Fuzzy Super-Geometry and Topological Many-body States

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There exists one to one correspondence between the states on fuzzy two-sphere and those of the lowest Landau level (LLL) on two-sphere. Such correspondence also holds in their super-symmetric version [Hasebe, Kimura, NPB 709 (2005) 94]. However, such correspondence is at the one-particle level. In this talk, I will discuss the correspondence at many-body level.

The one-particle correspondence between the fuzzy sphere and the LLL is the one between the Schwinger boson operator and the $SU(2)$ coherent state

$$(a^\dagger, b^\dagger) \longleftrightarrow (u = \sqrt{\frac{1+x_3}{2}}, v = \frac{x_1 + ix_2}{\sqrt{1+x_3}}). \tag{1}$$

We insert this correspondence to many-body wavefunction of the quantum Hall effect, the Laughlin-Haldane wavefunction, $\Phi_{LH} = \prod_{i<j}(u_i v_j - v_i u_j)^m$, to have

$$|VBS\rangle = \prod_{\langle i,j\rangle}(a_i^\dagger b_j^\dagger - b_i^\dagger a_j^\dagger)^M|0\rangle. \tag{2}$$

This wavefunction is known as the valence bond solid state [Affelck et al. PRL 59 (1987) 799]. The VBS state has the remarkable properties, such as unique groundstate of parent Hamiltonian, hidden non-local order, and gapped excitations. The supersymmetric version of the Laughlin-Haldane wavefunction with the $UOSp(1|2)$ supersymmetry was already constructed in [Hasebe, PRL 94 (2005) 206802]. We further apply the correspondence to the supersymmetric Laughlin-Haldane wavefunction to have

$$|SVBS\rangle = \prod_{\langle i,j\rangle}(a_i^\dagger b_j^\dagger - b_i^\dagger a_j^\dagger - r f_i^\dagger f_j^\dagger)^M|0\rangle. \tag{3}$$

Here, $f_i^\dagger f_j^\dagger$ term can be interpreted as doped hole-pair, and r is the doping parameter. Thus, it is ecpected that, depending on the amount of hole doping, the supersymmetric VBS states have the following properties,

- In charge sector, the states exhibit the superconducting property with finite r.
- In the spin sector, the states exhibit non-local hidden order.

On 1D chain, we gave a detail calculation of the superconducting and hidden string orders, and confirmed that the supersymmetric VBS states indeed exhibit the above properties. We also discussed the gapped excitations on the supersymmetric valence bond solid states by constructing the parent supersymmetric Hamiltonian.