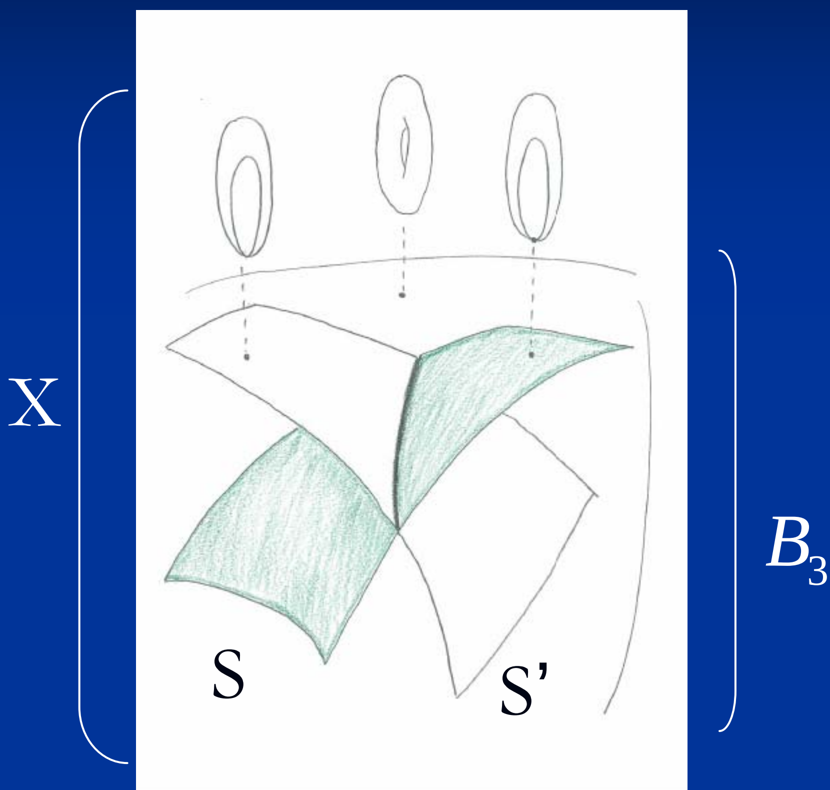
F-理論のコンパクト化の理論的進展と現象論への応用

夏の基研研究会　2009/07/07 (Tue.)

Taizan Watari　(IPMU)

IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

F-theory / elliptic fibered CY4



elliptic fiber: 2 independent 1-cycles $\{\alpha, \beta\}$

(p,q) string in Type IIB

$= $ M2-brane wrapped on $(p\alpha + q\beta)$.

(p,q) 7-brane = locus in B_3 where

$(p\alpha + q\beta)$ degenerates.

$$\Delta = 4f^3 + 27g^2 = 0.$$

discriminant locus

$$X: \quad y^2 = x^3 + f\,x + g.$$

$$f \in \Gamma(B_3; O(-4K_{B3})),$$

$$g \in \Gamma(B_3; O(-6K_{B3})).$$

non-Abelian gauge theory in a case

$$(\Delta = 0) = nS + S' + \cdots.$$

何が難しいのか？

- Type IIB を SL(2,Z) で 7-brane まわりにひねる。

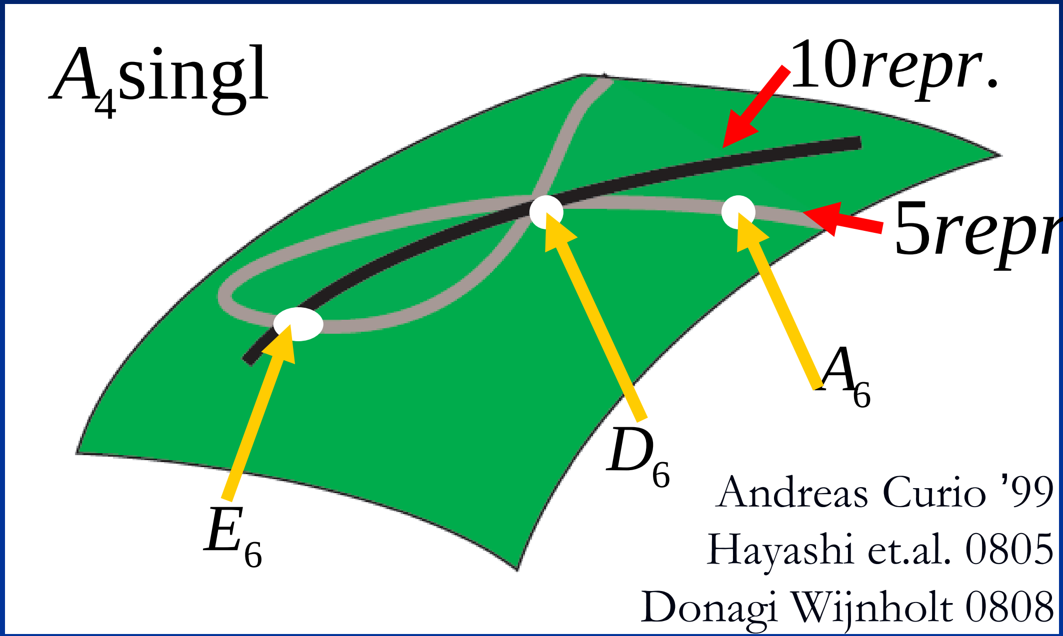
$$(\alpha,\beta)' M_{p,q} = (\alpha,\beta),$$
$$(B,C)' M_{p,q} = (B,C),$$
$$(1:\tau)' M_{p,q} = (1:\tau),$$

$$\begin{pmatrix} r \\ s \end{pmatrix}' = M_{p,q} \begin{pmatrix} r \\ s \end{pmatrix}.$$

$$M_{p,q} = \begin{pmatrix} 1-pq & p^2 \\ -q^2 & 1+pq \end{pmatrix}.$$

- それだけ？
  - 7-brane 直上で場は定義されているのか？
    (p,q) 7-brane 上では $(pB+qC)$ は定義できている。
  - 互いに non-local [$(p,q) \propto (p',q')$ でない] 7-brane が交差するところ(codimension-2)では？Codim.-3 では？
  - 特異点まわりの幾何はどうなっている？

F-theory w/ unbroken SU(5) symmetry



$$y^2 = x^3 + a_5 xy + a_4 zx^2$$
$$+ a_3 z^2 y + a_2 z^3 x + a_0 z^5 + \cdots.$$

Bershadsky et.al. '96

$$10: \quad a_5 = 0,$$

$$5: \; P^{(5)} := a_0 a_5^2 - a_2 a_5 a_3 + a_4 a_3^2 = 0.$$

$$\Delta \propto z^5 (a_5^4 P^{(5)} + O(z)).$$
$$5S + S'$$

Charged Matter chiral multiplets: hol sections of a line bundle on a curve

Hayashi et.al. 0805.

Yukawa: recombination of M2-branes on 2-cycles

Tatar Watari '06
Donagi-Wijnholt 0802
Beasley et.al. 0802
Hayashi et.al. 0805.

$E_6 \rightarrow A_4$ up-type Yukawa generated.

$D_6 \rightarrow A_4$ down-type Yukawa generated

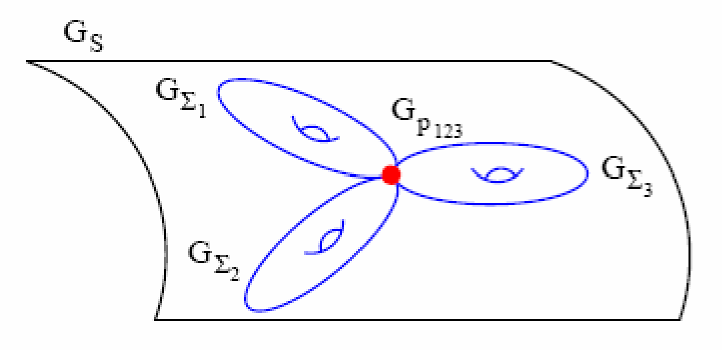
yet to be done (topic of this talk)

- Yukawa from codim.-3 singularity, but F-theory does not have any microscopic formulations.
- How to deal with SU(5) neutral fields (like RH neutrinos)

a side remark

- Harvard group's claim "triple intersection at E-type singularity": actually not generic.

(林くんのポスター)



(taken from Beasley Heckman Vafa 0802)

References

- R. Tatar (U. Liverpool), 渡利(Berkeley) th/0602238
- Donagi Wijnholt 0802.2969
- Beasley Heckman Vafa 0802.3391
- 林博貴(本郷)、Tatar, 戸田幸伸(IPMU), 渡利(本郷)、山崎雅人(本郷) 0805.1057

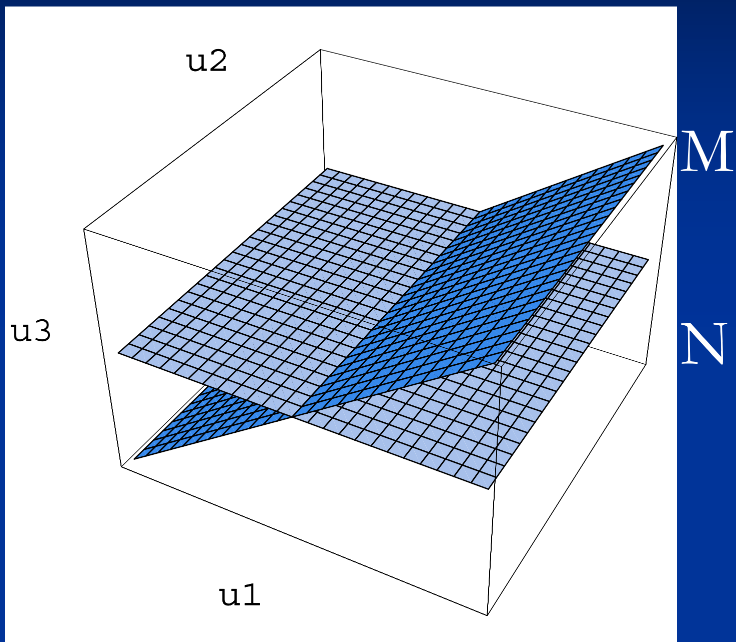
- 林, 川野輝彦(本郷), Tatar, 渡利(IPMU) 0901.4941
- Tatar, 土屋陽一(本郷)、渡利 0905.2289

plan of this talk

- Theoretical progress
  - Field Theory Local Models
  - Heterotic--F duality revisited
- Some results in phenomenology
  - proton decay
  - right-handed neutrinos: Yukawa couplings and Majorana masses
  - cubic couipling of NMSSM

Field-theory Local Models

intersecting D7—D7 in field theory



U(N+M) Super Yang-Mills

$$\Rightarrow \begin{cases} A_M \quad (M = 0, \cdots, 7) \\ \Phi = (X^8 + iX^9)/\alpha'. \end{cases}$$

intersecting configuration

$$\Leftrightarrow \quad \langle \Phi \rangle (u_1, u_2) \neq 0.$$

Higgsed, and $SU(N) \times SU(M)$ unbroken symmetry

D7—D7 open string in bifundamental $(N, \bar{M})$ representation:

$\Rightarrow$ localized mode under the bg. $\langle \Phi \rangle$.

$$\delta\Phi = f(u_2)\exp[-|u_1|^2],$$

$$\delta A_{\bar{1}} = f(u_2)\exp[-|u_1|^2].$$

(the same config. in F-theory language)

$X \to B_3$ (elliptic fibration)

$(\Delta = 0) = N\,S + M\,S' + \cdots.$

S, S': cpx codim.-1 in B_3.

U(N+M) Super Yang-Mills

$$\begin{cases} A_M & (M = 0, \cdots, 7) \\ \Phi & \text{(locally cpx scalar)} \end{cases}$$

local geometry of X around $S \cdot S'$:

$$y^2 = x^2 + z^N (z - z_0(u_1, u_2))^M + \cdots.$$

$$\Longleftrightarrow \quad \langle \Phi \rangle (u_1, u_2) = \mathrm{diag}(0^N, z_0(u_1, u_2)^M).$$

A_{N+M-1} singularity $\longrightarrow$ $A_{N-1} + A_{M-1}$

$SU(N+M)$ sym. $\longrightarrow$ $SU(N) \times SU(M)$

D7—D7 open string in bifundamental $(N, \bar{M})$ representation:

$\Rightarrow$ localized mode under the bg. $\langle \Phi \rangle$.

$$\delta\Phi = f(u_2) \exp[-|u_1|^2],$$

$$\delta A_{\bar{1}} = f(u_2) \exp[-|u_1|^2].$$

- deformation of $E_{6,7,8}$: use $E_{6,7,8}$ gauge theory with Higgs vaccum value $\langle \varphi \rangle$.
- φ field: section of $K_S = \wedge^2 T^* S$ on S, not on $N_{S|B_3}$.

  Katz Vafa '97, Donagi Wijnholt 0802, Beasley Heckman Vafa 0802.

- action on 7+1 dim.

$$S \propto \int_{\mathbb{R}^{3,1} \times S} [tr(F_{MN} F^{MN}) + (\text{supersymmetric})] + O(g_s l_s^4)$$

Beasley et.al. 0802, Hayashi et.al. 0901.

- meaning of this action
  - capture some D.O.F of F-theory (土屋くんのポスター)
  - effective action. unknown part of F-theory shoved into higher order terms. (hidden under the carpet)

eff. theory below the KK scale

- need to see the entire (e.g. A_4) singularity locus S.
- $X \to B_3$ looks locally an ALE fibration on $U_\alpha \subset S$.

ALE fiber space of ADE type

ADE gauge theory

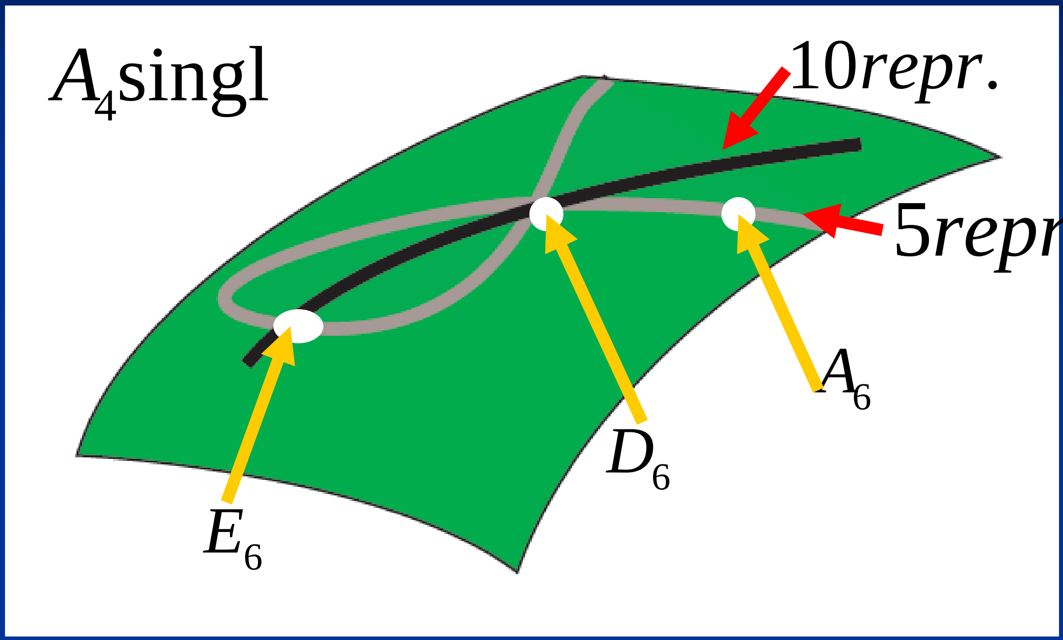
partially deformed ADE singularity

partially Higgsed ADE gauge theory

fiber spaces glued together between two adjacent patches U_α

gauge theories glued together between two adjacent patches U_α

field theory local models



D_5, A_5 gauge theories along codim.-2 curves

E_6 gauge theory (for up-type Yukawa)

$D_6 = SO(12)$ gauge theory (for down-type Yukawa)

$A_6 = SU(7)$ gauge theory

around codim.-3 points

- How to translate local geometry into $\langle\varphi\rangle$?
- How to glue local models together? (林くん、土屋くんのポスター)
- to calculate Yukawa, use $\Delta W = \int_S tr(\varphi \wedge F) \Leftarrow \int_X \Omega \wedge G.$

Heterotic-F duality revisited

$Het/T^2 - F/K3$ duality

Het: Narain compactification

F: cpx str of elliptic fibration on $\mathbb{P}^1$

$\Gamma \backslash O(2,18;\mathbb{R})/O(2)\times O(18)$

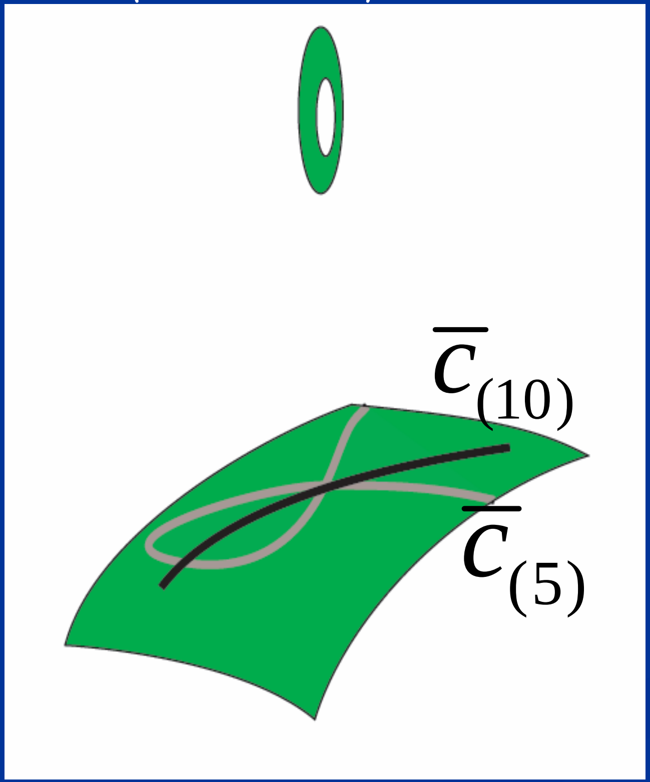
moduli space

Fiber this duality adiabatically, to obtain

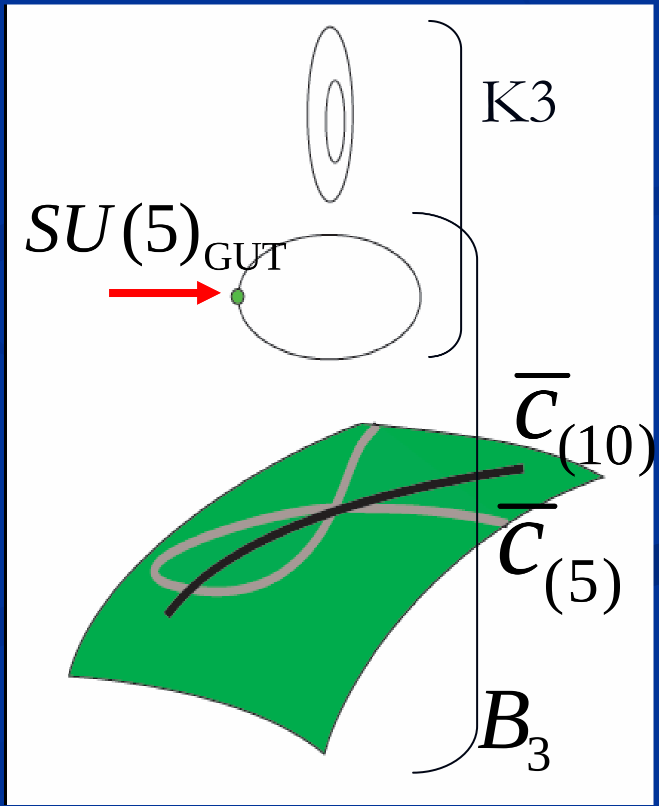
$\mathrm{Het}/(T^2 \to Z \to B_2)$ ⟷ $F/(K3 \to X \to B_2)$

e.g. $E_8 \to \langle SU(5)_{\mathrm{bdl}} \rangle \times SU(5)_{\mathrm{GUT}}$.

Z



B_2



X

moduli map I
(Het sugra / F stable degen. limit)

- moduli D.O.F counting, 90's
  - follow symmetry breaking / singularity deformation on both sides

Morrison Vafa '96, Bershadsky et.al. '96, '97

  - [cpx str of Z + bdle moduli (Het)] to [cpx str of X (F)]
- moduli map [incl. mixing and normalization]
  - 1[2] vanishing Wilson line(s) to codim.-2[3] singularity

Line bundles on matter curves: determined by these data

Codim.-3 singularities: detected by Δ in any cpx str in F-theory

Hayashi et.al. 0805, 0901

moduli map II
(Het sugra / F stable degen. limit)

- Het side $(E_8 \times E_8)$ w/ rank V $= N \leq 5$ bndle
  - N Wilson lines $(A_8 - iA_9)$ on T^2 = N points on T^2 =zero points of elliptic fcn $s = a_5 yx + a_4 x^2 + a_3 y + a_2 x + a_0$ on the elliptic curve (fiber) $y^2 = x^3 + f_0 x + g_0$
  - N pnts $s = 0$ in each fiber defines C_V in Z.

Friedman Morgan Witten '97

- F side K3 (elliptic fibration on $\mathbb{P}^1$)

$$y^2 = (x^3 + f_0 z^4 x + g_0 z^6) + (a_5 yx + a_4 zx^2 + a_3 z^2 y + a_2 z^3 x + a_0 z^5).$$

↓

$$\tilde{y}^2 = \tilde{x}^3 + f\,\tilde{x} + g, \begin{cases} f = f_0 z^4 + a_2 z^3 + \left(-\frac{1}{3} a_4^2 + \frac{1}{2} a_5 a_3\right) z^2 + \cdots \\ g = g_0 z^6 + \left(a_0 - \frac{1}{3} a_4 f_0\right) z^5 + \cdots. \end{cases}$$

Katz Mayr Vafa '97
Curio Donagi '98
Hayashi et.al. 0805

F-theory in field theory

- SUSY condition of the background

$$\begin{cases} \omega \wedge F - \dfrac{|\alpha|^2}{2}[\varphi, \overline{\varphi}] = 0, \\ \overline{\partial}\varphi = 0, \\ F^{0,2} = 0 \end{cases} \quad \text{on } S = B_2.$$

e.g. $E_8 \to \langle SU(5)_{\text{bdl}} \rangle \times SU(5)_{\text{GUT}}.$

$$adj. \to (adj.,1) + (1, adj.) + [(5,10) + (\wedge^2 5, \overline{5})] + h.c.$$

$$A = \langle A \rangle + \delta A = \langle A \rangle + \psi, \quad \varphi = \langle \varphi \rangle + \delta\varphi = \langle \varphi \rangle + \chi.$$

$$\begin{cases} \omega \wedge D\psi - \dfrac{|\alpha|^2}{2}\rho(\varphi)\chi = 0, \\ \overline{\partial}\chi - \rho(\varphi)\psi = 0, \\ \overline{\partial}\psi = 0. \end{cases}$$

$C_{\rho(V)}$ (spectral surface)

$$\det(\xi \times Id - \rho(\varphi_{12})) = 0$$

divisor in $K_S = \wedge^2 T^* S$

F-theory in field theory

- SUSY condition of the background

$$\begin{cases} \omega \wedge F - \dfrac{|\alpha|^2}{2}[\varphi, \overline{\varphi}] = 0, \\ \overline{\partial}\varphi = 0, \\ F^{0,2} = 0 \end{cases} \quad \text{on } S = B_2.$$

(in Heterotic theory)

$$\begin{cases} \omega \wedge \omega \wedge F = 0, \\ F^{0,2} = 0 \end{cases} \quad \text{on } Z.$$

$\Longleftrightarrow$ $\varphi_{12} \leftrightarrow A_{\overline{3}}$

e.g. $E_8 \to \langle SU(5)_{\mathrm{bdl}} \rangle \times SU(5)_{\mathrm{GUT}}.$

$$adj. \to (adj.,1) + (1, adj.) + [(5,10) + (\wedge^2 5, \overline{5})] + h.c.$$

$$A = \langle A \rangle + \delta A = \langle A \rangle + \psi, \quad \varphi = \langle \varphi \rangle + \delta\varphi = \langle \varphi \rangle + \chi.$$

$$\begin{cases} \omega \wedge D\psi - \dfrac{|\alpha|^2}{2}\rho(\varphi)\chi = 0, \\ \overline{\partial}\chi - \rho(\varphi)\psi = 0, \\ \overline{\partial}\psi = 0. \end{cases}$$

$C_{\rho(V)}$ (spectral surface)

$$\det(\xi \times Id - \rho(\varphi_{12})) = 0$$

divisor in $K_S = \wedge^2 T^* S$

a new understanding of Het-F duality

Hayashi et.al. 0901

- moduli map

Het/elliptic CY3 F w/ D=8 field theory F / elliptic CY4

(K3/del Pezzo fibration)

vector bdle Higgs bdle

$$(C_{\rho(V)}, \mathcal{N}_{\rho(V)})^{\text{Het}} = (C_{\rho(V)}, \mathcal{N}_{\rho(V)})^{\text{F}}.$$

→ ALE fibration deformed ADE

(Hitchin/Katz—Vafa map)

- spectral surface $C_{\rho(V)}$ in F-theory description
  - one for each one of $adj. \to (adj.,1) + (1, adj.) \oplus_i (U_i, R_i)$
    (unlike discriminant locus)
  - was the missing link in the Het – IIB duality.

(林くん、土屋くんのポスター)

Proton Decay

proton decay

Tatar TW 0806

- Friedman-Witten '02 for M/G2:
  - decay rate "diverges" when matter localizes

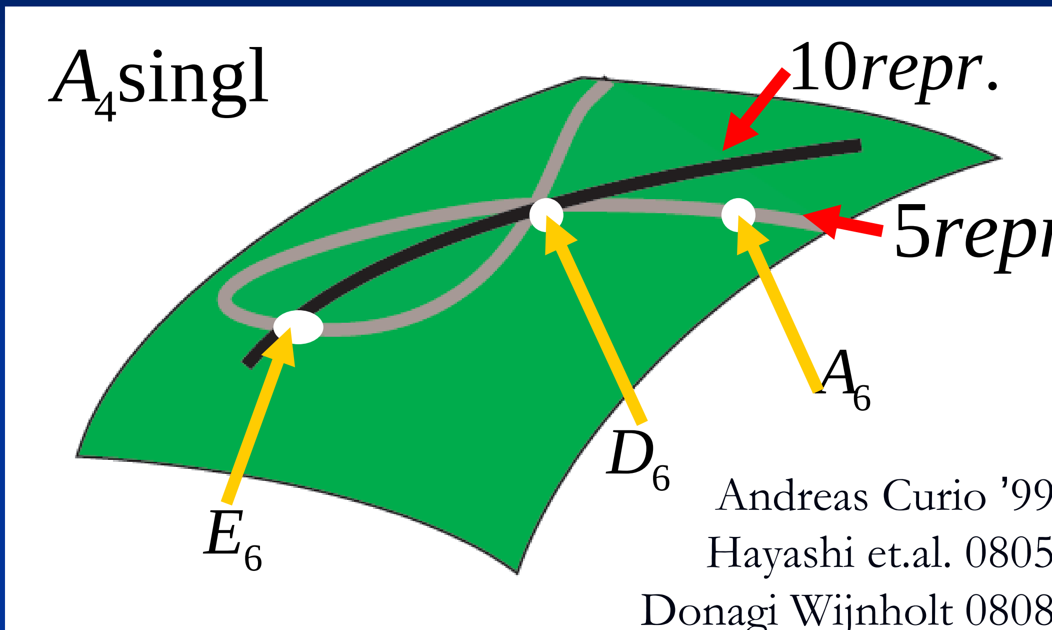
$$\Delta L \sim \frac{g_{GUT}^2}{M_{KK}^2} 10^{\dagger}1010^{\dagger}10 \times \left(\frac{M_*}{M_{KK}}\right) + \frac{g_{GUT}^2}{M_{KK}^2} 10^{\dagger}10\bar{5}^{\dagger}\bar{5}.$$

  - enhanced $\Gamma(p \to \pi^0 e^+_{L/R}) / \Gamma(p \to \pi^+ \bar{\nu})$.
- In F-theory, all charged matter are along the curves, not localized at points. codim.-2.

$$\Delta L \sim \frac{g_{GUT}^2}{M_{KK}^2} 10^{\dagger}1010^{\dagger}10 \times \ln\left(\frac{M_{str}}{M_{KK}}\right) + \frac{g_{GUT}^2}{M_{KK}^2} 10^{\dagger}10\bar{5}^{\dagger}\bar{5}.$$

only log enhancement.

F-theory w/ unbroken SU(5) symmetry



$$y^2 = x^3 + a_5 xy + a_4 z x^2 + a_3 z^2 y + a_2 z^3 x + a_0 z^5 + \cdots.$$

Bershadsky et.al. '96

10: $\quad a_5 = 0,$

5: $P^{(5)} := a_0 a_5^2 - a_2 a_5 a_3 + a_4 a_3^2 = 0.$

$$\Delta \propto z^5 (a_5^4 P^{(5)} + O(z)).$$

$$5S + S'$$

Charged Matter: **hol sections of a line bundle** on a curve

despite all these singularities

Hayashi et.al. 0805.

Right-handed Neutrinos and NMSSM

What do we expect for RHneutrinos?

- SU(5) singlets.
- have neutrino Yukawa couplings
- (hopefully) have Majorana masses; at least one of them is below $(\lambda v)^2 / \sqrt{\Delta m_{\oplus}^2} \sim 10^{15}$ GeV.

- We do not know how many. (2 or more, but not necessarily 3)

- In Het $E_8 \times E_8$ on a CY3, gauge-field moduli have couplings suitable for neutrino Yukawas.

Witten '86

Tatar et.al. 0905.

- In Het—F duality, gauge field moduli are mapped to cpx str moduli of F-theory. (90's)
- F-theory cpx str moduli have neutrino Yukawa couplings [use the field theory local models]

$$\Delta W = \lambda 5 \Phi \bar{5}.$$

- F-theory flux compactification: Majorana masses for the cpx str moduli:

$$M_R \sim M_{cs} \sim 1/(R^3 M_*^2) \sim M_{GUT} \sqrt{\alpha_{GUT}} \varepsilon^n.$$

Singlet Field in the NMSSM

- Anothoer type of moduli $H^{1,2}(X;C)$
  - usually there are not many, often none.
- Those moduli also have $\Delta W = \lambda 5 \Phi \overline{5}.$
- But, no masses from flux compactification, or no trilinear terms from $\Delta W = \int_X \Omega \wedge G^{(4)}.$
- Candidate for the NMSSM singlet, with a suppressed cubic coupling?

$$\Delta W = SH\overline{H}.$$

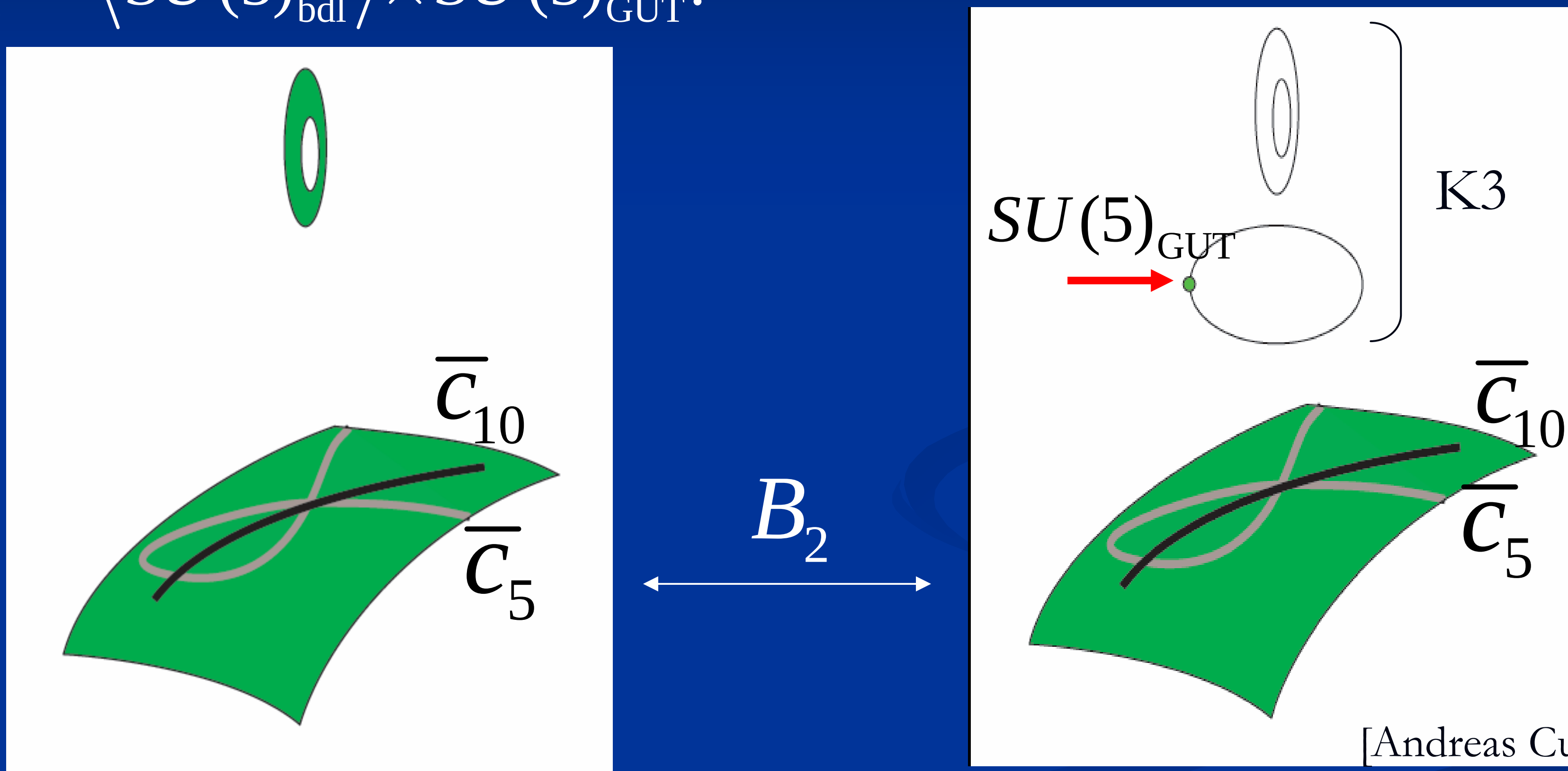
back up slides

Use Heterotic F duality

Hayashi et.al. 0805

$$\text{Het}/(T^2 \to Z \to B_2) \Longleftrightarrow F/(K3 \to X \to B_2)$$

e.g. $E_8 \to \langle SU(5)_{\text{bdl}} \rangle \times SU(5)_{\text{GUT}}$.

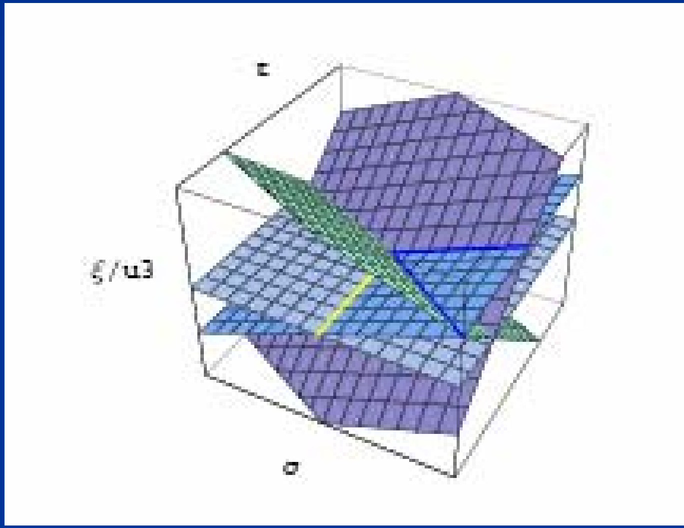


$$10's = H^1(Z;V) \cong H^0(B_2; R^1\pi_{Z*}V) \cong H^0(c_{10}; K_{c_{10}}^{1/2} \otimes L_\gamma).$$

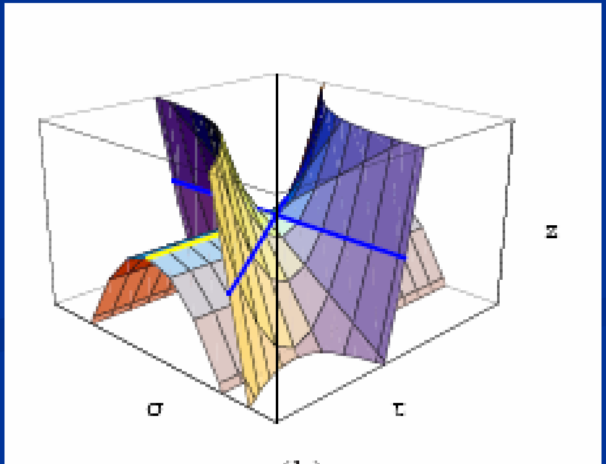
Curio, Diaconescu Ionesei '98

$$\bar{5}'s = H^1(Z;\wedge^2 V) \cong H^0(B_2; R^1\pi_{Z*}\wedge^2 V) \cong H^0(c_5; ?).$$

spectral surface vs discriminant locus



spectral surface



(b)

discriminant locus

$$D_6 \longleftarrow A_4$$

Charged Matter Yukawa Couplings

field theory local models

- Yukawa couplings from singularity
- F-theory lacks microscopic formulation
- effective field theory approach: Donagi-WIjholt 0802, Beasley-Heckman-Vafa 0802

something like DBI dealing with intersecting D7-branes

$$
\begin{aligned}
y^2 &= x^3 + a_5 yx + a_4 zx^2 + a_3 z^2 y + (a_2 z^3 + f_0 z^4)x + (a_0 z^5 + g_0 z^6), \\
&= (x^3 + f_0 z^4 x + g_0 z^6) + (a_5 xy + a_4 zx^2 + a_3 z^2 y + a_2 z^3 x + a_0 z^5),
\end{aligned}
$$

field theory local models

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local model around $(a_4, a_5) = (0, 0)$ Hayashi et.al. 0901.

$$\begin{aligned} y^2 &= x^3 + a_5 yx + a_4 z x^2 + a_3 z^2 y + (a_2 z^3 + f_0 z^4)x + (a_0 z^5 + g_0 z^6), \\ &= (x^3 + f_0 z^4 x + g_0 z^6) + (a_5 xy + a_4 z x^2 + a_3 z^2 y + a_2 z^3 x + a_0 z^5), \end{aligned}$$

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local model around $(a_4, a_5) = (0,0)$ Hayashi et.al. 0901.

$$\tilde{y}^2 \simeq \tilde{x}^3 + \tilde{x}\left[\left(-\frac{\tilde{a}_4^2}{3} + \frac{\tilde{a}_5}{2}\right)\tilde{z}^2 - \frac{2}{3}\left(\frac{\tilde{a}_5}{2}\right)^2 \tilde{a}_4 \tilde{z} - \frac{1}{3}\left(\frac{\tilde{a}_5}{2}\right)^4\right] \tag{58}$$
$$+ \left[\frac{1}{4}\tilde{z}^4 + \left(\frac{2}{27}\tilde{a}_4^3 - \frac{1}{6}\tilde{a}_4\tilde{a}_5\right)\tilde{z}^3 + \frac{1}{3}\left(\frac{\tilde{a}_5}{2}\right)^2\left(\frac{2}{3}\tilde{a}_4^2 - \frac{\tilde{a}_5}{2}\right)\tilde{z}^2 + \frac{2}{9}\left(\frac{\tilde{a}_5}{2}\right)^4 \tilde{a}_4\tilde{z} + \frac{2}{27}\left(\frac{\tilde{a}_5}{2}\right)^6\right].$$

$$\tilde{z} = \frac{z}{a_3}, \qquad \tilde{y} = \frac{1}{a_3^3}\left(y - \frac{1}{2}(a_5 x + a_3 z^2)\right), \qquad \tilde{x} = \frac{1}{a_3^2}\left(x + \frac{1}{3}\left(\left(\frac{a_5}{2}\right)^2 + a_4 z\right)\right),$$

$\tilde{a}_4 \equiv a_4/a_3$ and $\tilde{a}_5 \equiv a_5/a_3$.

field theory local models

- Yukawa couplings from singularity
- F-theory lacks microscopic formulation
- effective field theory approach: Donagi-WIjholt 0802, Beasley-Heckman-Vafa 0802

local model around $(a_4, a_5) = (0, 0)$ Hayashi et.al. 0901.

$$\begin{aligned} Y^2 &= X^3 \\ &+ X\left[\left(-\frac{(3\sigma)^2}{3} - \frac{9\tau(\tau+\sigma)}{2}\right)Z'^2 - \frac{2}{3}\left(\frac{9\tau(\tau+\sigma)}{2}\right)^2(3\sigma)Z' - \frac{1}{3}\left(\frac{9\tau(\tau+\sigma)}{2}\right)^4\right] \\ &+ \left[\frac{1}{4}Z'^4 + \left(\frac{2}{27}(3\sigma)^3 + \frac{1}{6}(3\sigma)9\tau(\tau+\sigma)\right)Z'^3\right. \\ &\quad + \frac{1}{3}\left(\frac{9\tau(\tau+\sigma)}{2}\right)^2\left(\frac{2}{3}(3\sigma)^2 + \frac{9\tau(\tau+\sigma)}{2}\right)Z'^2 \\ &\quad \left. + \frac{2}{9}\left(\frac{9\tau(\tau+\sigma)}{2}\right)^4 3\sigma Z' + \frac{2}{27}\left(\frac{9\tau(\tau+\sigma)}{2}\right)^6\right]. \end{aligned} \tag{72}$$

$$\left(\frac{a_4}{a_3}, \frac{a_5}{a_3}\right) = (3\sigma c, -3\tau(3\tau+3\sigma)c^2).$$

field theory local models

- Yukawa couplings from singularity
- F-theory lacks microscopic formulation
- effective field theory approach: Donagi-WIjholt 0802, Beasley-Heckman-Vafa 0802

local model around $(a_4, a_5) = (0, 0)$ Hayashi et.al. 0901.

Standard form of deformation of E6 singularity to A4 (SU(5))

U(2) doublet background (for SU(5)-10 repr.)

$$\begin{aligned} 3\tau &= -(\tilde{a}_4/2) + \sqrt{(\tilde{a}_4/2)^2 - \tilde{a}_5}, \\ -(3\tau + 3\sigma) &= -(\tilde{a}_4/2) - \sqrt{(\tilde{a}_4/2)^2 - \tilde{a}_5}. \end{aligned}$$

For SU(5)-5 repr.

$$\rho_{U=\wedge^2 \mathbf{2}}(2\alpha \langle \varphi_{12} \rangle) = +3\sigma$$

solve the 0-mode wavefcn under this background, Yukawa is from overlap integration

u, d, e-Yukawa matrix

- Yukawa matrix (up/down type) generated at a singularity: approx. rank-1.
  - good for the 3rd generagion Yukawas.

Beasley et.al. 0806.
Hayashi et.al. 0901.

- All the contributions from singularities add up.
  - problem, if all the Yukawa eigenvalues are O(1).
- Heckman—Vafa 0811 assumes only one singularit pt.
  - subleading eg.values suppressed in powers of

Heckman-Vafa 0811.

$$(M_{KK}/M_{str})^2 \sim (\alpha_{GUT})^{1/2} \sim 0.2,$$

  - refined analysis: $(dM_{KK})^2 \sim \sqrt{\alpha_{GUT}}/\pi.$

Tatar et.al. 0905.

The GUT coupling sets the largest hierarchy possible.