Expectation values of chiral primary operaters and holographic interface CFT

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We consider the expectation values of chiral primary operators in the presence of the interface in the 4 dimensional $\mathcal{N}= 4$ super Yang-Mills theory. This interface is derived from D3-D5 system in type IIB string theory. These expectation values are computed classically in the gauge theory side. On the other hand, this interface is a holographic dual to type IIB string theory onAdS$_5\times$S^5 spacetime with a probe D5-brane. The expectation values are computed by GKPW prescription in the gravity side. We find non-trivial agreement of these two results: the gauge theory side and the gravity side.

1 $\mathcal{N}$=4 super Yang-Mills theory

Our gauge theory is composed of gauge fields A_μ, $\mu = 0, 1, 2, 3$, fermion fields ψ and scalar fields ϕ_i, $i = 4, 5, \cdots, 9$. The action is

$$S = \frac{2}{g^2}\int d^4x \mathrm{tr}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D_\mu\phi_i D^\mu\phi^i + \frac{i}{2}\bar{\psi}\Gamma^\mu D_\mu\psi + \frac{1}{2}\bar{\psi}\Gamma^i[\phi,\psi_i] + \frac{1}{4}[\phi_i,\phi_j][\phi^i,\phi^j]\Big] \quad (1)$$

In the presence of the interface, there is a non-trivial classical vacuum solution.

$$A_\mu = 0, \qquad \phi_i = -\frac{1}{x_3}t_i \oplus 0_{(N-k)\times(N-k)}\ (x_3 > 0)\ i = 4, 5, 6, \qquad \phi_i = 0\ i = 7, 8, 9. \qquad (2)$$

where matrices t_i $i = 4, 5, 6$ are the generators of SU(2) algebra of the k-dimensional irreducible representation.

The physical quantity we calculate here is the expectation value of the chiral primary operator.

$$\mathcal{O}_\ell = \frac{(8\pi^2)^{\Delta/2}}{\lambda^{\Delta/2}\sqrt{\Delta}}C^{I_1I_2\cdots I_\Delta}\mathrm{Tr}(\phi_{I_1}\phi_{I_2}\cdots\phi_{I_\Delta}). \qquad (3)$$

We substitute the classical solution (**??**) to the definition of the chiral primary operators and get the gauge theory result.

$$\langle\mathcal{O}_{2\ell}(\xi)\rangle = C_\ell\frac{(2\pi^2)^\ell}{\sqrt{2\ell}\lambda^\ell}(k-1)^{2\ell}k\frac{1}{\xi^{2\ell}}. \qquad (4)$$

2 Type IIB superstring theory in $\mathbf{AdS}_5 \times \mathbf{S}^5$

We consider type IIB superstring theory. Its geomertry and an exited 4-form are given by

$$ds^2_{AdS_5\times S^5} = \frac{1}{y^2}(dy^2 + \eta_{\mu\nu}dx^\mu dx^\nu) + ds^2_{S^5} \tag{5}$$

$$C_4 = -\frac{1}{y^4}dx^0 dx^1 dx^2 dx^3 + \cdots. \tag{6}$$

In this approach we added a prove D5-brane whose action is given by

$$S = T_5 \int d^6\zeta \sqrt{\det(G+\mathcal{F})} + iT_5 \int \mathcal{F}\wedge C_4 \tag{7}$$

where T_5 is the tension of the D5-brane, ζ's are the coordinates on the D5-brane worldvolume and G and $\mathcal{F}$ are the induced metric and the field strength of the worldvolume gauge field.

The equation of motion of (**??**) has the solution in which the D5-brane is wrapped on $x_3 = \kappa y$ in AdS and on S^2 in S^5.

Using GKPW relation, we calculate the expectation values of the chiral primary operators.

$$\left\langle e^{\int d^4x x_0(x)\mathcal{O}(x)}\right\rangle \cong e^{-S_{\rm cl}(s_0)}. \tag{8}$$

$$\begin{aligned}\langle\mathcal{O}(x)\rangle &= -\frac{\delta S_{\rm cl}}{\delta s_0(x)}\bigg|_{s_0=0} \\ &= C_\ell \frac{2^{3+\Delta/2}\pi^{5/2}\Gamma(\Delta+1/2)}{N\sqrt{\Delta}\Gamma(\Delta)}\frac{1}{\xi^\Delta}\int_0^\infty du \frac{u^{\Delta-2}}{[(1-\kappa u)^2+u^2]^{\Delta+1/2}}.\end{aligned} \tag{9}$$

3 Comparison

We compare these results in the limit $k \gg 1, \lambda/k^2 \ll 1$. The both theories give the same results

$$C_\ell \frac{(2\pi^2)^\ell}{\lambda^\ell\sqrt{2\ell}}k^{2\ell+1}\frac{1}{\xi^{2\ell}} \tag{10}$$

in the leading term. This results confirms the AdS/CFT correspondence. Furthermore we already obtained the 1-loop correction from gravity side calculation.

$$-\frac{\delta S^{(1)}}{\delta s_0(x)} = C_\ell \frac{(2\pi^2)^\ell}{\lambda^\ell\sqrt{2\lambda}}k^{2\ell+1}\frac{1}{\xi^{2\ell}}\left\{1+\frac{\lambda}{\pi^2k^2}\left(\frac{3}{2}+\frac{(2\ell-2)(2\ell-3)}{4(2\ell-1)}\right)\right\} \tag{11}$$

To confirm this result from gauge theory side calculation is an intersting future work.

References

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