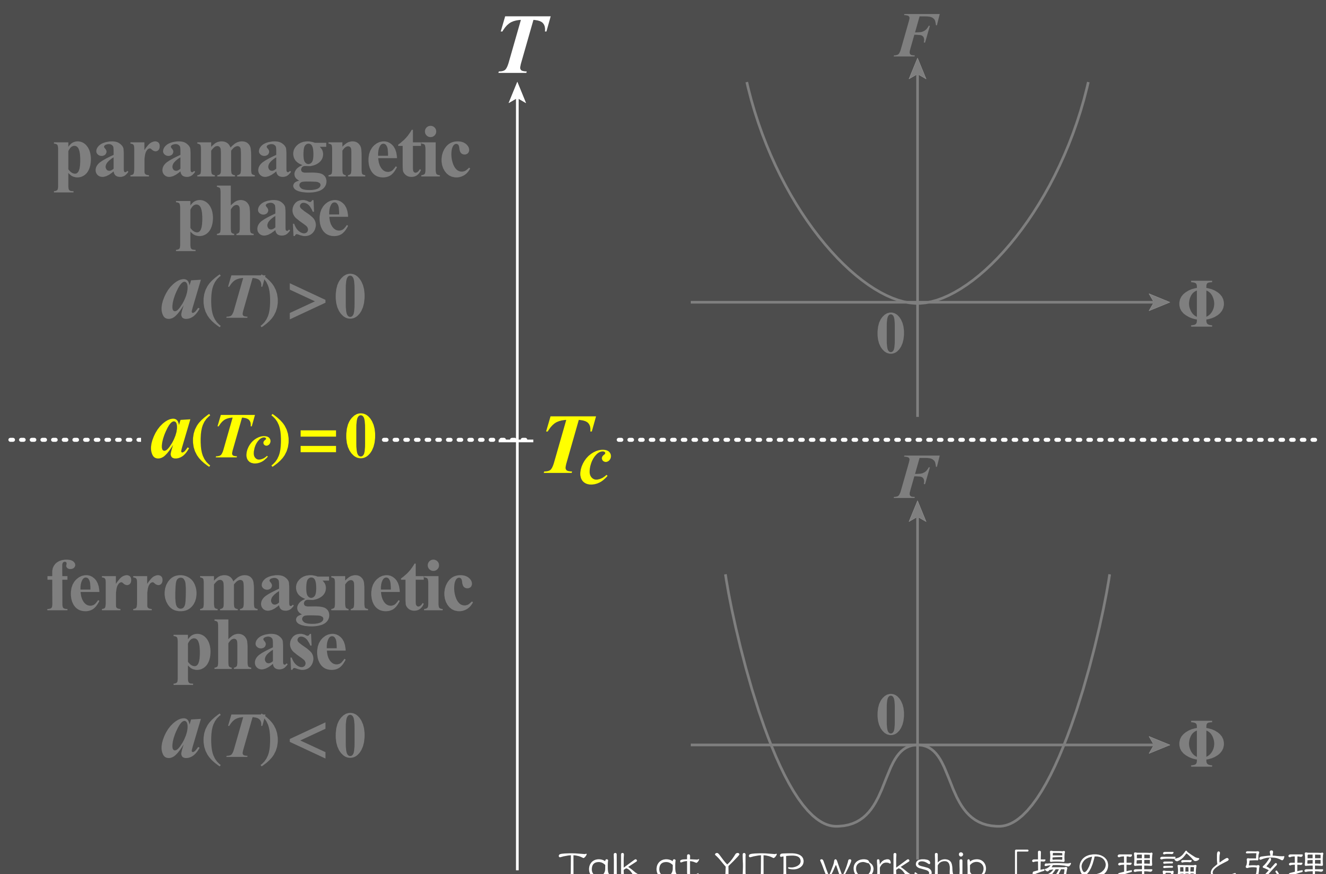
$(b>0)$

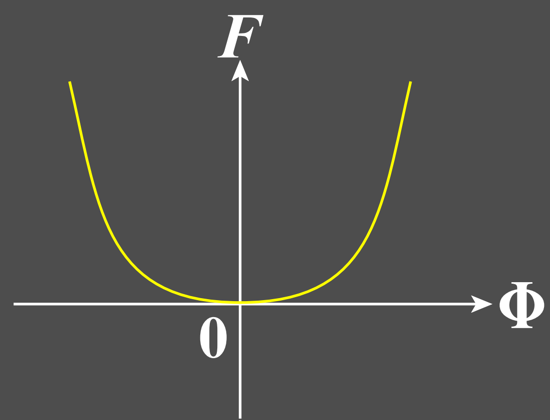


Lifshitz Point

critical point

$$F = a\Phi^2 + b\Phi^4 + c\Phi^6 + \cdots$$
$$(b>0) \qquad + \alpha(\nabla\Phi)^2 + \beta(\nabla^2\Phi)^2 + \cdots$$





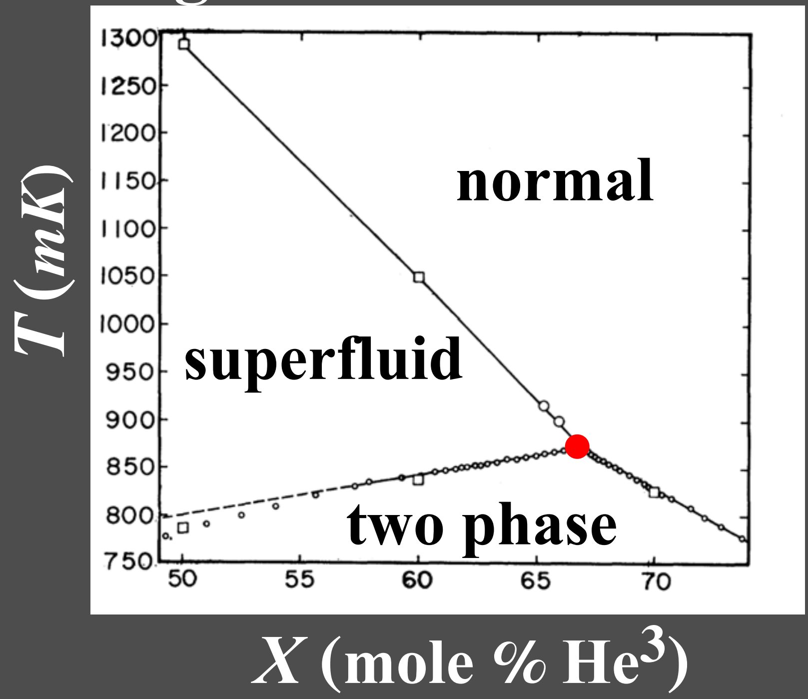
Lifshitz Point

tricritical point

$$F = a\Phi^2 + b\Phi^4 + c\Phi^6 + \cdots$$

$(c>0)$ $$+ \alpha(\nabla\Phi)^2 + \beta(\nabla^2\Phi)^2 + \cdots$$

phase diagram in He^3-He^4 mixtures



Graf, Lee and Reppy 1967,
PRL19(1967)417.

Lifshitz Point

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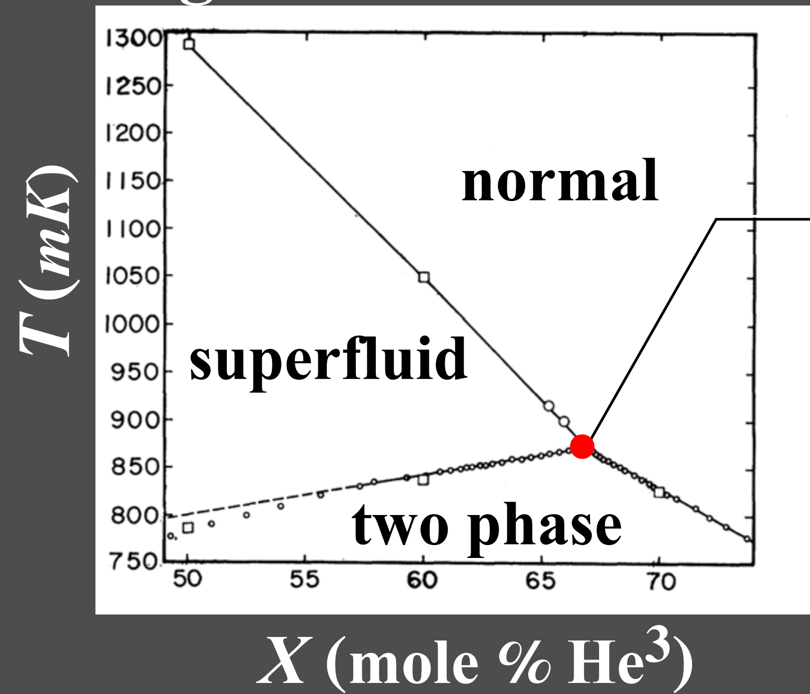


tricritical point
$a = b = 0$

Graf, Lee and Reppy 1967,
PRL19(1967)417.

Lifshitz Point

Lifshitz point

$$F = a\Phi^2 + b\Phi^4 + c\Phi^6 + \cdots$$

$$+\alpha(\nabla\Phi)^2 + \beta(\nabla^2\Phi)^2 + \cdots$$

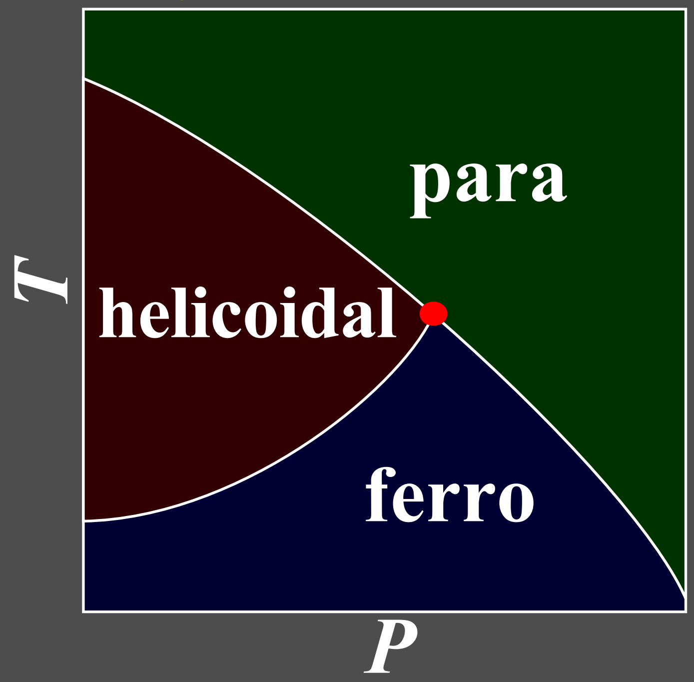
$(b, \beta > 0)$

Lifshitz Point

Lifshitz point

$$F = a\Phi^2 + b\Phi^4 + c\Phi^6 + \cdots$$
$$(b,\beta>0) \quad + \alpha(\nabla\Phi)^2 + \beta(\nabla^2\Phi)^2 + \cdots$$

Schematic phase diagram of a magnetic system with a Lifshitz point



Hornreich, Luban, Shtrikman, 1975, PRL35(1975)0678

Lifshitz Point

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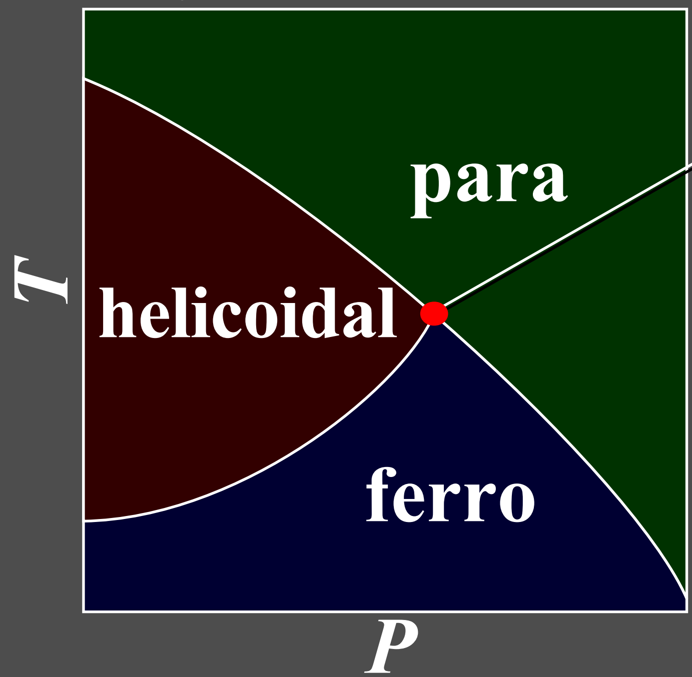


Lifshitz point
$a = \alpha = 0$

Hornreich, Luban, Shtrikman, 1975, PRL35(1975)0678

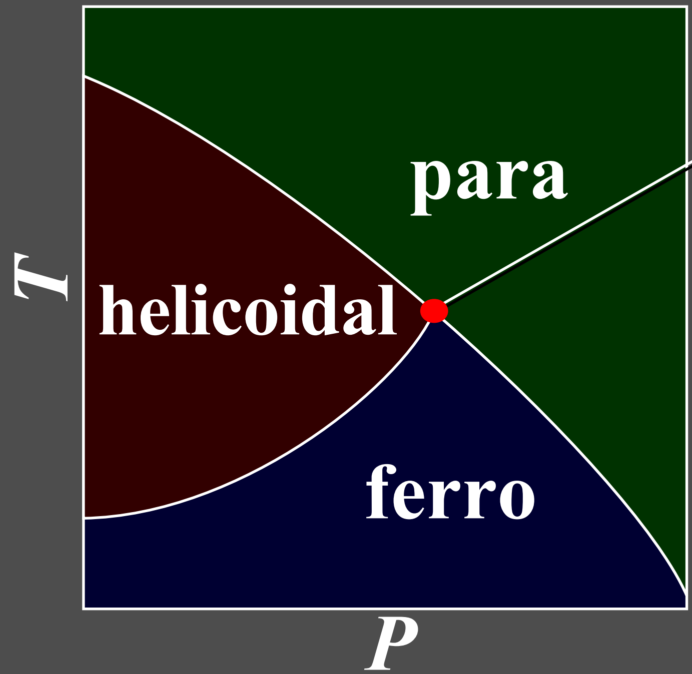
Lifshitz Point

Lifshitz point

$$F = a\Phi^2 + b\Phi^4 + c\Phi^6 + \cdots + \alpha(\nabla\Phi)^2 + \overbrace{\beta(\nabla^2\Phi)^2}^{z=2\text{ anisotropy}} + \cdots$$

$(b, \beta > 0)$

Schematic phase diagram of
a magnetic system with a Lifshitz point



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Lifshitz Point

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$$F = a\Phi^2 + b\Phi^4 + c\Phi^6 + \cdots + \alpha(\nabla\Phi)^2 + \overbrace{\beta(\nabla^2\Phi)^2}^{z=2\ \text{anisotropy}} + \cdots$$

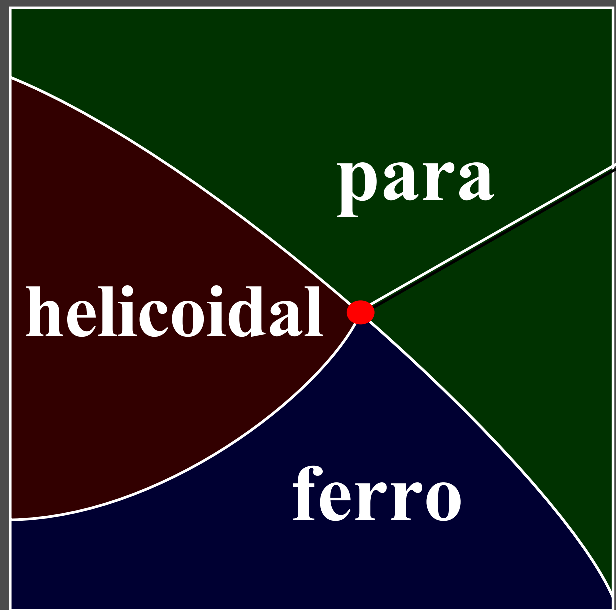
$(b,\beta > 0)$

Schematic phase diagram of
a magnetic system with a Lifshitz point

Lifshitz points may be found in various systems:

magnetic material
liquid crystal
high-Tc superconductor
finite density QCD
• • •

T



P

Lifshitz point
$a = \alpha = 0$

Hornreich, Luban, Shtrikman, 1975, PRL35(1975)0678

Lifshitz Point

3d ANNNI model

ferro antiferro

$$H_{\mathrm{ANNNI}} = -J_1 \sum_{<nm>} \sigma_n \cdot \sigma_m + J_2 \sum_n \sigma_n \cdot \sigma_{n+2x}$$

Lifshitz Point

3d ANNNI model

ferro antiferro

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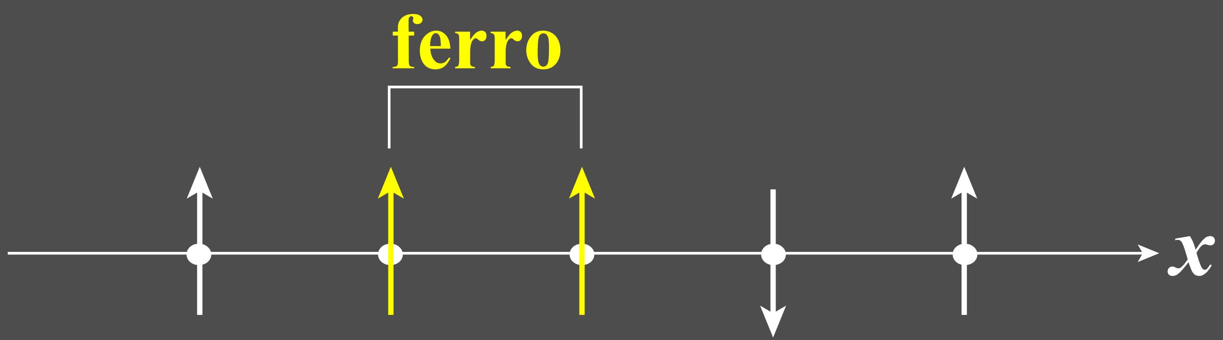
nearest-neighbor



Lifshitz Point

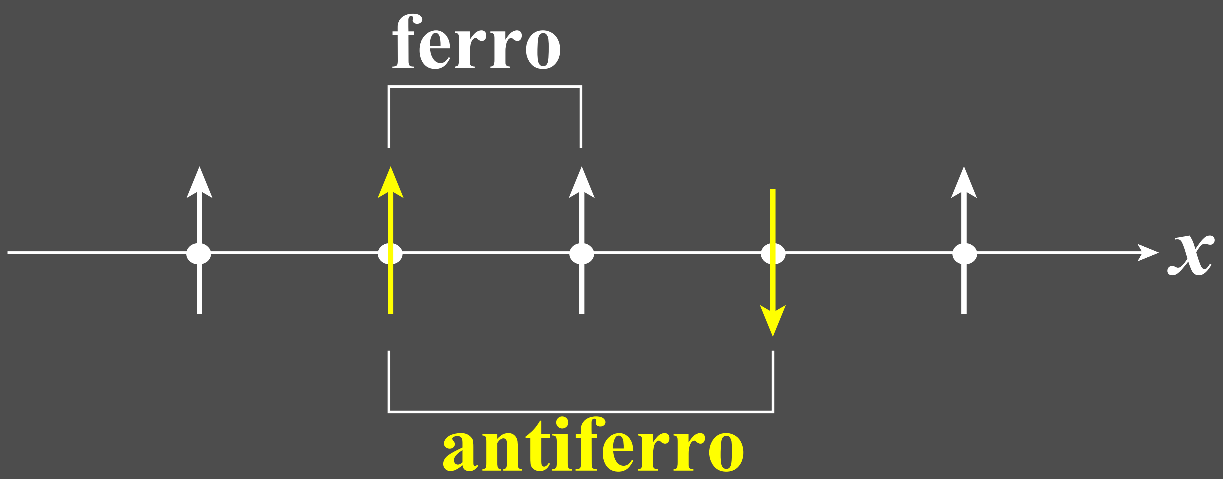
3d ANNNI model

ferro antiferro anisotropy

$$H_{\mathrm{ANNNI}} = -J_1 \sum_{<nm>} \sigma_n \cdot \sigma_m + J_2 \sum_{n} \sigma_n \cdot \sigma_{n+2x}$$

nearest-neighbor

next-nearest-neighbor

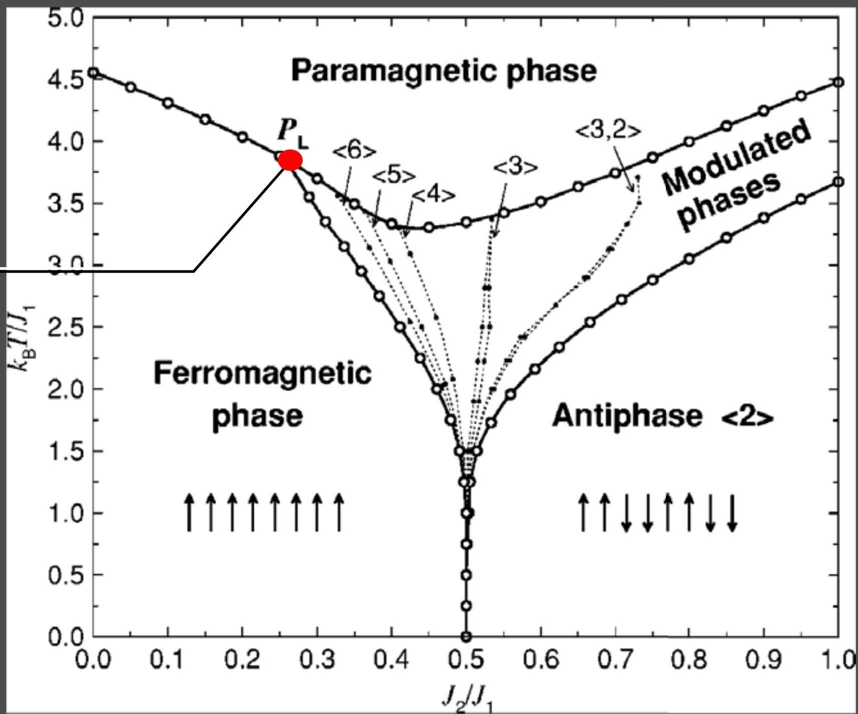


Lifshitz Point

3d ANNNI model

$$H_{\mathrm{ANNNI}} = \overbrace{-J_1 \sum_{<nm>} \sigma_n \cdot \sigma_m}^{\text{ferro}} + \overbrace{J_2 \sum_n \sigma_n \cdot \sigma_{n+2x}}^{\text{antiferro}}$$

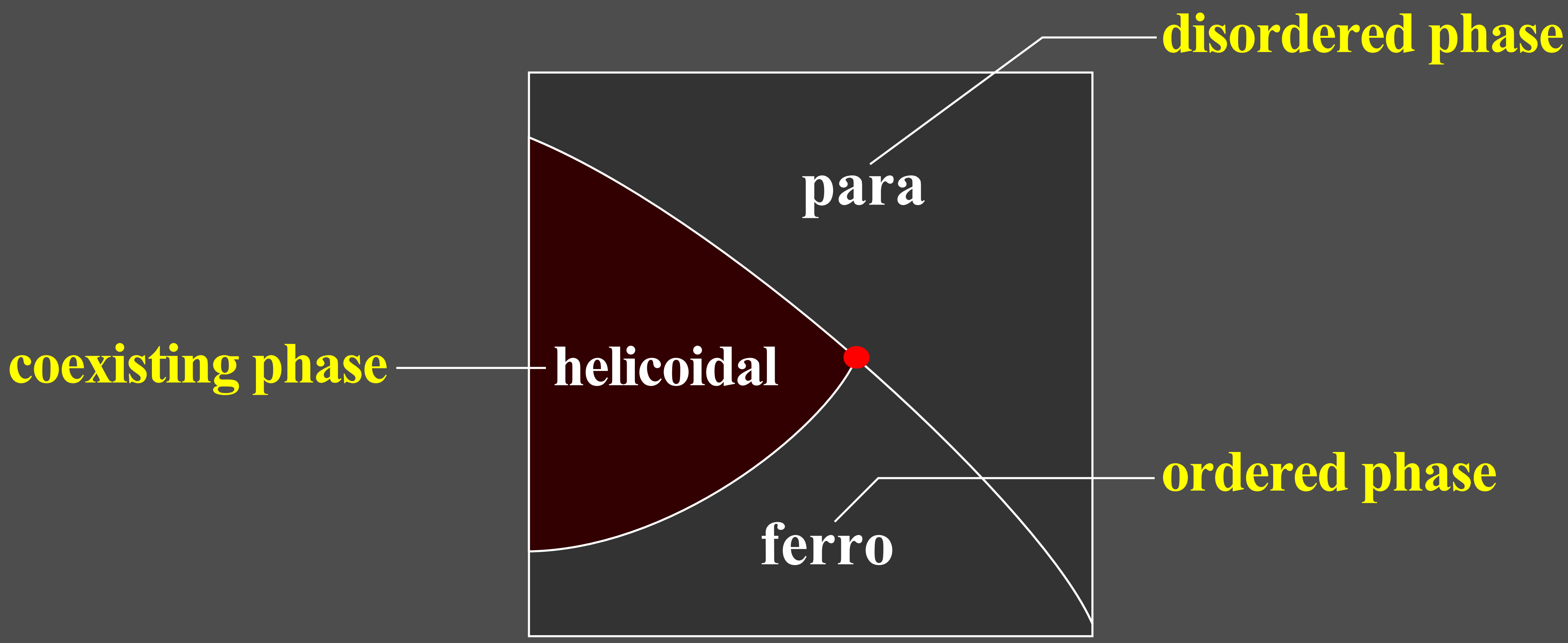
Lifshitz point



Gendiar, Nishino 2005,
PRB71(2005)02404

Lifshitz Point

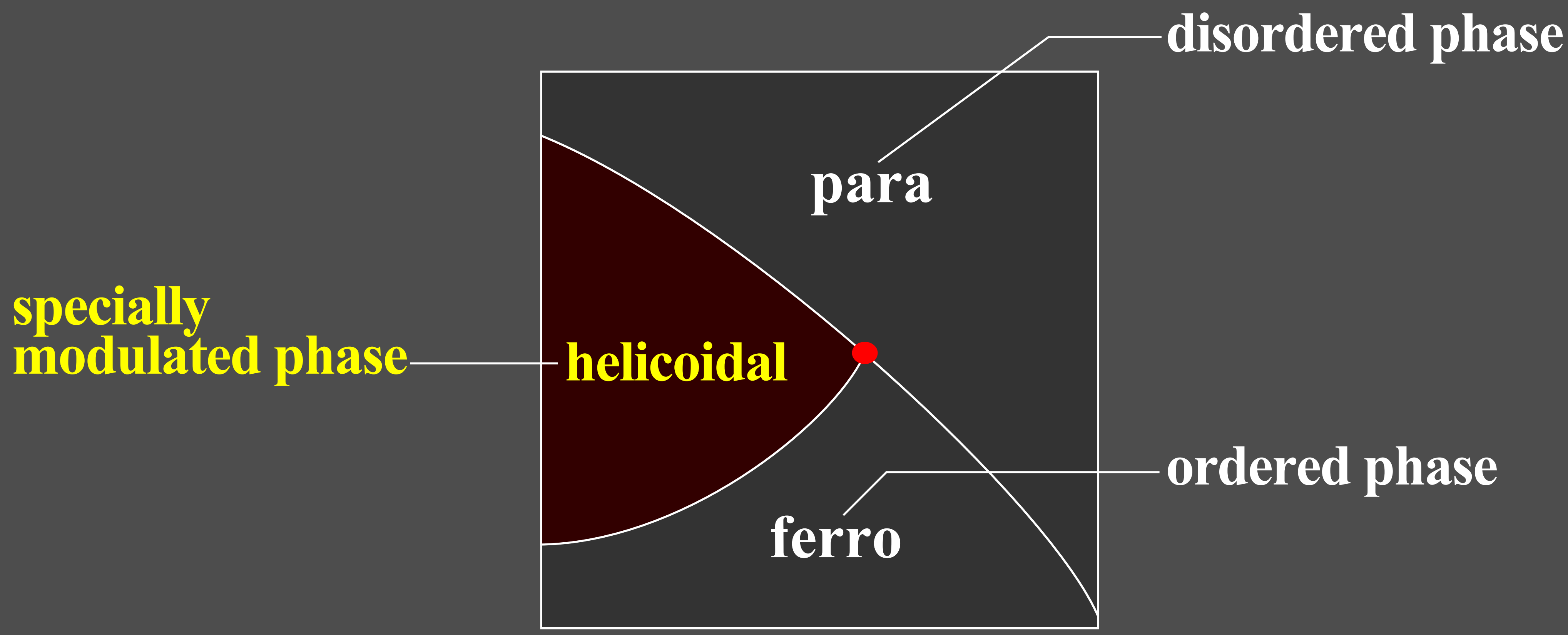
Coexisting or Competition among order-disorder phases



Lifshitz Point

Coexisting or Competition among order-disorder phases

⇒ spatially modulated phase

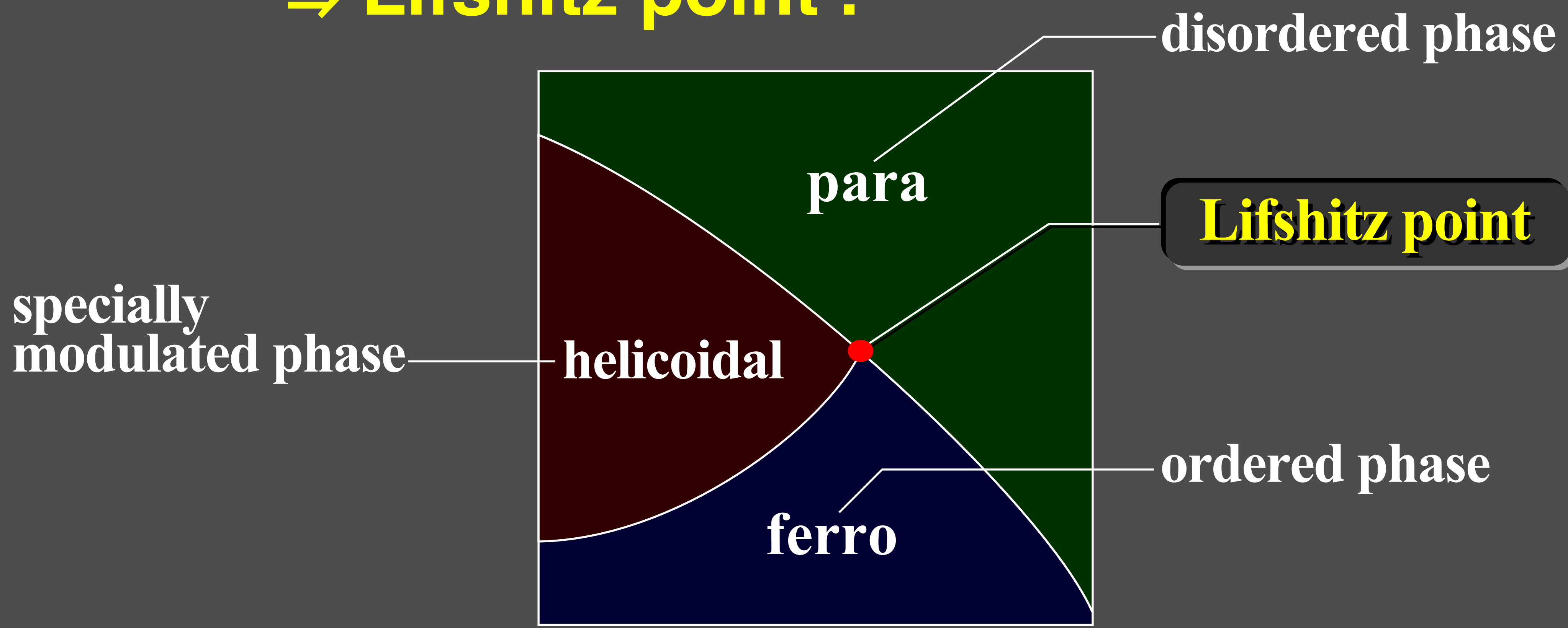


Lifshitz Point

Coexisting or Competition among order-disorder phases

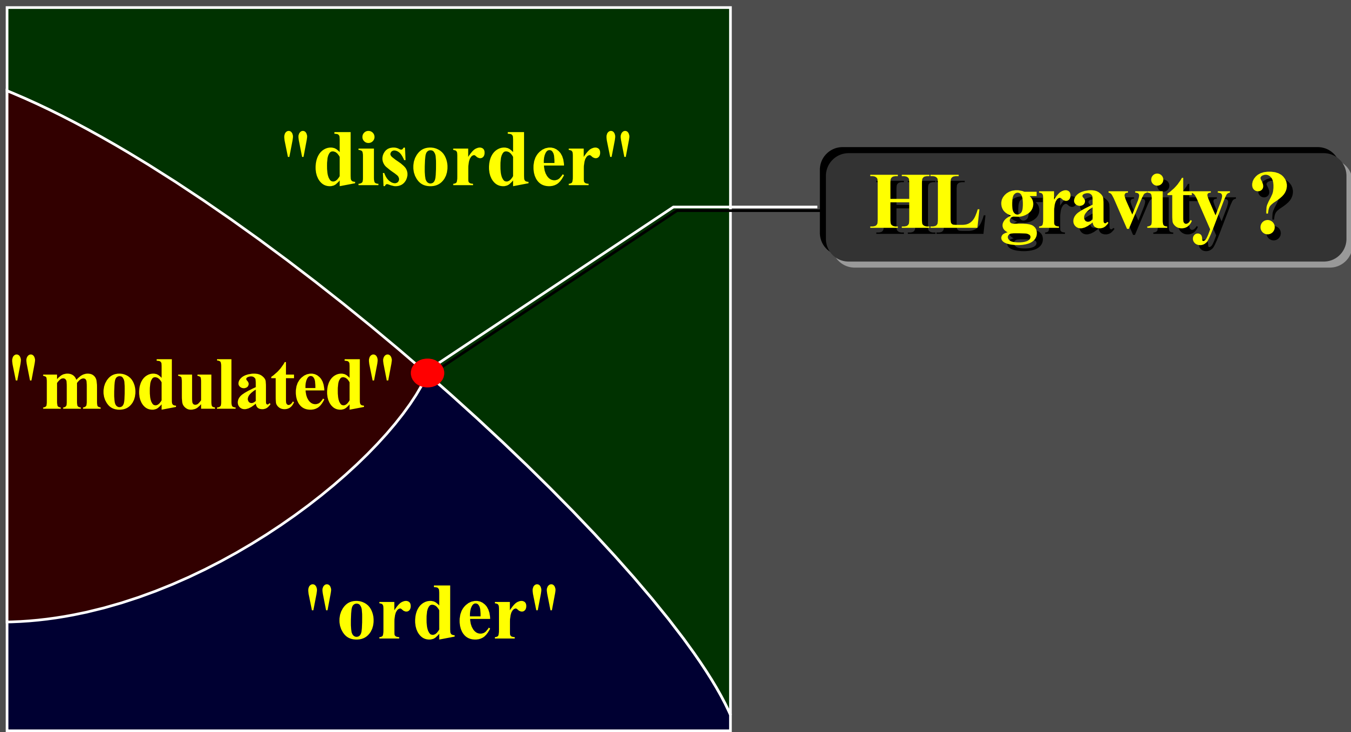
⇒ spatially modulated phase

⇒ Lifshitz point?



Lifshitz Point

Hořava-Lifshitz gravity could be realized if the phase structure of Quantum Gravity is schematically given by



Detailed Balance

Hořava required the detailed balance for the potential.

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"Detailed balance condition"

$$V_{\mathrm{HL}}(g) = \left(\frac{1}{\sqrt{g}}\frac{\delta W[g]}{\delta g_{ij}}\right) \mathcal{G}_{ijkl} \left(\frac{1}{\sqrt{g}}\frac{\delta W[g]}{\delta g_{kl}}\right)$$

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$W[g]$ **must contain** ∂^3 **with** $z=3$.

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a unique choice:

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a unique choice:

$$W[g] = S_{3d\,\mathrm{CS}}$$

3d gravitational Chern-Simons action

$$S_{3d\mathrm{CS}} = \int_\Sigma \mathrm{Tr}[\Gamma\wedge d\Gamma + \Gamma\wedge\Gamma\wedge\Gamma]$$

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$W[g]$ must contain ∂^3 with $z=3$.
a unique choice:

3d Einstein's Gravity

$$W[g] = S_{3d\,\mathrm{CS}} + S_{3d\,\mathrm{EG}}$$

3d gravitational Chern-Simons action

$$S_{3d\,\mathrm{CS}} = \int_\Sigma \mathrm{Tr}[\Gamma\wedge d\Gamma + \Gamma\wedge\Gamma\wedge\Gamma]$$

relevant deformation

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Detailed Balance

Is the detailed balance condition necessary?

Detailed Balance

Is the detailed balance condition necessary?

merely simplicity?

or

a profound physical meaning?

Detailed Balance

Suppose that $\boxed{\Psi_0[\phi] = \exp\left\{-W[\phi]\right\}}$ **is a vacuum wavefunction with** $E_0 = 0$**.**

Detailed Balance

Suppose that $\boxed{\Psi_0[\phi] = \exp\left\{-W[\phi]\right\}}$ is a vacuum wavefunction with $E_0 = 0$. **Then, Hamiltonian $\mathcal{H}$ may be reconstructed as**

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$$\mathcal{H} = \left(-\frac{\delta}{\delta\phi} + \frac{\delta W[\phi]}{\delta\phi}\right)\left(\frac{\delta}{\delta\phi} + \frac{\delta W[\phi]}{\delta\phi}\right)$$

Obviously, $\mathcal{H} = Q^\dagger Q \geq 0, \quad Q\Psi_0 = 0.$

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$$= \left(i\Pi_\phi + \frac{\delta W[\phi]}{\delta\phi}\right)\left(-i\Pi_\phi + \frac{\delta W[\phi]}{\delta\phi}\right)$$

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$$= (\Pi_\phi)^2 + \left(\frac{\delta W[\phi]}{\delta\phi} \right)^2$$

Detailed Balance

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$\Pi_\phi \equiv \dot{\phi}$

$$\boxed{\mathcal{L} = (\dot{\phi})^2 - \left(\frac{\delta W[\phi]}{\delta\phi}\right)^2}$$

Detailed Balance

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$\Pi_\phi \equiv \dot{\phi}$

$$\boxed{\mathcal{L} = (\dot{\phi})^2 - \underbrace{\left(\frac{\delta W[\phi]}{\delta\phi}\right)^2}_{\text{detailed balance!}}}$$

Detailed Balance

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Obviously, $\mathcal{H} = Q^\dagger Q \geq 0, \quad Q\Psi_0 = 0.$

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$$= (\Pi_\phi)^2 + \left(\frac{\delta W[\phi]}{\delta\phi}\right)^2 - \frac{\delta^2 W[\phi]}{\delta\phi^2}$$

$\Pi_\phi \equiv \dot{\phi}$

$$\boxed{\mathcal{L} = (\dot{\phi})^2 - \left(\frac{\delta W[\phi]}{\delta\phi}\right)^2 - \frac{\delta^2 W[\phi]}{\delta\phi^2}}$$

?

detailed balance!

Detailed Balance

Thus, we can introduce the "time" coordinate from the vacuum wavefunction!

$$\Psi_0[\phi] = \exp\left\{-W[\phi]\right\}$$ vacuum wavefunction

$$\Updownarrow \quad \mathcal{H} = Q^\dagger Q \geq 0, \quad Q\Psi_0 = 0$$

$$\mathcal{L} = (\dot{\phi})^2 - \left(\frac{\delta W[\phi]}{\delta \phi}\right)^2$$ Lagrangian

Detailed Balance

Examples: a relativistic scalar in D+1 dim.

Vacuum wavefunction

$$\Psi_0[\phi] = \exp\left\{-\int d^Dk\ |k|\ \tilde{\phi}^2\right\}$$

vacuum wavefunction
of harmonic oscillator

Detailed Balance

Examples: a relativistic scalar in D+1 dim.

Vacuum wavefunction

$$\Psi_0[\phi] = \exp\left\{ -\int d^D k \, |k| \, \tilde{\phi}^2 \right\}$$

vacuum wavefunction
of harmonic oscillator

$\Updownarrow$

$$\mathcal{L} = (\dot{\phi})^2 - \left(\frac{\delta W[\phi]}{\delta \phi}\right)^2 = (\dot{\phi})^2 - (\partial \phi)^2$$

Lagragian of a relativistic scalar!

Detailed Balance

Examples: a Lifshitz scalar in D+1 dim.

Vacuum wavefunction

D-dim. scalar action!

$$\Psi_0[\phi] = \exp\left\{ - \int d^D x \, (\partial\phi)^2 \right\}$$

Detailed Balance

Examples: a Lifshitz scalar in D+1 dim.

Vacuum wavefunction

$$\Psi_0[\phi] = \exp\left\{ - \underbrace{\int d^D x \, (\partial\phi)^2}_{D\text{-dim. scalar action!}} \right\}$$

$\Updownarrow$

Lifshitz 1941, *Zh.Eksp.Teor.Fiz.11(1941)255&269.*

$$\mathcal{L} = (\dot{\phi})^2 - \left(\frac{\delta W[\phi]}{\delta\phi}\right)^2 = (\dot{\phi})^2 - \underbrace{(\partial^2\phi)^2}_{z=2\ \text{Lifshitz scalar!}}$$

Detailed Balance

Examples: Yang-Mills theory in 3+1 dim.

Vacuum wavefunction at IR

$$\Psi_0[A_i] \approx \exp\Big\{ -S_{\mathrm{3dYM}}[A_i] \Big\}$$

Greensite 1979, NPB158(1979)469;
Arisue, Kato and Fujiwara 1983, PTP70(1983)229;
Mansfield 1994, NPB418(1994)113;
Kawamura, Maeda and M.S 1997, PTP97(1997)939.

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$\Updownarrow$

$$\mathcal{L} = \mathrm{Tr}(E_i E_i) - \mathrm{Tr}(D_i F_{ik} D_j F_{jk})$$

$z=2$ Lifshitz Yang-Mills theory

Detailed Balance

Examples: Einstein Gravity in 3+1 dim.

Solutions to Wheeler-DeWitt eq.

Detailed Balance

Examples: Einstein Gravity in 3+1 dim.

Solutions to Wheeler-DeWitt eq.

3d gravitational CS action

$$\Psi[g] = \exp\left\{ -S_{3d\,CS}[g] \right\}$$

in Ashtekar formalism

Kodama 1990, PRD42(1990)2548.

Detailed Balance

Examples: Einstein Gravity in 3+1 dim.

Solutions to Wheeler-DeWitt eq.

3d gravitational CS action

$$\Psi[g] = \exp\left\{ -S_{3\mathrm{dCS}}[g] \right\}$$

in Ashtekar formalism

Kodama 1990, PRD42(1990)2548.

3d Einstein's Gravity action

$$\Psi[g] \approx \exp\left\{ -S_{3\mathrm{dEG}}[g] \right\}$$

in the strong coupling limit

Horiguchi, Maeda and M.S. 1995&1996, PLB344(1995)105; PRD54(1996)1500;

M.S. 2009, Phys.Rev.D79(2009)124038, arXiv:0905.4213.

Detailed Balance

Hořava-Lifshitz Gravity:

$$\Psi_{\mathrm{HL}}[g] = ?$$

$$\Updownarrow$$

$$\mathcal{L}_{\mathrm{HL}} = \frac{2}{\kappa^2} K_{ij}\,\mathcal{G}^{ijkl} K_{kl} - \frac{\kappa^2}{2w^4} C^{ij}\,\mathcal{G}_{ijkl}\,C^{kl}$$

Detailed Balance

Hořava-Lifshitz Gravity:

$$\Psi_{\mathrm{HL}}[g] = \exp\{-\underbrace{S_{\mathrm{3dCS}}[g] - S_{\mathrm{3dEG}}[g]}\}$$

3d topological massive gravity
which is probably a finite theory and will governs the renormalization of HL gravity!

$$\Updownarrow$$

$$\mathcal{L}_{\mathrm{HL}} = \frac{2}{\kappa^2} K_{ij} \mathcal{G}^{ijkl} K_{kl} - \frac{\kappa^2}{2w^4} C^{ij} \mathcal{G}_{ijkl} C^{kl}$$

Detailed Balance

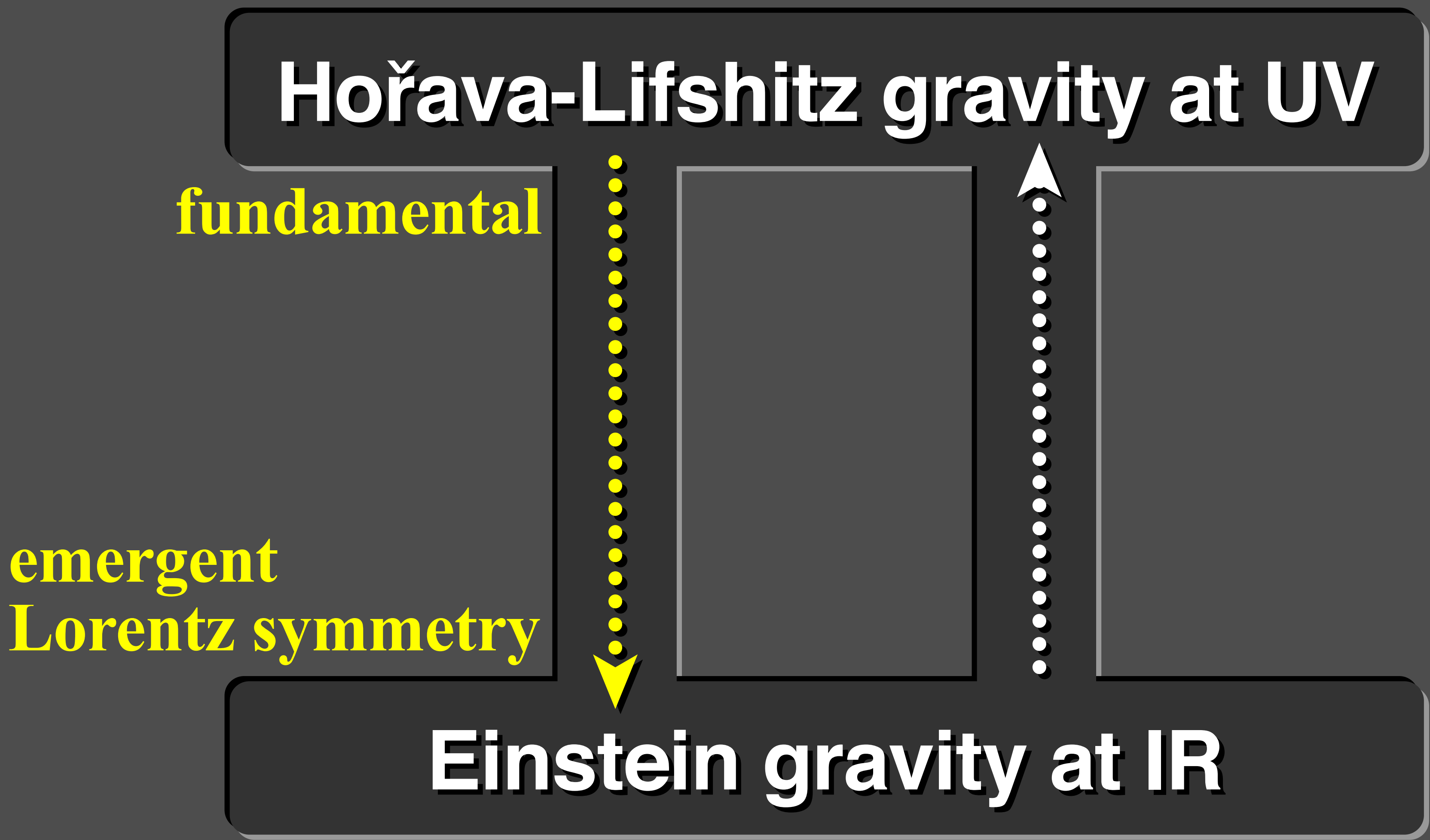
Two different perspective:

Hořava-Lifshitz gravity at UV

Einstein gravity at IR

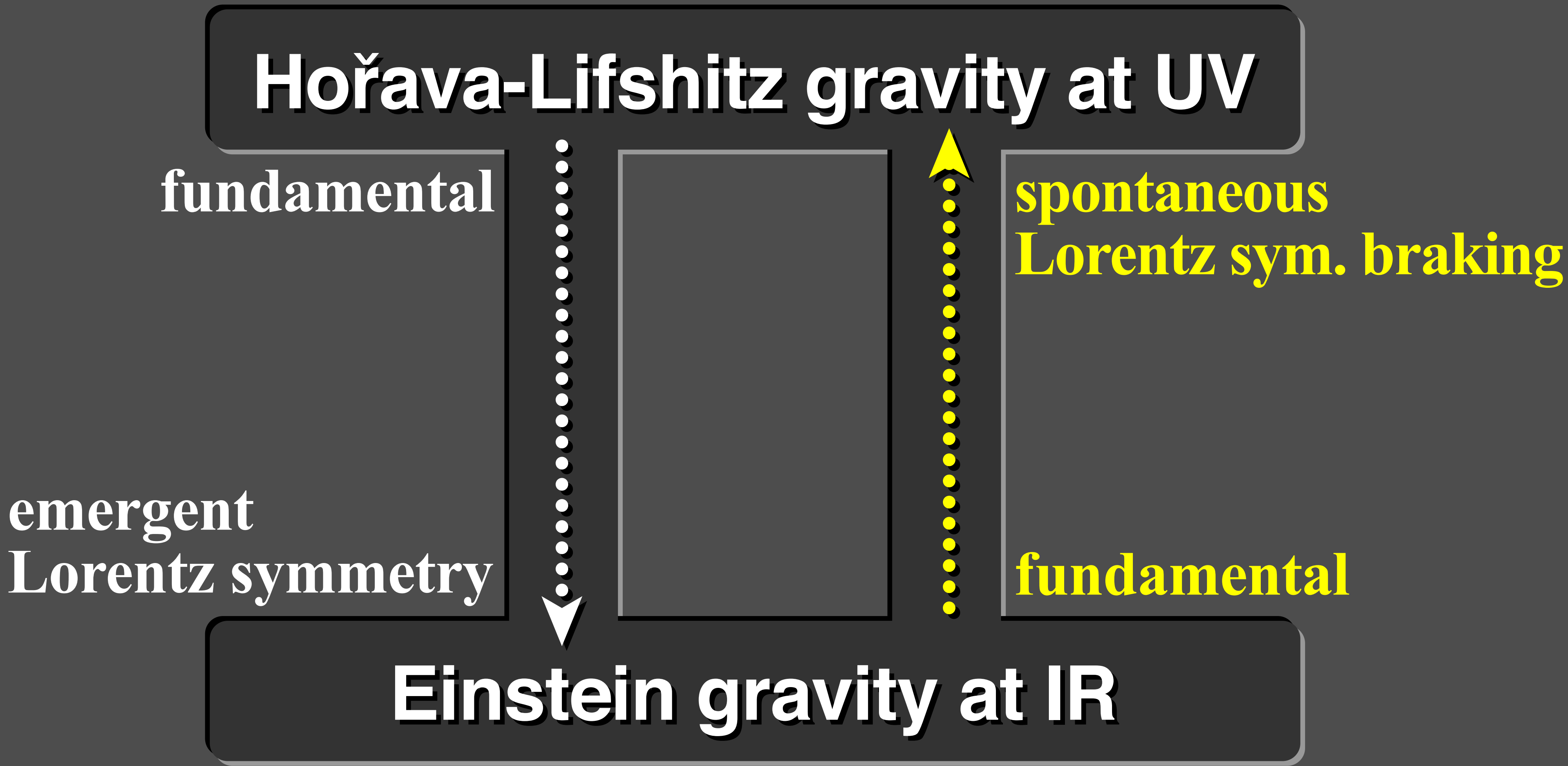
Detailed Balance

Two different perspective:



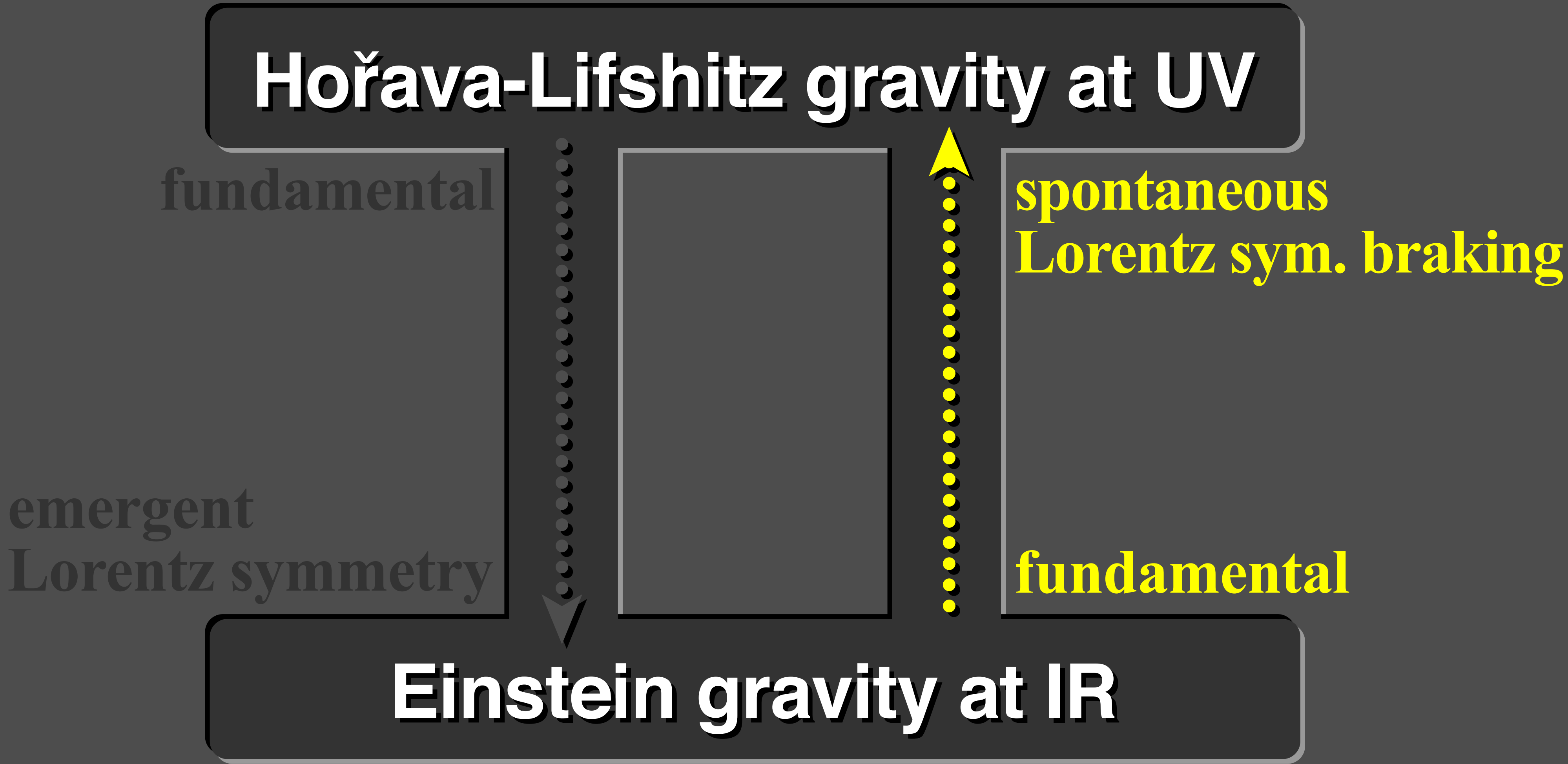
Detailed Balance

Two different perspective:



Detailed Balance

The detailed balance seems to support this perspective!



Let's enjoy Hořava-Lifshitz Gravity!

► **Cosmological Implications**

► **Black Hole Physics**

► **Theoretical Aspects**

Let's enjoy Hořava-Lifshitz Gravity!

► Cosmological Implications

Drastic changes of causal structure & singularities?

- **cosmology without inflation?**
  no horizon/flatness problems?
  scale-invariant fluctuation?
  bounce solutions?

The detailed balance condition should be relaxed?

Let's enjoy Hořava-Lifshitz Gravity!

► Cosmological Implications

Drastic changes of causal structure & singularities?

- cosmology without inflation?
  no horizon/flatness problems?
  scale-invariant fluctuation?
  bounce solutions?

The detailed balance condition should be relaxed?

► Black Hole Physics

- new black hole solutions & thermodynamics
- drastic change of singularities? ($\Leftarrow \partial^6$ terms)
- no BH information paradox?

$$S_{\mathrm{BH}} = \frac{c^3 A}{4 G_N \hbar} \to \infty \,? \quad \text{at UV}$$

Let's enjoy Hořava-Lifshitz Gravity!

▸ Theoretical Aspects

- beyond the power-counting renormalizability?
- Is the detailed balance condition preserved in renormalization and really necessary?
- Does Einstein gravity really appear at IR?
- $\lambda \to 1$ at IR?   Is the HL gravity asymptotically safe?
- Does an extra scalar mode cause any trouble?
- z>3 HL gravity?
- Lifshitz matter?
- nonrelativistic AdS/CFT?

⋮

Appendices

Spectral Dimension

Spectral Dimension:

Causal Dynamical Triangulations approach
Ambjorn, Jurkiewicz and Loll 2005, PRL95(2005)171301

$$d_s = 1 + \frac{D}{z} = \begin{cases} 2 & \text{at UV } (z = D = 3) \\ 4 & \text{at IR } (z = 1,\ D = 3) \end{cases}$$

One way to understand the spectral dimension is to remember the thermal behavior of the system.

Stefan-Boltzmann law: $F \propto T^4$ — spacetime dimensions

$$F \propto \int [d\phi] \exp\left\{ -\int_0^\beta d\tau \int d^D x \left\{ -\phi\left(\partial_\tau{}^2 + \partial_x{}^{2z}\right)\phi \right\} \right\}$$

$$\propto T^{1+D/z}$$

$[x] = -1/z$

$[\tau] = -1$

Dimensional Reduction

Dimensional reduction with random forces:

Systems with random forces may be described by some lower-dimensional theories without random forces.

Imry and Ma 1975, PRL35(1975)1399,
Parisi and Sourlas 1979, PRL43(1979)744.

Stochastic Quantization

Parisi-Wu stochastic quantization in D+1 dim:

Langevin equation

$$\underbrace{\frac{\partial\phi(x,\tau)}{\partial\tau}}_{\text{fictitious time}} = -\underbrace{\frac{\delta S_D[\phi]}{\delta\phi(x)}}_{\text{detailed balance}}\Bigg|_{\phi=\phi(x,\tau)} + \overbrace{\eta(x,\tau)}^{\text{random force}}$$

D-dim. action

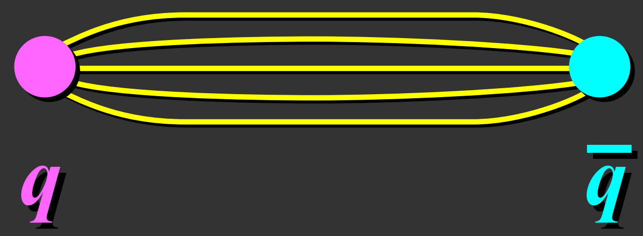
In the equilibrium limit $\tau\to\infty$, the system becomes identical to the D-dim. quantum system governed by the action $S_D[\phi]$.

Parisi and Wu 1981, Sci. Sin.24(1981)483

Dimensional Reduction

Dimensional reduction in QCD:

quark confinement



Nambu, PRD10(1974)4262

This confinement picture suggests that the system could be described by a 1+1-dim. theory!

!

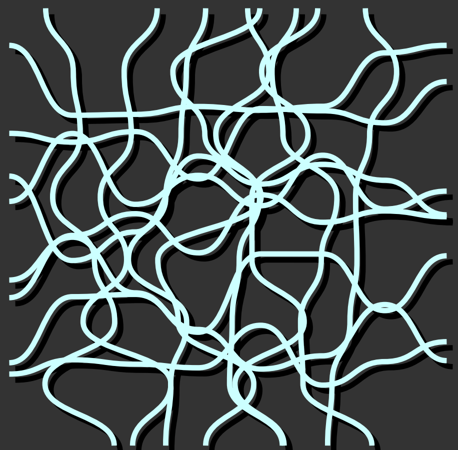
The quarks are trivially confined in 2-dim.

Dimensional Reduction

The idea of DR in QCD is supported by

QCD vacuum

B



spaghetti state

Nielsen and Olesen 1978 & 1979,
NPB144(1978)376;
NPB160(1979)380.

The QCD vacuum will be given by random configuration of magnetic fluxes (spaghetti state).

$\Downarrow$ **dimensional reduction**

2-dim. QCD ??

Dimensional Reduction

The idea of DR in QCD is also supported by

Vacuum wavefunction at IR

$$\Psi_0[A_i] \approx \exp\left\{-S_{3\mathrm{dYM}}[A_i]\right\}$$

Greensite 1979, NPB158(1979)469;
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Mansfield 1994, NPB418(1994)113;
Kawamura, Maeda and M.S 1997, PTP97(1997)939

Wilson loop

$$\underbrace{\langle 0|\, W[C]\, |0\rangle}_{\text{}} = \int [dA_i]_{3\mathrm{d}}\, \Psi_0[A_i]^*\, W[C]\, \Psi_0[A_i]$$

$$\approx \int [dA_i]_{3\mathrm{d}}\, W[C]\, \exp\left\{-2S_{3\mathrm{dYM}}[A_i]\right\}$$

3d→2d DR

$$\approx \int [dA_i]_{2\mathrm{d}}\, W[C]\, \exp\left\{-2S_{2\mathrm{dYM}}[A_i]\right\}$$

→ **Area low!**

Holographic Principle

Dimensional reduction in Quantum Gravity:

holographic principle

All information inside a 3d volume of space should be encoded on a boundary to the region.

- **high energy scattering at Planckian energies**

't Hooft 1990, *NPB335(1990)138*

- **black hole entropy**

$$S_{\mathrm{BH}} = \frac{c^3 \overbrace{A}^{\text{Area}}}{4G_{\mathrm{N}}\hbar}$$

't Hooft 1993, *gr-qc/9310026;*
Susskind 1995, *J.Math.Phys.36(1995)6377*

Problem of Time

- ►What is "time" in (quantum) gravity?

Wheeler-DeWitt equation

$$H\Psi[g]=0 \longrightarrow \frac{\partial}{\partial t}\Psi[g]=0$$

No time evolution!