A low-energy effective Yang-Mills theory for quark and gluon confinement

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In order to discuss both quark confinement and gluon confinement, we derive a novel low-energy effective model of $SU(2)$ Yang-Mills theory in the gauge-independent way (without fixing the original $SU(2)$ gauge symmetry).

(1) The effective gluon propagator belonging to the Gribov-Stingl type follows from this effective model, irrespective of the gauge choice:

$$\tilde{D}_{\rm cc}(p) = \frac{d_0 + d_1 p^2}{c_0 + c_1 p^2 + c_2 p^4} \quad (d_0 = 1). \tag{1}$$

It is remarkable that the effective model respects the gauge invariance of the original Yang-Mills theory, which allows one to take any gauge fixing in computing physical quantities of interest.

(2) In MA gauge, the model exhibits both quark confinement and gluon confinement simultaneously in the following sense:

• gluon confinement: The Schwinger function (Euclidean Green's functions) violates the reflection positivity.

$$\Delta(t) := \int_{-\infty}^{+\infty} \frac{dp_4}{2\pi} e^{ip_4 t} \tilde{D}_{\rm cc}(\vec{p} = 0, p_4) < 0 \quad \text{for large} \quad t > 0. \tag{2}$$

• quark confinement: The Wilson loop average satisfies the area law (i.e., the linear quark-antiquark potential). The string tension σ is given for $\tilde{D}_{\rm GG}(p) = p^2 \tilde{D}_{\rm cc}(p)$ by the formula:

$$\sigma = \frac{1}{8} g^2 \int_{p_1^2 + p_2^2 \le M^2} \frac{dp_1 dp_2}{(2\pi)^2} \tilde{D}_{\rm GG}(p_1, p_2, 0, 0) = \frac{1}{8} \frac{g^2}{4\pi} \int_{|p|^2 \le M^2} d|p|^2 \frac{d_0 |p|^2 + d_1 |p|^4}{c_0 + c_1 |p|^2 + c_2 |p|^4}, \tag{3}$$

where M is the (dynamical) mass of off-diagonal gluons.

(3) However, the exact agreement with the Gribov-Stingl form is reproduced only when one includes a gauge-invariant nonlocal mass term or a mass term that breaks nilpotency of the BRST symmetry. Otherwise, we have $c_0 = 0$.

Numerical simulations [Mendes, Cucchieri and Mihara, e-Print: hep-lat/0611002] show

$$c_0 = 0.064(2)\text{GeV}^2,\ c_1 = 0.125(9),\ c_2 = 0.197(9)\text{GeV}^{-2},\ d_1 = 0.13(1)\text{GeV}^{-2}. \tag{4}$$

This indeed leads to a good estimate for the string tension according to (3) for $\alpha(\mu) = g^2(\mu)/(4\pi) \simeq 1.0$ at $\mu = M \simeq 0.97$GeV:

$$\sigma \simeq (0.5\text{GeV})^2. \tag{5}$$

This talk is based on references, K.-I. Kondo, e-Print: arXiv:1103.3829 [hep-th], Phys.Rev. D (Rapid Communication), in press. and a detailed paper in preparation.