Factorization of the Effective Action in the IIB Matrix Model

Yuhma Asano
(Kyoto University)

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Contents

1. Introduction

In ordinary theories, the effective actions are local.

But is it the case with the fluctuations of space-time?

⇨ We claim that the effective actions should be factorized universally!

e.g. wormholes realize a factorized action

quantum-fluctuated space-time

emerging wormholes

factorization $$S_{eff} = \sum_i c_i s_i + \sum_{i,j} c_{ij} s_i s_j + \sum_{i,j,k} c_{ijk} s_i s_j s_k + \cdots$$

where $$s_i = \int_{\mathcal{M}} d^D x \sqrt{-g(x)}\, O_i(x)$$

$O_i(x)$: scalar local operator e.g. $1, R, F_{\mu\nu}F^{\mu\nu}, \cdots$

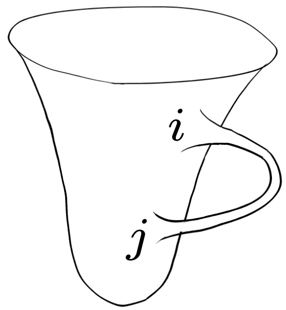
1. Introduction

Factorization by wormholes

Consider a universe interacted by wormholes. [Coleman '88]

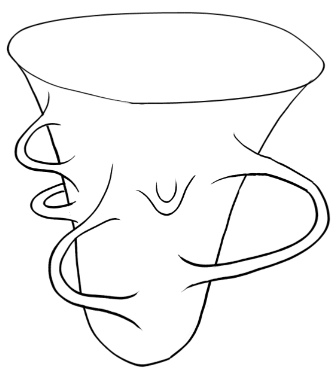
The low-energy effect of attaching a wormhole to the universe can be written as

$$\int \mathcal{D}g\, c_{ij} \int dx^4\, dy^4 \sqrt{g(x)}\sqrt{g(y)} O^i(x) O^j(y) e^{-S}$$



If k wormholes exist, there are $k!$ identical configurations.

$$\frac{1}{k!}\int \mathcal{D}g \left[c_{ij} \int dx^4\, dy^4 \sqrt{g(x)}\sqrt{g(y)} O^i(x) O^j(y)\right]^k e^{-S}$$



Thus after summing up them, we obtain

$$\int \mathcal{D}g \exp\left[c_{ij} \int dx^4\, dy^4 \sqrt{g(x)}\sqrt{g(y)} O^i(x) O^j(y)\right] e^{-S}$$

$$\Delta S_{eff} = \sum_{i,j} c_{ij} s_i s_j$$ Factorized action

1. Introduction

Factorized actions are nice:

- It seems non-local.
  However, we find by Fourier trf. it only means that the coupling constants get dynamical.

$$Z = \int \mathcal{D}\phi\, e^{i(\sum_i c_i s_i + \sum_{i,j} c_{ij} s_i s_j + \cdots)} = \int d\lambda\, f(\lambda) \int \mathcal{D}\phi\, e^{i \sum_i \lambda_i s_i}$$

It is effectively local if λ's are dominated in the integral.

quantum 

$\longrightarrow$ effective $\displaystyle\sum_i c_i s_i + \sum_{i,j} c_{ij} s_i s_j + \cdots$ $\longrightarrow$ real world $\displaystyle\sum_i \lambda_i s_i$

with dominated λ

- The naturalness problem can be solved also for the Lorentzian multiverse.

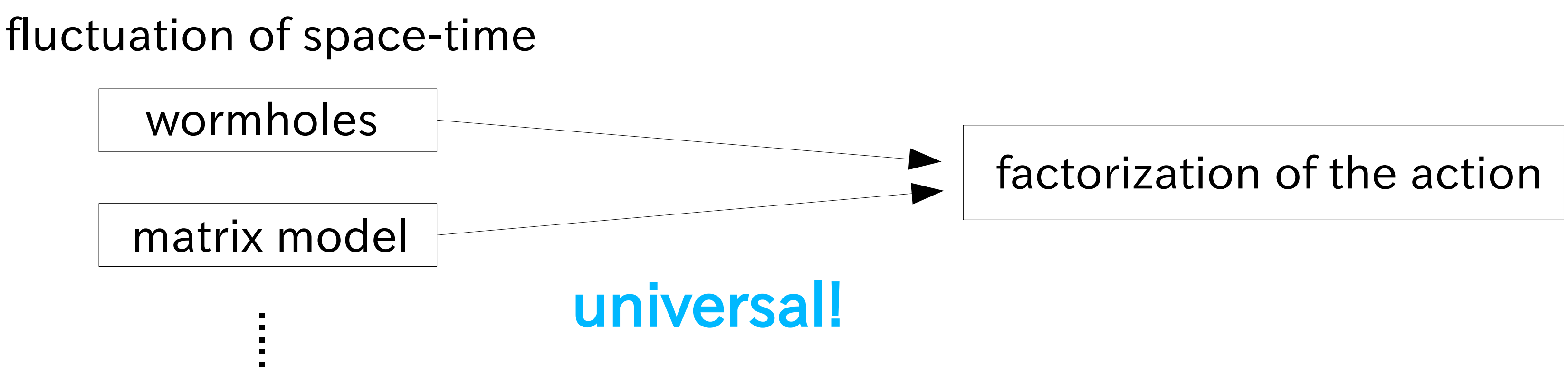
[Kawai-Okada '11]

1. Introduction

On the other hand, IIB matrix model is considered to be the non-perturbative formulation of the String theory.

It is expected to describe the fluctuation of space-time.

Therefore it would naturally occur that the factorization of the action of the matrix model.



1. Introduction

The action is obtained by the matrix regularization of the Green-Schwarz action with Schild gauge.

$$S_{Schild} = \int d^2\sigma \sqrt{g} \left[\frac{1}{4}\{X_\mu, X_\nu\}\{X^\mu, X^\nu\} + \frac{1}{2}\bar{\Psi}\Gamma^\mu\{X_\mu, \Psi\} \right]$$

$$\{A, B\} := \frac{\varepsilon^{ij}}{\sqrt{g}} \partial_i A \partial_j B$$

It is also obtained by dimensionally reducing 10D $\mathcal{N}$=1 SYM to a point.

$$S = \frac{1}{g^2} \mathrm{Tr} \left[\frac{1}{4}[A_a, A_b][A^a, A^b] + \frac{1}{2}\bar{\Psi}\Gamma^a[A_a, \Psi] \right]$$

A_a : 10D Lorentz vec. Ψ : 10D Majorana-Weyl spn.

⇒ Matrices are interpreted as space-time coordinates. (original interpretation)

2. Derivative Interpretation

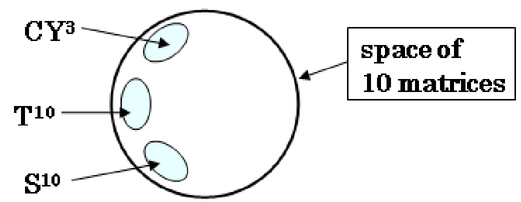
There is another interpretation.
Matrices can also be interpreted as covariant derivatives.

$$A_a \sim i\nabla_a$$

[Hanada-Kawai-Kimura '05]

Good properties:

- The diffeomorphism invariance should be manifest.
- Any manifold (in any dimension ≦10) can be expressed by matrices.



- Einstein equations follow from the equations of motion of the matrix model. $R_{ab} = 0$ (for vacuum)
- Gauge transformation, local Lorentz transformation and diffeomorphism are included in $U(N)$ of the matrix model.

N: matrix size

2. Derivative Interpretation

The action of the covariant derivative on a representation of $G = Spin(D-1,1)$ results in

$$f \to \nabla_a f \quad \cdots \text{scalar to vector}$$

$$f_b \to \nabla_a f_b \quad \cdots \text{vector to tensor}$$

Therefore, the action changes the representation.
It cannot be considered as a matrix in general.

Is the interpretation $A_a \sim i\nabla_a$ possible?

2. Derivative Interpretation

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Therefore, the action changes the representation. It cannot be considered as a matrix in general.

Is the interpretation $A_a \sim i\nabla_a$ possible? ⇨ YES

Because the action of the regular representation is

$$V_{vec} \otimes V_{reg} \cong V_{reg} \oplus \cdots \oplus V_{reg}$$

V_{vec} : space of the vec. rep.

$$V_{reg} = \{f : G \to \mathbb{C}\}$$
$$f(g) \xrightarrow{h} f(h^{-1}g)$$
$$g, h \in G$$

by the Clebsch-Gordan coefficients ${R_{(a)}}^b(g^{-1})$ (group element in the vector representation).

2. Derivative Interpretation

$$V_{vec} \otimes V_{reg} \cong V_{reg} \oplus \cdots \oplus V_{reg}$$

$R_{(a)}{}^{b}(g^{-1})$: group element in the vector representation

$a = 0, 1, \cdots, D-1$

The action of $h \in G$ (Lorentz trf.) on $v_a(g) \in V_{vec} \otimes V_{reg}$ is

$$v_a(g) \to R_a{}^b(h) v_b(h^{-1}g)$$

$R_a{}^b(h)$: group element of the vector rep.

$$\begin{aligned} R_{(a)}{}^{b}(g^{-1}) v_b(g) &\to R_{(a)}{}^{b}(g^{-1}) R_b{}^c(h) v_c(h^{-1}g) \\ &= R_{(a)}{}^{c}((h^{-1}g)^{-1}) v_c(h^{-1}g) \end{aligned}$$

$\Longrightarrow$ $R_{(a)}{}^{b}(g^{-1}) v_b(g)$ is in the regular rep.

$\cdots D$ regular reps.

(a)'s are mere labels that are invariant under the local Lorentz transformation.

2. Derivative Interpretation

Take $f(x,g) \in \mathcal{V}_{reg}$ $\cdots$ space of the fld. of the regular rep.

The action of the covariant derivative is

$$\nabla_a f(x,g) = e_a{}^{\mu}(x)\left(\partial_\mu - \frac{i}{2}\omega_\mu{}^{bc}(x)O_{bc}\right)f(x,g)$$

Then if we define $\nabla_{(a)} := R_{(a)}{}^{b}(g^{-1})\nabla_b$, each of it becomes

$$\nabla_{(a)} f(x,g) = R_{(a)}{}^{b}(g^{-1}) e_a{}^{\mu}(x)\left(\partial_\mu - \frac{i}{2}\omega_\mu{}^{bc}(x)O_{bc}\right)f(x,g)$$

$\in \mathcal{V}_{reg}$

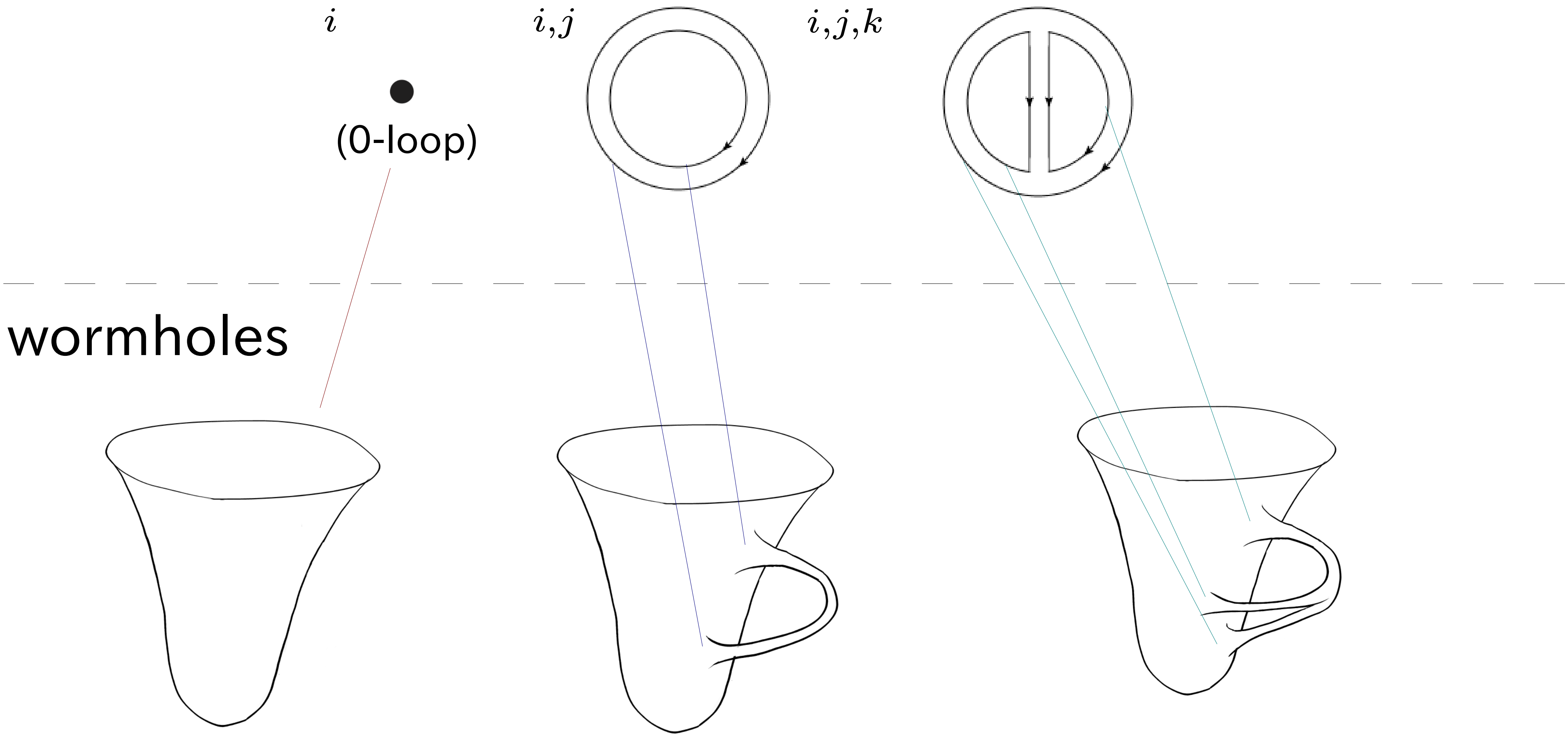
$\Rightarrow$ $\nabla_{(a)} : \mathcal{V}_{reg} \to \mathcal{V}_{reg}$ $\cdots$ D matrices !

We've obtained the covariant derivative as a matrix.

3. Factorization

In the end, we find that the effective action is in the form of

$$S_{eff} = \sum_i c_i s_i + \sum_{i,j} c_{ij} s_i s_j + \sum_{i,j,k} c_{ijk} s_i s_j s_k + \cdots \quad \text{(loop expansion)}$$



3. Factorization

We use the background field method. Let us decompose the matrices as

$$\langle x,g|A_a|y,h\rangle =: A_{(a)}(x,g;y,h) = \underbrace{A^0_{(a)}(x,g;y,h)}_{\text{BG fields}} + \underbrace{\phi_{(a)}(x,g;y,h)}_{\text{fluctuation which will be integrated out}}$$

To see the effective action, expand $A^0{}_{(a)}$ around the flat metric

$$A^0_{(a)}(x,g;y,h) = \Big[i\partial_{(a)} + B_{(a)}(x,g) + \frac{1}{2}\{h_{(a)}{}^b(x,g), i\partial_b\} + \frac{1}{4}\{\varpi_{(a)}{}^{bc}(x,g), O_{bc}\} + \cdots \Big] \delta(x-y)\delta_{gh}$$

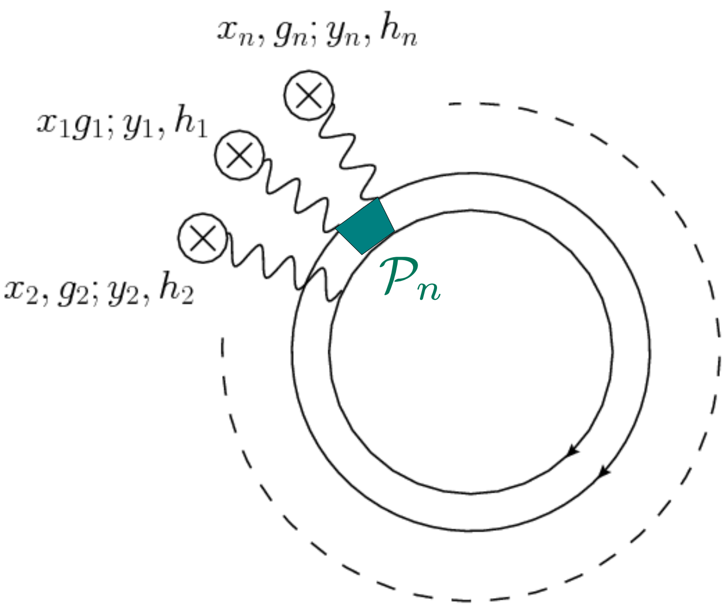
while not expand $\phi_{(a)}$ but treat it as a bi-local field for the convenience of the calculation.

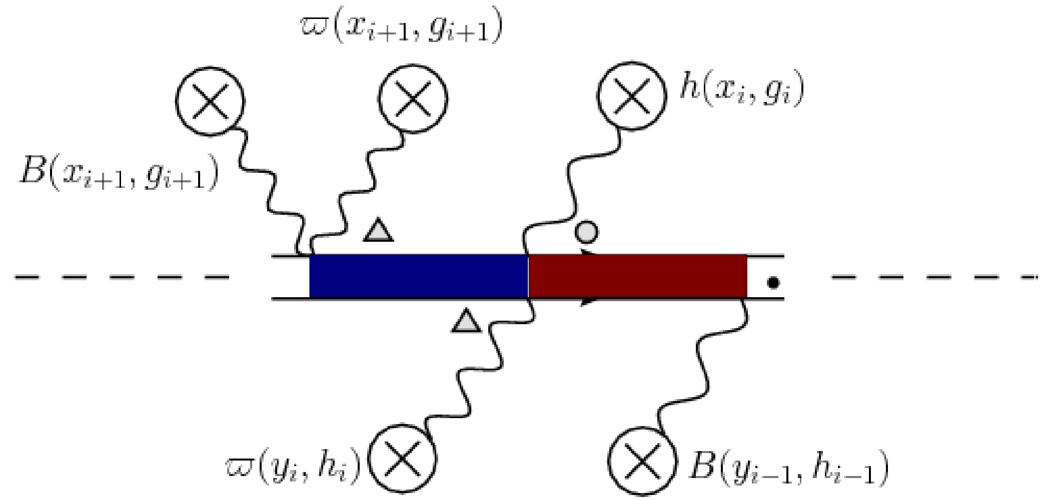
We would like to know the general form of the effective action. It is enough to consider a scalar matrix ϕ whose quadratic part is given by

$$S_{\phi^2} = \frac{1}{2}\,\mathrm{Tr}\left[[A^{0(a)}, \phi][A^0_{(a)}, \phi] \right]$$

3. Factorization

Example: The effect of insertions





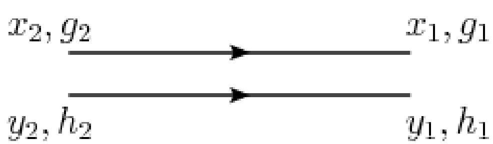
$$\underbrace{\left[iB_{(a)}(y_{i-1}, h_{i-1}) \frac{-i}{2} \{ h^{(a')d'}(x_i, g_i), i\frac{\partial}{\partial x_i^{d'}} \} \mathcal{G}(x_{i-1}, g_{i-1}; y_{i-1}, h_{i-1} | x_i, g_i; y_i, h_i) \right]}_{\mathcal{P}_{i-1}}$$

$$\times \Bigg[\underbrace{\frac{-i}{4} \{ \varpi_{(a')}{}^{b'c'}(y_i, h_i), O^{(h_i)}_{b'c'} \} \frac{-i}{4} \{ \varpi^{(a'')b''c''}(x_{i+1}, g_{i+1}), O^{(g_{i+1})}_{b''c''} \} \mathcal{G}(x_i, g_i; y_i, h_i | x_{i+1}, g_{i+1}; y_{i+1}, h_{i+1})}_{\mathcal{P}_i} \Bigg]$$

- The Lorentz invariance of the vertices in each index loop
- The Poincaré invariance of the propagators in each index loop

3. Factorization

- The Poincaré invariance of propagators



$$\xi^{(a)} := R^{(a)}{}_b(g^{-1})x^b, \ \eta^{(a)} := R^{(a)}{}_b(h^{-1})y^b$$

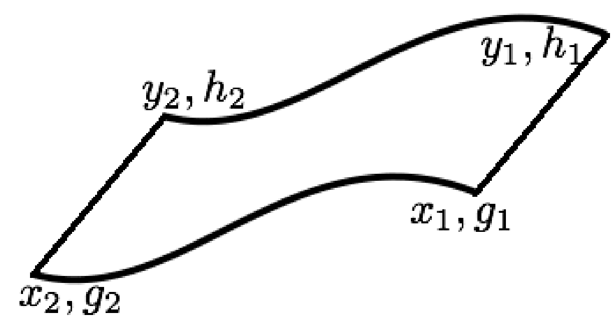
$$\mathcal{G}(x_1, g_1; y_1, h_1 | x_2, g_2; y_2, h_2) := \langle \phi(x_1, g_1; y_1, h_1)\phi^*(x_2, g_2; y_2, h_2)\rangle$$
$$= G(\xi_1 - \xi_2)\delta((\xi_1 - \eta_1) - (\xi_2 - \eta_2))\delta_{g_1 g_2}\delta_{h_1 h_2}$$

Because

$$S_{\phi^2} = -\frac{1}{2}\int d^D x\, d^D y\, dg\, dh \Bigg[\left(\frac{\partial}{\partial \xi_{(a)}} + \frac{\partial}{\partial \eta_{(a)}} - i\mathcal{A}^{(a)}(y, h; x, g) \right) \phi^*(x, g; y, h)$$
$$\times \left(\frac{\partial}{\partial \xi^{(a)}} + \frac{\partial}{\partial \eta^{(a)}} - i\mathcal{A}_{(a)}(x, g; y, h) \right) \phi(x, g; y, h) \Bigg]$$

contains kinetic terms only of $\xi + \eta$. $\Longrightarrow$ $G(\xi_1 - \xi_2)$

ξ - η , g and h doesn't propagate.



Since this is a scalar propagator,

it is **Poincaré invariant in x and y, respectively**.

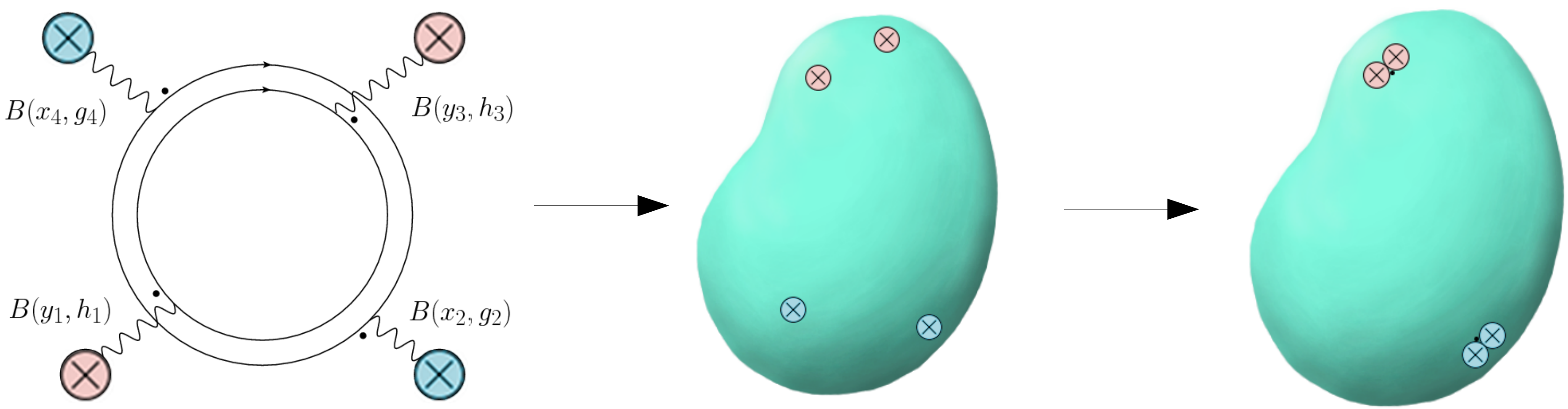
3. Factorization

- Consider in the level of the derivative expansion (Taylor series expansion of flds.),

※ (I) and J stand for sets of indices.

$$A_{(I)J}(x_i) = \sum_{s=0}^{\infty} \frac{1}{s!} \hat{A}_{(I)J\,a_1\cdots a_s}(x_n)(x_i^{a_1} - x_n^{a_1})\cdots(x_i^{a_s} - x_n^{a_s})$$

- The SO(D-1,1) x SO(D-1,1) covariant tensor turns into SO(D-1,1) x SO(D-1,1) invariant tensor after the integration over the coordinates x, y and the elements of SO(D-1,1) g and h.



Factorization occurs

$$\int d^D x\, d^D y\, (\hat{A}(x)\cdots\hat{A}(x)) \times (\hat{A}(y)\cdots\hat{A}(y)) \to \sum_{i,j} c_{ij}\, s_i s_j$$

3. Factorization

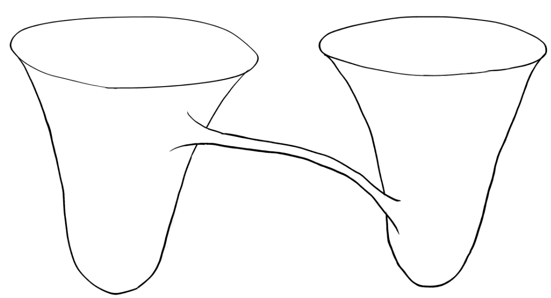
- The effective action factorized by the wormhole interaction and that by the matrix model have the same structure.

  This would be a universal property.

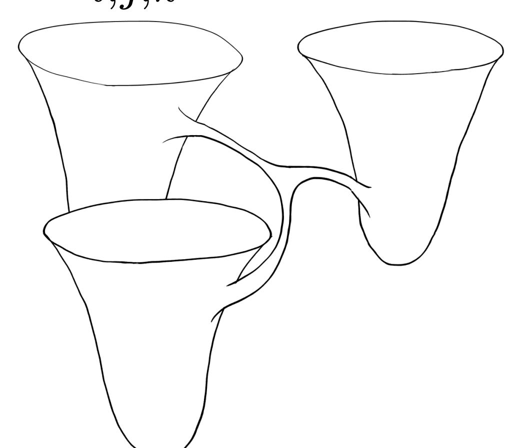
- We can consider multiverses naturally.

  Then, the factorized action turns to be a local action with dynamical coupling λ, and the naturalness problem can be analyzed for the Lorentzian multiverse.

$$\sum_{i,j} c_{ij} s_i s_j$$



$$\sum_{i,j,k} c_{ijk} s_i s_j s_k$$



... etc.

4. Summary and Future Works

- The effective action of IIB Matrix Model in the derivative interpretation is in the form of

$$S_{eff} = \sum_i c_i s_i + \sum_{i,j} c_{ij} s_i s_j + \sum_{i,j,k} c_{ijk} s_i s_j s_k + \cdots$$

  This form would be universal for the low-energy effective theories describing quantum gravity.

- Factorized actions haven't been studied well yet. It should be studied further.

- The relation between the original and the derivative interpretation must be a certain duality.
- We want to reduce the vast degree of freedom.
  ⋯ noncommutative geometry?
- Further investigation of the components of $U(N)$ symmetry in the derivative interpretation.
  ⋯ higher spin gauge symmetry?

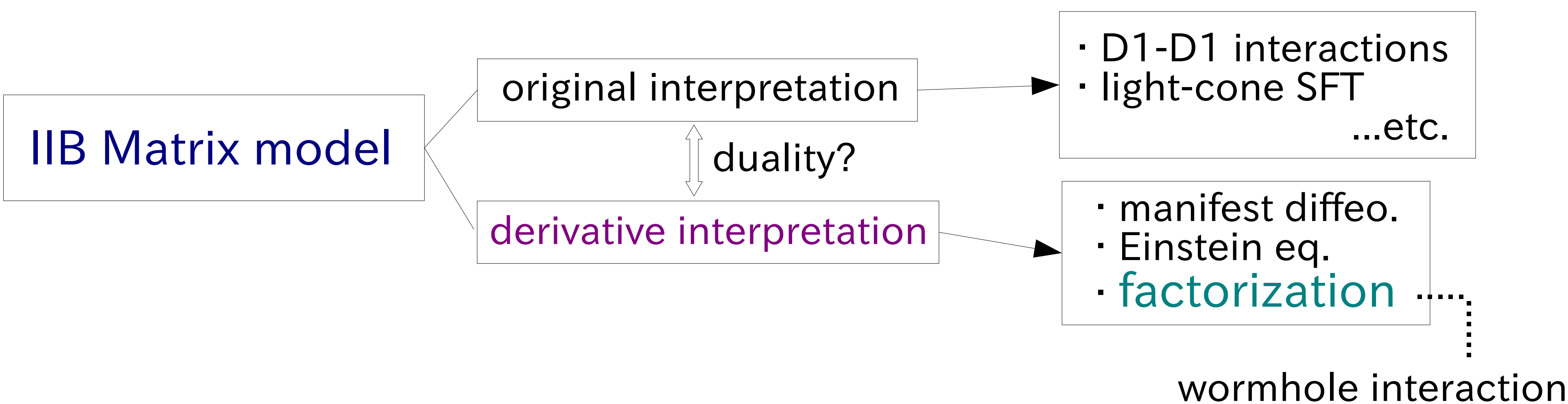
Backups

Introduction of IIB M.M.

The aim is

“factorization of the action
from the Lorentzian IIB matrix model.”

$$S_{eff} = \sum_i c_i s_i + \sum_{i,j} c_{ij} s_i s_j + \sum_{i,j,k} c_{ijk} s_i s_j s_k + \cdots$$



U(N) transformation

$$\delta A_a = i[\Lambda, A_a], \quad \delta\Psi = i[\Lambda, \Psi]$$

$$\Lambda(x,g;y,h) = \Big[\lambda^{\{0\}}(x,g) + \frac{1}{4}\{\lambda^{\{0\}bc}(x,g), O_{bc}\} + \frac{1}{2}\Big\{\lambda^{\{1\}b}(x,g), i\nabla_b\Big\} + \cdots\Big]\delta(x-y)\delta_{gh}$$

This implies

$$\delta\tilde{A}^{\{0\}}_a(x) = \nabla_a\lambda^{\{0\}}(x)$$

gauge transformation

$$\delta e_a{}^{\mu}(x) = \lambda^{\{0\}}{}_a{}^b(x)\, e_b{}^{\mu}(x)$$
$$\delta\omega_\mu{}^{bc}(x) = e^a_\mu\nabla_a\lambda^{\{0\}bc}(x)$$
$$\delta\tilde{A}^{\{0\}}_c(x) = \lambda^{\{0\}}{}_c{}^b(x)\tilde{A}^{\{0\}}_b(x)$$

Local Lorentz transformation

$$\delta e_a{}^{\mu}(x) = \left(\nabla_a\lambda^{\{1\}b}(x)\right) e_b{}^{\mu}(x)$$
$$\delta\omega_\mu{}^{bc}(x) = -\lambda^{\{1\}\nu}(x)\, R_{\nu\mu}{}^{bc}(x)$$
$$\delta\tilde{A}^{\{0\}}_a(x) = -\lambda^{\{1\}\nu}(x)\nabla_\nu\,\tilde{A}^{\{0\}}_a(x)$$

Diffeomorphism

U(N) transformation

$$\delta A_a = i[\Lambda, A_a]$$

Local Lorentz

$$\frac{i}{4}[\{\lambda^{\{0\}bc}, O_{bc}\}, i\nabla_a] = \frac{1}{2}\{\lambda_a^{\{0\}c}, i\nabla_c\} + \frac{1}{4}\{\nabla_a\lambda^{\{0\}bc}, O_{bc}\}$$
$$= \frac{1}{2}\{\lambda_a^{\{0\}d} e_d^{\ \mu}, i\partial_\mu\}$$
$$+\frac{1}{4}\{\nabla_a\lambda^{\{0\}bc} + \lambda_a^{\{0\}d} e_d^{\ \mu}\omega_\mu^{\ bc}, O_{bc}\}$$

$$\Longrightarrow \quad \delta e_a^{\ \mu} = \lambda^{\{0\}\ d}_{\ \ a}\, e_d^{\ \mu}$$
$$\delta\omega_\mu^{\ bc} = e^a_{\ \mu}\nabla_a\lambda^{\{0\}bc}$$

Diffeo.

$$\frac{i}{2}[\{\lambda^{\{1\}b}, i\nabla_b\}, i\nabla_a] = \frac{1}{2}\{\nabla_a\lambda^{\{1\}b}, i\nabla_b\} + \frac{1}{4}\{\lambda^{\{1\}b}R_{ab}^{\ \ cd}, O_{cd}\}$$

$$\Longrightarrow \quad \delta e_a^{\ \mu} = (\nabla_a\lambda^{\{1\}b})\, e_b^{\ \mu}$$
$$\delta\omega_\mu^{\ cd} = -\lambda^{\{1\}\nu}\, R_{\nu\mu}^{\ \ \ cd}$$

Full action of M.M.

The right action is

$$S = \frac{1}{4}\mathrm{Tr}\Big[[A^0_{(a)}, A^0_{(b)}]^2 + 4[A^{0(a)}, A^{0(b)}][A^0_{(a)}, \phi_{(b)}]$$
$$+2[A^0_{(a)}, \phi_{(b)}]^2 + 2[A^{0(a)}, A^{0(b)}][\phi_{(a)}, \phi_{(b)}] - 2[A^0_{(a)}, \phi_{(b)}][A^{0(b)}, \phi^{(a)}]$$
$$+4[A^{0(a)}, \phi^{(b)}][\phi_{(a)}, \phi_{(b)}] + [\phi_{(a)}, \phi_{(b)}]^2 + \text{fermion}\Big]$$

We have to fix the gauge. However it is not necessary for analyzing the factorization of the effective action.

Lorentz invariance of vertices

For calculating 1-loop amplitudes, we need only the quadratic part of the action.

$$\xi^{(a)} := R^{(a)}{}_b(g^{-1})x^b,\ \eta^{(a)} := R^{(a)}{}_b(h^{-1})y^b$$

$$\begin{aligned}
S_{\phi^2} &= \frac{1}{2}\,\mathrm{Tr}\left[[A^{0(a)},\phi][A^0_{(a)},\phi]\right] \\
&= -\frac{1}{2}\int d^D x\, d^D y\, dg\, dh \Bigg[\left(\frac{\partial}{\partial \xi_{(a)}} + \frac{\partial}{\partial \eta_{(a)}} - i\mathcal{A}^{(a)}(y,h;x,g)\right)\phi^*(x,g;y,h) \\
&\qquad\qquad \times \left(\frac{\partial}{\partial \xi^{(a)}} + \frac{\partial}{\partial \eta^{(a)}} - i\mathcal{A}_{(a)}(x,g;y,h)\right)\phi(x,g;y,h)\Bigg]
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_{(a)}(x,g;y,h) := {}& B_{(a)}(x,g) - B_{(a)}(y,h) \\
&+\frac{1}{2}\{h_{(a)}{}^b(x,g), i\frac{\partial}{\partial x^b}\} + \frac{1}{2}\{h_{(a)}{}^b(y,h), i\frac{\partial}{\partial y^b}\} \\
&+\frac{1}{4}\{\varpi_{(a)}{}^{bc}(x,g), O^{[g]}_{bc}\} + \frac{1}{4}\{\varpi_{(a)}{}^{bc}(y,h), O^{[h]}_{bc}\} + \cdots
\end{aligned}$$

$\cdots$ Lorentz invariant in $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively

Interpretation of the Action

This effective action seems to be non-local.

However, an observer in a universe observes the nature described by a local effective action if λ's are dominated:

$$Z = \int \mathcal{D}\phi \, e^{i(\sum_i c_i s_i + \cdots)} = \int d\lambda \, f(\lambda) \int \mathcal{D}\phi \, e^{i \sum_i \lambda_i s_i}$$

We find that "the sum of the products of local actions" turns to "the sum of the several values of couplings" by Fourier transformation.

It also means that the values of the couplings may be determined to be some special values by dynamics!

If we understand the mechanisms of the cosmological time-evolution such as the Inflation, what the dark matter is, and so on, we will be able to compute the partition function as a function of couplings.