On correlation function of Wilson loop and local operator with large R-charge and its gravity dual

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Introduction

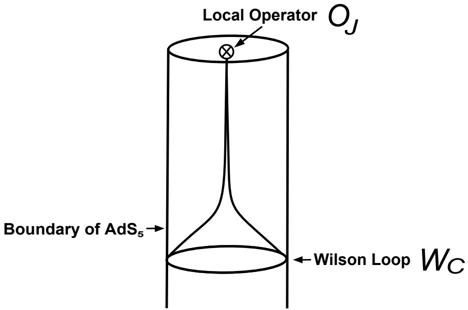
$\boxed{\text{AdS/CFT correspondence}}$

$\mathcal{N} = 4$ SYM $\Leftrightarrow$ type IIB superstring

Wilson loop $\Leftrightarrow$ area of world-sheet

Wilson loop + heavy local operator
$\Leftrightarrow$ area of deformed world-sheet
$\Rightarrow$ Zarembo's solution (1/2 BPS Wilson Loop)

today's talk ... 1/4 BPS Wilson Loop



$$\langle W_C O_J \rangle \sim \exp[-(S_{\text{bulk}} + \text{boundary term})]$$

Plan of Talk

1. Introduction
2. Calculation in Gauge Theory
   - Operators
   - Correlation Function
3. Calculation in String Theory
   - Action and Solution of EOM
   - Supersymmetry
   - Semi-Classical Evaluation of Path Integral
4. Another Saddle Point
   - Another Saddle Point
5. Summary and Future Works

Operators

$\boxed{\text{1/4 BPS Wilson loop operator}}$

$$W_C = \frac{1}{N}\mathrm{tr}\mathcal{P}\exp\oint_C \left(iA_\mu \dot{x}^\mu + \Phi_i \Theta_i |\dot{x}|\right) d\sigma$$

$$x(\sigma) = (r\cos\sigma, r\sin\sigma, 0, 0) \quad r \text{ ... radius of the loop}$$
$$\Theta(\sigma) = (\sin\theta_0\cos\sigma, \sin\theta_0\sin\sigma, \cos\theta_0, 0, 0, 0)$$

$\boxed{\text{chiral primary operator with R-charge } J}$

$$O_J = \mathrm{tr}(\Phi_3 + i\Phi_4)^J \quad \rightarrow 1/2 \text{ BPS}$$

$$\boxed{W_C O_J \rightarrow 1/8 \text{ SUSY}}$$

K. Zarembo, Nucl. Phys. B643 (2002) 157-171
N. Drukker, JHEP 09 (2006) 004
G. W. Semenoff and D. Young, Phys. Lett. B643 (2006) 195-204

Correlation Function

$\boxed{\text{correlation function}}$ sum of ladder diagrams

$$\frac{\langle W_C O_J \rangle}{\langle W_C \rangle} \propto \frac{r^J}{(\ell^2 + r^2)^J} \sqrt{\lambda'} J \frac{I_J(\sqrt{\lambda'})}{I_1(\sqrt{\lambda'})}$$

$\sqrt{\lambda'} \equiv \sqrt{\lambda} \cos \theta_0 \quad \lambda$ ... 't Hooft coupling

$I_J(\sqrt{\lambda'})$ ... J-th modified Bessel function

r ... radius of the loop

ℓ ... distance $W_C \leftrightarrow O_J$

G. W. Semenoff and K. Zarembo, Nucl. Phys. B616 (2001) 34-46
G. W. Semenoff and D. Young, Phys. Lett. B643 (2006) 195-204

saddle point analysis

$$\sqrt{\lambda'} \to \infty,\ J \to \infty,\ j' \equiv J/\sqrt{\lambda'} \text{ fixed} \quad \left(j' \equiv \frac{j}{\cos\theta_0}\right)$$

$$I_J(\sqrt{\lambda'}) = \frac{1}{2\pi i}\int_{\infty-\pi i}^{\infty+\pi i} \exp\sqrt{\lambda'}(\cosh x - j'x)dx$$
$$\sim \exp\sqrt{\lambda'}[\sqrt{j'^2+1} + j'\ln(\sqrt{j'^2+1} - j')]$$

saddle points ... $x = \ln(\sqrt{j'^2+1} + j'), -\ln(\sqrt{j'^2+1} + j') + \pi i$

$$\boxed{\text{Result: } \langle W_C O_J \rangle \sim \frac{r^J}{(\ell^2+r^2)^J}\exp\sqrt{\lambda'}[\sqrt{j'^2+1} + j'\ln(\sqrt{j'^2+1} - j')]}$$

K. Zarembo, Phys. Rev. D66 (2002) 105021

Action and Solution of EOM

string action

$$S = \frac{1}{4\pi\alpha'} \int G_{MN}(\dot{X}^M \dot{X}^N + X'^M X'^N)\, d\tau_{\mathrm{E}} d\sigma$$

$\mathrm{AdS}_5 \otimes \mathrm{S}^5$ metric

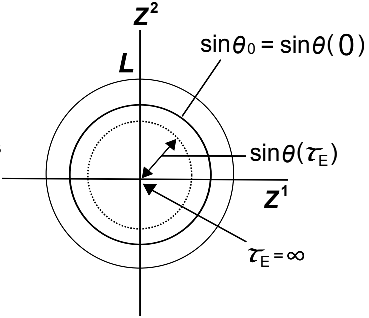
$$ds^2 = L^2 \left[\cosh^2\rho dt_{\mathrm{E}}^2 + d\rho^2 + \sinh^2\rho(d\varphi_1^2 + \sin^2\varphi_1(d\varphi_2^2 + \sin^2\varphi_2 d\varphi_3^2)) + \right.$$
$$\left. + d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta(d\chi_1^2 + \sin^2\chi_1(d\chi_2^2 + \sin^2\chi_2 d\chi_3^2)) \right]$$

$\boxed{\text{ansatz}}$

$$\mathrm{AdS}_5: \ t_\mathrm{E} = t_\mathrm{E}(\tau_\mathrm{E}),\ \rho = \rho(\tau_\mathrm{E}),\ \varphi_1 = \varphi_2 = \frac{\pi}{2},\ \varphi_3 = \sigma$$

$$\mathrm{S}^5: \ \theta = \theta(\tau_\mathrm{E}),\ \phi = \sigma,\ \chi_1 = \chi_2 = \frac{\pi}{2},\ \chi_3 = \chi_3(\tau_\mathrm{E})$$

$$Z^1 = L \sin\theta \cos\phi$$
$$Z^2 = L \sin\theta \sin\phi$$
$$Z^3 = L \cos\theta \sin\chi_3$$
$$Z^4 = L \cos\theta \cos\chi_3$$



$$S = \frac{\sqrt{\lambda}}{2}\int \left(\cosh^2\rho\, \dot{t}_{\mathrm{E}}^{\,2} + \dot{\rho}^2 + \sinh^2\rho + \dot{\theta}^2 + \sin^2\theta + \cos^2\theta\, \dot{\chi}_3^2 \right) d\tau_{\mathrm{E}}$$

$\boxed{\text{EOM}}$ $(j \equiv J/\sqrt{\lambda})$

$$\frac{d}{d\tau_{\mathrm{E}}}(\cosh^2\rho\, \dot{t}_{\mathrm{E}}) = 0 \quad \Rightarrow \cosh^2\rho\, \dot{t}_{\mathrm{E}} = j$$

$$\ddot{\rho} - \sinh\rho\cosh\rho(\dot{t}_{\mathrm{E}}^2 - 1) = 0$$

$$\ddot{\theta} + \sin\theta\cos\theta(\dot{\chi}_3^2 - 1) = 0$$

$$\frac{d}{d\tau_{\mathrm{E}}}(\cos^2\theta\, \dot{\chi}_3) = 0 \quad \Rightarrow \cos^2\theta\, \dot{\chi}_3 = ij$$

$\boxed{\text{Virasoro constraint}}$

$$\cosh^2\rho\, \dot{t}_{\mathrm{E}}^{\,2} + \dot{\rho}^2 + \dot{\theta}^2 + \cos^2\theta\, \dot{\chi}_3^2 = \sinh^2\rho + \sin^2\theta$$

solution of EOM

$$t_{\mathrm{E}} = j\tau_{\mathrm{E}} - \frac{1}{2}\ln\frac{\cosh(\sqrt{j^2+1}\tau_{\mathrm{E}}+\xi)}{\cosh(\sqrt{j^2+1}\tau_{\mathrm{E}}-\xi)}$$

$$\sinh\rho = \frac{\sqrt{j^2+1}}{\sinh(\sqrt{j^2+1}\tau_{\mathrm{E}})}$$

$$\sin\theta = \frac{\sqrt{j^2+1}}{\cosh(\sqrt{j^2+1}(\tau_{\mathrm{E}}+\tau_0))}$$

$$\chi_3 = ij\tau_{\mathrm{E}} + \frac{i}{2}\ln\frac{\sinh(\sqrt{j^2+1}(\tau_{\mathrm{E}}+\tau_0)-\xi)}{\sinh(\sqrt{j^2+1}(\tau_{\mathrm{E}}+\tau_0)+\xi)} + \frac{i}{2}\ln\frac{\sinh(\sqrt{j^2+1}\tau_0+\xi)}{\sinh(\sqrt{j^2+1}\tau_0-\xi)}$$

$$(\,\xi \equiv \ln(\sqrt{j^2+1}+j)\,)$$

Supersymmetry

Killing spinor equation for $AdS_5 \otimes S^5$

$$\left(D_M + \frac{\varepsilon}{2L}\Gamma_\star \hat{\Gamma}_M\right)\begin{pmatrix}\epsilon_1 \\ \epsilon_2\end{pmatrix} = 0$$

D_M ... covariant derivative

ϵ_1, ϵ_2 ... 32-components Majorana-Weyl spinors

$\hat{\Gamma}_M$ ... 10-dimensional gamma matrices

$\Gamma_\star$ ... a product of all gamma matrices in AdS_5

ε ... $\varepsilon = i\sigma_2$: $\varepsilon\begin{pmatrix}\epsilon_1 \\ \epsilon_2\end{pmatrix} = \begin{pmatrix}\epsilon_2 \\ -\epsilon_1\end{pmatrix}$

solution of Killing spinor equation

$$\begin{pmatrix}\epsilon_{1\mathrm{E}}\\ \epsilon_{2\mathrm{E}}\end{pmatrix}=\exp\left(\frac{1}{2}\rho\Gamma_\star\Gamma_1\varepsilon\right)\exp\left(\frac{1}{2}t_{\mathrm{E}}\Gamma_\star\Gamma_{\mathrm{E}}\varepsilon\right)\exp\left(\frac{1}{2}\varphi_3\Gamma_{14}\right)\exp\left(-\frac{1}{2}\theta\gamma_1\gamma_\star\varepsilon\right)\times$$
$$\times\exp\left(\frac{1}{2}\phi\gamma_{12}\right)\exp\left(-\frac{1}{2}\chi_3\gamma_5\gamma_\star\varepsilon\right)\begin{pmatrix}\eta_{1\mathrm{E}}\\ \eta_{2\mathrm{E}}\end{pmatrix}$$

Γ_μ $(\mu=\mathrm{E},1,2,3,4)$ ... gamma matrices in AdS_5
γ_m $(m=1,2,3,4,5)$ ... gamma matrices in S^5
$\gamma_\star$ ... a product of all gamma matrices in S^5
$\eta_{1\mathrm{E}}$, $\eta_{2\mathrm{E}}$ ... constant spinors

projection

$$\frac{i}{\sqrt{\det g}}\partial_{\tau \mathrm{E}} X^M \partial_\sigma X^N \; e^a_M \hat{\Gamma}_a \; e^b_N \hat{\Gamma}_b \; \sigma_3 \begin{pmatrix} \epsilon_{1\mathrm{E}} \\ \epsilon_{2\mathrm{E}} \end{pmatrix} = \begin{pmatrix} \epsilon_{1\mathrm{E}} \\ \epsilon_{2\mathrm{E}} \end{pmatrix}$$

g ... induced metric on world-sheet

e^a_M ... vielbein

σ_3 ... σ_3: $\sigma_3 \begin{pmatrix} \epsilon_{1\mathrm{E}} \\ \epsilon_{2\mathrm{E}} \end{pmatrix} = \begin{pmatrix} \epsilon_{1\mathrm{E}} \\ -\epsilon_{2\mathrm{E}} \end{pmatrix}$

three projections

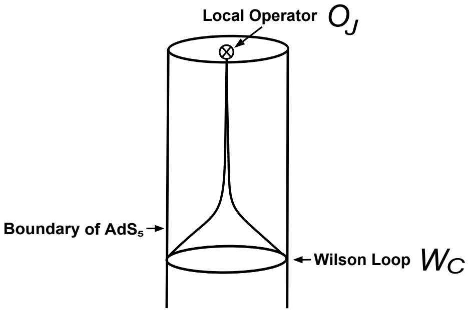
$$(\Gamma_{\mathrm{E}} + i\gamma_5)\begin{pmatrix}\eta_{1\mathrm{E}}\\ \eta_{2\mathrm{E}}\end{pmatrix} = 0$$

$$(\Gamma_{14} + \gamma_{12})\begin{pmatrix}\eta_{1\mathrm{E}}\\ \eta_{2\mathrm{E}}\end{pmatrix} = 0$$

$$(\sin\theta_0\gamma_1\Gamma_4 + \cos\theta_0\Gamma_{14})\begin{pmatrix}\eta_{1\mathrm{E}}\\ -\eta_{2\mathrm{E}}\end{pmatrix} = i\begin{pmatrix}\eta_{1\mathrm{E}}\\ \eta_{2\mathrm{E}}\end{pmatrix}$$

Our solution is 1/8 BPS.

Semi-Classical Evaluation of Path Integral



$$\langle W_C O_J \rangle \sim \exp[-(S_{\text{bulk}} + \text{boundary term})]$$

Coordinates transformations, global $\rightarrow$ Poincaré:

$$\frac{re^{t_{\mathrm{E}}}}{\cosh\rho} = Y, \;\; re^{t_{\mathrm{E}}}\tanh\rho = R$$

Using isometry:

$$\tilde{Y} = \frac{\ell^2 + r^2}{\ell^2 + R(\tau_{\mathrm{E}})^2 + Y(\tau_{\mathrm{E}})^2} Y(\tau_{\mathrm{E}})$$

$$(\tilde{X}^1, \tilde{X}^2) = \frac{\ell^2 + r^2}{\ell^2 + R(\tau_{\mathrm{E}})^2 + Y(\tau_{\mathrm{E}})^2} R(\tau_{\mathrm{E}})(\cos\sigma, \sin\sigma)$$

$$\tilde{X}^3 = 0$$

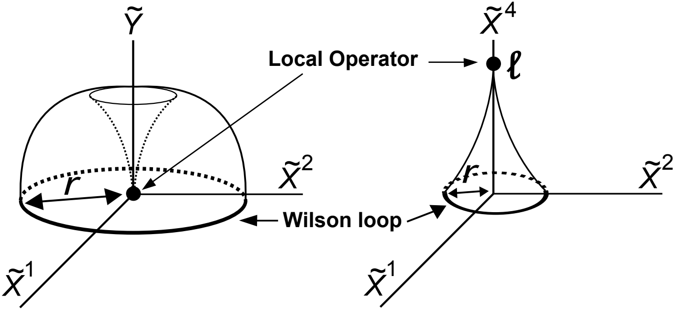
$$\tilde{X}^4 = -\frac{(\ell^2 + r^2)\ell}{\ell^2 + R(\tau_{\mathrm{E}})^2 + Y(\tau_{\mathrm{E}})^2} + \ell$$

Table: Boundaries

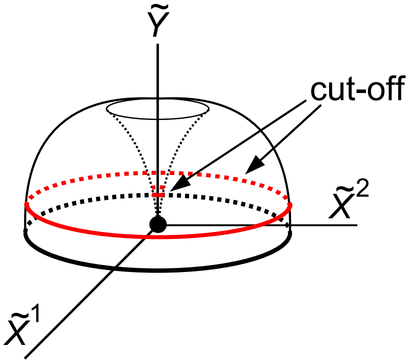
	$\tau_{\mathrm{E}} \to 0$	$\tau_{\mathrm{E}} \to \infty$
$\tilde{Y}(\tau_{\mathrm{E}})$	0	0
$(\tilde{X}^1(\tau_{\mathrm{E}}), \tilde{X}^2(\tau_{\mathrm{E}}))$	$r(\cos\sigma, \sin\sigma)$	0
$\tilde{X}^3$	0	0
$\tilde{X}^4(\tau_{\mathrm{E}})$	0	ℓ

r ... radius of the Wilson loop

ℓ ... distance $W_C \leftrightarrow O_J$



$\tilde{Y}$
$\tilde{X}^4$
Local Operator
ℓ
r
$\tilde{X}^2$
Wilson loop
r
$\tilde{X}^2$
$\tilde{X}^1$
$\tilde{X}^1$


$\tilde{Y}$
cut-off
$\tilde{X}^2$
$\tilde{X}^1$

evaluation of bulk action

$$S_{\text{bulk}} = \frac{\sqrt{\lambda}}{2}\int_{\tau_-}^{\tau_+} \left(\cosh^2\rho \dot{t_{\text{E}}}^2 + \dot{\rho}^2 + \sinh^2\rho + \dot{\theta}^2 + \sin^2\theta + \cos^2\theta\dot{\chi}_3^2 \right) d\tau_{\text{E}}$$

$$\sim \sqrt{\lambda}\left(\frac{1}{\tau_-} - \sqrt{j^2+\cos^2\theta_0}\right)$$

τ_-, τ_+ ... cut-off

boundary term

Drukker-Gross-Ooguri prescription

$$\left.\frac{\partial L}{\partial \dot{u}}u\right|_{\tau=\tau_-} = -\frac{\sqrt{\lambda}}{\tau_-} \qquad \left(u \equiv \frac{1}{\tilde{Y}}\right)$$

L ... Lagrangian

N. Drukker, D. Gross, and H. Ooguri, Phys. Rev. D60 (1999) 125006

$\boxed{\text{vertex operator} \leftrightarrow O_J}$

$$J \ln \frac{\tilde{Y}}{\tilde{Y}^2 + \tilde{R}_\ell^2}\bigg|_{\tau_+} = J[-\ln(\ell^2 + r^2) + j\tau_+ + \ln r + \ln(\sqrt{j^2+1} - j)],$$

$$J \ln(\cos\theta e^{i\chi_3})|_{\tau_+} = J[-j\tau_+ + \ln(\sqrt{j'^2+1} - j') - \ln(\sqrt{j^2+1} - j)]$$

$$\left(j' \equiv \frac{j}{\cos\theta_0}, \;\; \tilde{R}_\ell^2 \equiv (\tilde{X}^1)^2 + (\tilde{X}^2)^2 + (\tilde{X}^3)^2 + (\tilde{X}^4 - \ell)^2 \right)$$

R. Roiban and A. A. Tseytlin, Phys. Rev. D82 (2010) 106011

$$\boxed{\text{Result: } \frac{r^J}{(\ell^2+r^2)^J} \exp\sqrt{\lambda'}[\sqrt{j'^2+1}+j'\ln(\sqrt{j'^2+1}-j')]}$$

$$\text{Zarembo's result: } \exp\sqrt{\lambda}[\sqrt{j^2+1}+j\ln(\sqrt{j^2+1}-j)]$$

$$\left(\sqrt{\lambda'} \equiv \sqrt{\lambda}\cos\theta_0,\ \ j' \equiv \frac{j}{\cos\theta_0}\right)$$

K. Zarembo, Phys. Rev. D66 (2002) 105021
D. Berenstein, R. Corrado, W. Fischler, and J. Maldacena, Phys. Rev. D59 (1999) 105023

Another Saddle Point

another solution for EOM

$$\sin\theta = \frac{\sqrt{j^2+1}}{\cosh\sqrt{j^2+1}(-\tau_E+\tau_0)}$$

$$\chi_3 = ij\tau_E + \frac{i}{2}\ln\frac{\sinh(\sqrt{j^2+1}(-\tau_E+\tau_0)-\xi)}{\sinh(\sqrt{j^2+1}(-\tau_E+\tau_0)+\xi)} + \frac{i}{2}\ln\frac{\sinh(\sqrt{j^2+1}\tau_0+\xi)}{\sinh(\sqrt{j^2+1}\tau_0-\xi)}$$

$$(\ \xi \equiv \ln(\sqrt{j^2+1}+j)\ )$$

evaluation of bulk action

$$S_{\text{bulk}} \sim \sqrt{\lambda}\left(\frac{1}{\tau_-} + \sqrt{j^2 + \cos^2\theta_0}\right)$$

vertex operator

$$J\ln(\cos\theta e^{i\chi_3})|_{\tau_+} = \pi i J + J[-j\tau_+ + \ln(\sqrt{j'^2+1} + j') - \ln(\sqrt{j^2+1} - j)]$$

$$\boxed{\text{Result: } \frac{r^J}{(\ell^2 + r^2)^J}(-1)^J \exp\sqrt{\lambda'}[-\sqrt{j'^2+1} + j'\ln(\sqrt{j'^2+1} + j')]}$$

behavior of $\langle W_C O_J \rangle$

$$I_J(\sqrt{\lambda'}) = \frac{1}{2\pi i}\int_{\infty-\pi i}^{\infty+\pi i} \exp\sqrt{\lambda'}(\cosh x - j'x)dx$$

Another saddle point does not exist on the stationary phase contour:

$$x = -\ln(\sqrt{j'^2+1}+j') + \pi i.$$

$$\text{Result: } \frac{r^J}{(\ell^2+r^2)^J}(-1)^J \exp\sqrt{\lambda'}[-\sqrt{j'^2+1}+j'\ln(\sqrt{j'^2+1}+j')]$$

Summary and Future Works

Summary

·gravity solution for correlation function

·supersymmetry ... coincide

·the solution reproduces the large-R behavior of the correlation function

·another gravity solution $\leftrightarrow$ another saddle point of Bessel function

Future Works

·D-brane version

·exact calculation in gauge theory