Generation, Quark/Lepton Mass Hierarchy and Flavor Mixing from Point Interactions in an Extra Dimension

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soon with 高橋 亮（阪大→北海道大）
Generations
Generations

Who ordered the three same packages!?
Generations
Who ordered the three same packages!?

Mass Hierarchy
Mysteries of the Standard Model

- Generations
  Who ordered the three same packages!?

- Mass Hierarchy
  Including neutrinos, why so different the masses of the fermions are !?
Mysteries of the Standard Model

- Generations
  Who ordered the three same packages!?

- Mass Hierarchy
  Including neutrinos, why so different the masses of the fermions are!?

- Flavor Mixing
Generations
Who ordered the three same packages!?

Mass Hierarchy
Including neutrinos, why so different the masses of the fermions are !?

Flavor Mixing
Why so different the structure of flavor mixing is between quark and lepton !?
We want to realize a situation in which
Purpose

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- Mass hierarchy appear naturally between generations
Purpose

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- Mass hierarchy appear naturally between generations
- Different mixing structure will show up to quark and lepton
Purpose

We want to realize a situation in which

- Mass hierarchy appear naturally between generations
- Different mixing structure will show up to quark and lepton

in the context of 5d gauge theories on a circle.
Ideas & Features
Ideas & Features

- Extra dimension
Extra dimension

We produce the mass hierarchy from the differences in the position of extra dimension.
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\[ y = 0 \quad \text{and} \quad y = L \]
**Extra dimension**

We produce the mass hierarchy from the differences in the position of extra dimension.

\[ y = 0 \quad \text{to} \quad y = L \]
Extra dimension

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\[ y = 0 \quad \text{and} \quad y = L \]
Extra dimension

* We produce the mass hierarchy from the differences in the position of extra dimension.

\[ y = 0 \quad \rightarrow \quad y = L \]
Extra dimension

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Ideas & Features

- **Extra dimension**
  
  ★ We produce the mass hierarchy from the differences in the position of extra dimension.

- **Point Interaction**
Idea & Features

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- **Point Interaction**
  - Mode functions of the fermion degenerate due to point interactions.
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  (One 5D fermion --> Three 4d chiral zero modes)
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\[
\Psi(x, y)
\]

\[
y = 0 \quad y = L
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( One 5D fermion --> Three 4d chiral zero modes )

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\Psi(x, y) \quad y = 0 \quad y = L
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**Ideas & Features**

- **Extra dimension**
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- **Point Interaction**
  - Mode functions of the fermion degenerate due to point interactions.
    (One 5D fermion --> Three 4d chiral zero modes)

$$\Psi(x, y)$$

$$y = 0 \quad \downarrow \quad \psi_{L/R}(x) \quad \psi_{L/R}(x) \quad \psi_{L/R}(x)$$

$$y = L$$
Ideas & Features

- **Extra dimension**
  - We produce the mass hierarchy from the differences in the position of extra dimension.

- **Point Interaction**
  - Mode functions of the fermion degenerate due to point interactions.
  - **(One 5D fermion --> Three 4d chiral zero modes)**
  - Flavor mixing is determined by a configuration of extra dimension.
Setting

- 5d gauge theory on a circle
Setting

• 5d gauge theory on a circle
  with { 5d fermions (one generation)
  5d Higgs field & singlet scalar}
Setting

- 5d gauge theory on a circle

  with \{ 5d fermions (one generation) \\ 5d Higgs field & singlet scalar \}

<table>
<thead>
<tr>
<th>Gauge fields</th>
<th>Fermions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^a_M(x, y)$</td>
<td>$(u(x, y), e(x, y))$</td>
</tr>
<tr>
<td>$B_M(x, y)$</td>
<td>$(d(x, y), \nu(x, y))$</td>
</tr>
</tbody>
</table>

Higgs field
Singlet scalar

$H(x, y)$
$\Phi(x, y)$
Setting

- 5d gauge theory on a circle
  with { 5d fermions (one generation) 5d Higgs field & singlet scalar }

Gauge fields

\[ W^a_M(x, y) \]
\[ B_M(x, y) \]

Higgs field

\[ H(x, y) \]
\[ \Phi(x, y) \]

Fermions

\[
\begin{pmatrix}
  u(x, y) \\
  d(x, y) \\
  u'(x, y) \\
  d'(x, y)
\end{pmatrix}
\begin{pmatrix}
  e(x, y) \\
  \nu(x, y) \\
  e'(x, y) \\
  \nu'(x, y)
\end{pmatrix}
\]

5D Higgs (break the gauge sym.)

Gauge singlet scalar field (for fermion mass hierarchy)
Setting

- 5d gauge theory on a circle
  with \{ 5d fermions (one generation) \\
  5d Higgs field & singlet scalar \}

- Put point interactions on a circle
Setting

- 5d gauge theory on a circle
  - with \{ 5d fermions (one generation) \}
  - 5d Higgs field & singlet scalar

- Put point interactions on a circle
  - ★ Fermions feel several point interactions.
Setting

- **5d gauge theory on a circle**
  - with \{ 5d fermions (one generation) \}
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- **Put point interactions on a circle**
  - ★ Fermions feel several point interactions.
  - ★ Gauge fields & Higgs feel one point interaction.
Setting

- 5d gauge theory on a circle
  - with \{\begin{align*}
  & 5d \text{ fermions (one generation)} \\
  & 5d \text{ Higgs field} \& \text{ singlet scalar}
\end{align*}\}

- Put point interactions on a circle
  - Fermions feel several point interactions.
  - Gauge fields \& Higgs feel one point interaction.

- Impose boundary conditions
Setting

- **5d gauge theory on a circle**
  - with \( \{ \)
  - 5d fermions (one generation)
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- **Put point interactions on a circle**
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- **Impose boundary conditions**
  - ★ No flow of the probability current through the point interactions
Setting

- **5d gauge theory on a circle**
  with { 5d fermions (one generation)  
  5d Higgs field & singlet scalar }

- **Put point interactions on a circle**
  ★ Fermions feel several point interactions.
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- **Impose boundary conditions**
  ★ No flow of the probability current through the point interactions

compatible with several requirement as

★ action principle etc.
Boundary Conditions (BCs)
Boundary Conditions (BCs)

- **Gauge field**

\[
\begin{align*}
\partial_y A_\mu(x, y) &= 0 \\
A_y(x, y) &= 0 @ y = L_0, L_3
\end{align*}
\]
Boundary Conditions (BCs)

- **Gauge field**

\[
\begin{cases}
\partial_y A_\mu (x, y) = 0 \\
A_y (x, y) = 0
\end{cases}
\quad @ \quad y = L_0, L_3
\]

No flow of the probability current

\[y = L_0, L_3\]
Boundary Conditions (BCs)

- **Gauge field**

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\partial_y A_\mu(x, y) &= 0 \\
A_y(x, y) &= 0 \\
\end{align*}
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No flow of the probability current

\[y = L_0, L_3\]

- **4d spectrum**

\[A^{(n)}_\mu, A^{(n)}_y\]
Boundary Conditions (BCs)

- **Higgs field**

\[ \partial_y H(x, y) = 0 \quad @ \quad y = L_0, L_3 \]
Boundary Conditions (BCs)

- Higgs field

\[ \partial_y H(x, y) = 0 \quad @ \quad y = L_0, L_3 \]

No flow of the probability current

\( y = L_0 \quad y = L_3 \)
Boundary Conditions (BCs)

- Singlet scalar

\[
\begin{align*}
\Phi(x, L_0) + L_+ \partial_y \Phi(x, L_0) &= 0 \\
\Phi(x, L_3) - L_- \partial_y \Phi(x, L_3) &= 0
\end{align*}
\]

\((-\infty \leq L_\pm \leq +\infty)\)
Boundary Conditions (BCs)

- Singlet scalar

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\((-\infty \leq L_\pm \leq +\infty)\)

No flow of the probability current

\(y = L_0\) \hspace{1cm} y = L_3
Boundary Conditions (BCs)

♦ Fermions

\[ \Psi_R(x, y) = 0 \quad \text{@ point interactions} \]
\[ \text{or} \]
\[ \Psi_L(x, y) = 0 \quad \text{@ point interactions} \]
Boundary Conditions (BCs)

- Fermions

\[ \Psi_R(x, y) = 0 \text{ at point interactions} \]
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No flow of the probability current
**5d gauge theories on a circle with specified boundary conditions**
5d gauge theories on a circle
with specified boundary conditions

The low energy effective theory
5d gauge theories on a circle with specified boundary conditions → The low energy effective theory → 4d gauge theories
5d gauge theories on a circle with specified boundary conditions

The low energy effective theory

4d gauge theories

+ Generation
Results

5d gauge theories on a circle with specified boundary conditions

The low energy effective theory

4d gauge theories

+ Generation
+ Large mass hierarchy
5d gauge theories on a circle
with specified boundary conditions

The low energy
effective theory

4d gauge theories

+ Generation
+ Large mass hierarchy
+ Large/Small mixing
Wave functions of the fermion are triply-degenerated via the BCs.
Wave functions of the fermion are triply-degenerated via the BCs.

\[
\Psi(x, y) = \sum_{i=1}^{3} \psi_{0, L}^{(i)}(x) \mathcal{G}^{(i)}_{0}(y) + \cdots \\
\Psi_{R}(x, y) = 0 \text{ @ point interactions}
\]
Wave functions of the fermion are triply-degenerated via the BCs.

\[ \Psi(x, y) = \sum_{i=1}^{3} \psi_{0,L}^{(i)}(x) G_0^{(i)}(y) + \cdots \]

\[ \rightarrow (-\partial_y + M_F) G_0^{(i)}(y) = 0 \]
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Bulk mass
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Controlled by only one bulk mass!!
Wave functions of the fermion are triply-degenerated via the BCs.

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Controlled by only one bulk mass !!
Mass Hierarchy

- y-dependent VEV of the scalar can produce mass hierarchies
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\*y-dependent VEV of the scalar can produce mass hierarchies

\[ y \]

\[ L_0 \quad L_1 \quad L_2 \quad L_3 \]

\[ q_L^{(1)} \quad u_R^{(1)} \quad q_L^{(2)} \quad u_R^{(2)} \quad q_L^{(3)} \quad u_R^{(3)} \]
$y$-dependent VEV of the scalar can produce mass hierarchies

$$
\begin{align*}
\phi(y) \\
q_L^{(1)} \\
q_L^{(2)} \\
q_L^{(3)} \\
\end{align*}
$$

$$
\begin{align*}
L_0 & \quad L_1 & \quad L_2 & \quad L_3 \\
qu_R^{(1)} & \quad qu_R^{(2)} & \quad qu_R^{(3)} \\
\end{align*}
$$
**y-dependent VEV of the scalar can produce mass hierarchies**

- Constant VEV **can not produce** mass hierarchies.

\[
\begin{align*}
\phi(y) \\
L_0 & \quad L_1 & \quad L_2 & \quad L_3 \\
q_L^{(1)} & u_R^{(1)} & q_L^{(2)} & u_R^{(2)} & q_L^{(3)} & u_R^{(3)}
\end{align*}
\]
Mass Hierarchy

- y-dependent VEV of the scalar can produce mass hierarchies

- General boundary conditions for the singlet scalar can realize y-dependent VEV
y-dependent VEV of the scalar can produce mass hierarchies

General boundary conditions for the singlet scalar can realize y-dependent VEV

\[ \langle \Phi(x, y) \rangle = \phi(y) \]

Y.F., T.Nagasawa, S.Ohya and M.Sakamoto, PTP 126 (2011) 841
Mass Hierarchy

- Quark sector
Mass Hierarchy

- Quark sector

\[ \phi(y) \]

\[ y \]

\[ L_0 \]

\[ L_1 \]

\[ L_2 \]

\[ L_3 \]
Quark sector

Generation, Quark/Lepton Mass Hierarchy and Flavor Mixing from Point Interactions in an Extra Dimension

Mass Hierarchy
Quark sector

Mass Hierarchy
Quark sector

Mass Hierarchy
Flavor Mixing

- Off diagonal overlap integral leads flavor mixing
Flavor Mixing

- Off diagonal overlap integral leads flavor mixing

\[ R^{(1)} \quad L^{(1)} \quad R^{(2)} \quad L^{(2)} \quad R^{(3)} \quad L^{(3)} \]

\[ L_0 \quad L_1 \quad L_2 \quad L_3 \]

\[ \phi(y) \]
Flavor Mixing

- Off diagonal overlap integral leads flavor mixing

\[ M = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \]
Flavor Mixing

- Off diagonal overlap integral leads flavor mixing

\[ L^{(1)} \quad L^{(2)} \quad L^{(3)} \]

\[ L_0 \quad L_1 \quad L_2 \quad L_3 \]

\[ y \]

\[ \phi(y) \]
Flavor Mixing

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Flavor Mixing

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\[ \phi(y) \]

\[ L_0, L_1, L_2, L_3 \]

\[ R^{(1)}, R^{(2)}, R^{(3)} \]

\[ L'_0, L'_1, L'_2, L'_3 \]
Flavor Mixing

- Off diagonal overlap integral leads flavor mixing
Flavor Mixing

- Off diagonal overlap integral leads flavor mixing

\[ M = \begin{pmatrix}
  m_1 & m_{12} & 0 \\
  0 & m_2 & m_{23} \\
  m_{31} & 0 & m_3
\end{pmatrix} \]
Flavor Mixing

- Off diagonal overlap integral leads flavor mixing

\[
M = \begin{pmatrix}
  m_1 & m_{12} & 0 \\
  0 & m_2 & m_{23} \\
  m_{31} & 0 & m_3 \\
\end{pmatrix}
\]

Source of flavor mixing!!
Smallness of the neutrino masses lead large mixing structure.
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Smallness of the neutrino masses lead large mixing structure.
Smallness of the neutrino masses lead to large mixing structure.
Smallness of the neutrino masses lead large mixing structure.

Tiny mass !!

Large mixing !!
Minimal extension from an interval can lead the CKM-like matrix.
Flavor Mixing II

- Minimal extension from an interval can lead the CKM-like matrix.

\[ |V_{CKM}| = \begin{pmatrix} 0.976 & 0.216 & 0.00313 \\ 0.216 & 0.975 & 0.0498 \\ 0.0138 & 0.0480 & 0.999 \end{pmatrix} \]
Minimal extension from an interval can lead the CKM-like matrix.

\[
|V_{CKM}| = \begin{pmatrix}
0.976 & 0.216 & 0.00313 \\
0.216 & 0.975 & 0.0498 \\
0.0138 & 0.0480 & 0.999
\end{pmatrix}
\]

Good agreement !! But....
- Minimal extension from an interval can lead the CKM-like matrix.

$$|V_{CKM}| = \begin{pmatrix}
0.976 & 0.216 & 0.00313 \\
0.216 & 0.975 & 0.0498 \\
0.0138 & 0.0480 & 0.999
\end{pmatrix}$$

60% larger than real CKM......
Conclusion and Discussion
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5d gauge theories on a circle with specified boundary conditions

The low energy effective theory

4d gauge theories

+ Generation
+ Large mass hierarchy
+ Large/Small mixing
Challenges for the future
Conclusion and Discussion

- **Challenges for the future**
  - Reproduce PMNS matrix
  - Warped metric
  - CP phase from BCs
  - Constraint for extra dim. from FCNC