One loop divergences in quantum gravity coupled with nonlocal matter fields

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[1] One of most serious problems in quantum gravity is the non-renormalizability. For example one loop graviton self energy graph where the internal lines are matter fields induces the non-renormalizable divergences. Here we modify the propagator of scalar field at high momentum in order to realize the non-renormalizable divergences disappear. Therefore the Euclidean version of action we use is given

$$I = \int d^4x\sqrt{g}\ \Big[R + \phi f(\nabla^2)\phi\Big] \tag{1}$$

and $f(\nabla^2)$ is a function of ∇^2. If the scalar field is canonically coupled, $f(\nabla^2) = \nabla^2 + m^2$. But here we do not specify the form of $f(\nabla^2)$ at this moment. We then expand the metric around the flat space, $g_{ij} = \delta_{ij} + \kappa h_{ij}$, and treat the fluctuations as quantum fields. We then straightforwardly compute the one loop graviton self energy graph where the internal lines are the scalar field. We notice that the graviton-scalar-scalar three point vertex computed from $\phi f(\nabla^2)\phi$ is roughly $\phi f'(\partial^2)\partial_i\partial_j\phi$ where only the number of derivatives is relevant at high momentum when computing the degree of UV divergences. Since

$$\nabla^2\phi \ = \partial^2\phi + \kappa\Big(- h^{ij}\partial_i\partial_j\phi - (\partial_i h^{ij})\partial_j\phi + \tfrac{1}{2}(\partial_i h)\partial^i\phi\Big) + \mathcal{O}(\kappa^2), \tag{2}$$

as the propagator $1/f(\partial^2)$ is suppressed at high momentum, the vertices are on the other hand enhanced. The one loop self energy graph induces the quantum corrections to the cosmological constant, Newton constant and R^2. If the correction to R^2 is divergent, this divergent cannot be renormalized. We can easily compute the superficial degree of divergent for the correction to R^2, which we call $Deg(R^2)$, from the one loop graph which is zero, i.e. log divergent, or 4 when $f(\partial^2) \to \partial^{2n}$ or $f(\partial^2) \to e^{\partial^2}$ at high momentum respectively. Therefore a naive expectation does not work here. In order to have $Deg(R^2) < 0$, we conclude $f'(\partial^2) \to 0$, since the vertex $\sim \phi f'(\partial^2)\partial_i\partial_j\phi$ induces the correction to R^2. Therefore $f(\partial^2)$ should approach to a non-zero constant at high momentum, i.e.

$$f(\partial^2) \to c_0 + \frac{c_1}{\partial^2} + c_2 e^{\partial^2} \tag{3}$$

One such example is given $f(\nabla^2) = M^2\tanh(\nabla^2/M^2) + m^2$. Since the propagator goes to constant, we have to modify other couplings, for instance Yukawa coupling $y \to ye^{\partial^2/M^2}$ to maintain the renormalizability of Yukawa coupling. More detail and discussion can be found in my paper.

[1] This talk is based on my paper arXiv:1106.3624.