HIGHER SPIN GAUGE THEORIES AND THEIR CFT DUALS

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1. INTRODUCTION

Higher spin gauge theories and their applications to AdS/CFT correspondence

Higher spin gauge theories

- Higher spin gauge fields
  - A totally symmetric spin-*s* field
  - Yang-Mills (*s*=1), gravity (*s*=2), …

$$\phi_{\mu_1\cdots\mu_s} \sim \phi_{\mu_1\cdots\mu_s} + \partial_{(\mu_1}\xi_{\mu_2\cdots\mu_s)}$$

- Vasiliev theory
  - Non-trivial interacting theories on AdS space
  - Only equations of motion are known
- Toy models of string theory in the tensionless limit
  - Singularity resolution
  - Simplified AdS/CFT correspondence

AdS/CFT correspondence

- Maldacena conjecture '97

Superstring theory on AdS_{d+1}	⟺	*d* dim. conformal field theory (CFT)

- Difficulties to proof the conjecture
  - Strong/week duality
  - Superstrings on AdS have not been solved
- Simplified AdS/CFT

Higher spin gauge theory on AdS_{d+1}	⟺	*d* dim. CFT with higher spin currents

Examples

- AdS_4/CFT_3 [Klebanov-Polyakov '02]

  4d Vasiliev theory ⟺ 3d O(N) vector model

  - Evidences
    - Spectrum, RG-flow, correlation functions [Giombi-Yin '09, '10]
- AdS_3/CFT_2 [Gaberdiel-Gopakumar '10]

  3d Vasiliev theory ⟺ Large N minimal model

  - Evidences
    - Symmetry, partition function, RG-flow, correlation functions
  - A supersymmetric extension [Creutzig-YH-Rønne '11]

Plan of the talk

1. Introduction
2. Higher spin gauge theories
3. Higher spin holography
4. Conclusion

2. HIGHER SPIN GAUGE THEORIES

Higher spin gravity theories and Chern-Simons formulation

Field equation (free theory)

- A totally symmetric spin-*s* field

$$\phi_{\mu_1\cdots\mu_s}$$

  - Yang-Mills (*s*=1), Gravity (*s*=2), …
- Field equations for free theory [Fronsdal '78]

$$\mathcal{F}_{\mu_1\ldots\mu_s} \equiv \Box\,\phi_{\mu_1\ldots\mu_s} - \partial_{(\mu_1|}\partial^\lambda\phi_{|\mu_2\ldots\mu_s)\lambda} + \partial_{(\mu_1}\partial_{\mu_2}\phi_{\mu_3\ldots\mu_s)\lambda}{}^\lambda = 0$$

  - $\mathcal{F}_\mu = \partial^\nu F_{\nu\mu}$ (*s*=1), Linearized Ricci tensor (*s*=2)
- The higher spin gauge symmetry

$$\delta\phi_{\mu_1\ldots\mu_s} = \partial_{(\mu_1}\xi_{\mu_2\ldots\mu_s)},\ \ \xi^\lambda_{\lambda\mu_3\ldots\mu_s} = 0$$

  - Abelian gauge tfm. (*s*=1), Linearized diffeomorphism (*s*=2)

Action (free theory)

- The action for free theory

$$S = \frac{1}{2}\int d^D x \phi^{\mu_1 \dots \mu_s}\left(\mathcal{F}_{\mu_1 \dots \mu_s} - \frac{1}{2}\eta_{(\mu_1\mu_2}\mathcal{F}_{\mu_3 \dots \mu_s)\lambda}{}^{\lambda}\right)$$

  - Uniquely fixed by the gauge symmetry
  - Under the double-traceless constraint

$$\phi^{\lambda\sigma}_{\lambda\sigma\mu_5 \dots \mu_s} = 0$$

- Free theory on dS or AdS space [Fronsdal '79]

$$\partial_\mu \leftrightarrow \nabla_\mu, \ \mathcal{F}_{\mu_1 \dots \mu_s} \leftrightarrow \hat{\mathcal{F}}_{\mu_1 \dots \mu_s}$$

  - Derivatives are replaced by covariant derivative
  - The field strength receives corrections due to the curvature

Interacting theory

- Coleman-Mandula theorem
  - Any interacting theory is not possible with higher spin symmetry
  - Assumptions: mass gap, flat space, finitely many dof,…
- Vasiliev theory
  - Interacting theory by escaping assumptions
    - Defined on AdS space
    - With all higher spins (s=2,3,4,…)
  - Only equations of motion are known
- Higher spin AdS_3 gravity
  - Spin can be truncated (s=2,3,4,…N)
  - Chern-Simons description is possible

3d Einstein gravity

- Chern-Simons description [Achucarro-Townsend ’86, Witten ’88]
  - Action of SL(2) x SL(2) CS theory

$$S = S_{\rm CS}[A] - S_{\rm CS}[\tilde{A}]$$

$$S_{\rm CS}[A] = \frac{k_{\rm CS}}{4\pi} \int {\rm tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) , \ k_{\rm CS} = \frac{\ell}{4G}$$

  - Gauge transformation

$$\delta A = d\lambda + [A, \lambda], \ \delta \tilde{A} = d\tilde{\lambda} + [\tilde{A}, \tilde{\lambda}]$$

$$A = A^a_\mu J_a dx^\mu, \ J_a (a = 1, 2, 3) : {\rm sl}(2) \text{ generator}$$

- Einstein Gravity with $\Lambda < 0$
  - Dreibein: $e^a_\mu = \frac{\ell}{2}(A^a_\mu - \tilde{A}^a_\mu)$
  - Spin connection: $\omega_{\mu,a,b} = \frac{1}{2}\epsilon_{abc}\omega^c_\mu, \ \omega^c_\mu = \frac{1}{2}(A^c_\mu + \tilde{A}^c_\mu)$

Higher spin AdS_3 gravity

- G x G Chern-Simons theory
  - Higher spin gravity can be obtained by replacing SL(2) by G
- Embed gravitational sl(2) into g

$$\mathrm{sl}(N) = \mathrm{sl}(2) \oplus \left(\bigoplus_{s=3}^{N} \mathrm{g}^{(s)} \right) \qquad (\text{c.f. } 8 = 3 + 5 \ \ \text{for SL}(3))$$

Gravitational sl(2) Space-time spin *s*

- Examples

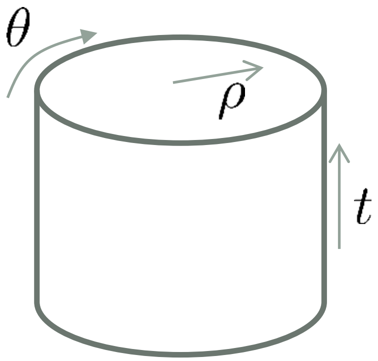
Group G	Theory
SL(*N*)	Higher spin gravity with *s*=2,3,…,*N*
SL(∞)	Bosonic Vasiliev theory
SL(*N*+1|*N*)	Higher spin *N*=2 supergravity
SL(∞+1|∞)	Supersymmetric Vasiliev theory [Blencowe '89]

Asymptotic symmetry

- Chern-Simons theory with boundary
  - DOF exist only at the boundary and described by WZNW model
- Classical asymptotic symmetry
  - Boundary conditions
    - Asymptotically AdS condition has to be assigned for AdS/CFT
    - The condition is equivalent to Drinfeld-Sokolov Hamiltonian reduction [Campoleoni, Fredenhagen, Pfenninger, Theisen '10, '11]
  - Examples

Group G	Symmetry	
SL(2)	Virasoro	Brown-Henneaux '86
SL(N)	W_N	Henneaux-Rey '10, Campoleni-Fredenhagen-Pfenninger-Theisen '10, Gaberdiel-Hartman '11
SL(N+1|N)	$\mathcal{N}$=2 W_{N+1}	Creutzig-YH-Rønne '11, Henneaux-Gómez-Park-Rey '12, Hanaki-Peng '12

Gauge fixings & conditions



- Coordinate system
  - t: time coordinate, (ρ,θ): coordinates of disk
  - Boundary at $\rho \to \infty$
- Solutions to the equations of motion
  - Gauge fixing & boundary condition $(A_\pm = A_\theta \pm A_t)$

$$A_+ = e^{-\rho V_0^{(2)}} a(t+\theta) e^{\rho V_0^{(2)}}, \ A_- = 0, \ A_\rho = e^{-\rho V_0^{(2)}} \partial_\rho e^{\rho V_0^{(2)}}$$

  - The condition of asymptotically AdS space
    - Metric should decay properly near the boundary (e.g. Kerr/CFT)

$$a(t+\theta) = V_1^{(2)} + \sum_{s \geq 2} L_s(t+\theta) V_{-s+1}^{(s)}, \ V_{n \neq -s+1}^{(s)} = 0$$

  - Same as the constraints for Drinfeld-Sokolov Hamiltonian reduction [Campoleoni, Fredenhagen, Pfenninger, Theisen ’10, ’11]

Asymptotic symmetry

- Residual gauge transformation (*t*=0)

$$\Lambda(\theta) = e^{-\rho V_0^{(2)+}} \lambda(\theta) e^{\rho V_0^{(2)+}}, \;\; \delta_\lambda a(\theta) = \partial_\theta \lambda(\theta) + [a(\theta), \lambda(\theta)]$$

  - λ(θ) not vanishing at the boundary generates physical symmetry
- Asymptotic symmetry
  - Generator

$$Q(\lambda) = -\frac{k}{2\pi} \int d\theta \, \mathrm{str} \left(\lambda(\theta) a(\theta)\right)$$

  - Poisson brackets

$$\{Q(\lambda), Q(\eta)\} = -\frac{k}{2\pi} \int d\theta \, \mathrm{str} \left(\eta(\theta) \delta_\lambda a(\theta)\right)$$

- Symmetry algebra
  - Same as the one from the Hamiltonian reduction
  - Virasoro symmetry with $c = 3\ell/2G$ as subalgebra

3. HIGHER SPIN HOLOGRAPHY

Proposals of simplified AdS/CFT correspondence and their evidences

AdS$_4$/CFT$_3$

- Klebanov-Polyakov conjecture '02

4d Vasiliev theory ⟺ 3d O(N) vector model

  - A weak/weak duality
- State counting

	Gauge invariant operator	Bulk fields
Vector-type model	$h^a \partial_{(\mu_1} \cdots \partial_{\mu_s)} h^a$	One higher spin field $\phi_{\mu_1 \cdots \mu_s}$
Matrix-type model	$\mathrm{tr}[\Phi \nabla^{l_1} \Phi \nabla^{l_2} \cdots \Phi]$	Many string states with fixed total spin

Evidences

- RG flow by a relevant operator $\mathcal{O} = \frac{\lambda}{2N}(h^a h^a)^2$

Flow	**O(*N*) model**	**B.C. for bulk scalar**
UV	Free theory ($\lambda = 0$)	Dirichlet (usual)
IR	Critical theory ($\lambda = \lambda^*$)	Neumann (alternative)

- Correlation functions
  - Some boundary correlation functions are computed explicitly from the bulk side [Giombi-Yin '09, '10]
  - A higher spin symmetry is enough to fix the CFT correlation functions [Maldacena-Zhiboedov '12, '12]

AdS_3/CFT_2

- Gaberdiel-Gopakumar conjecture ’10

3d Vasiliev theory ⟺ Large N minimal model

- Gravity side
  - A bosonic truncation of higher spin supergravity by Prokushkin-Vasiliev ’98
  - It includes massive scalar fields
- CFT side
  - Minimal model with respect to W_N-algebra (higher spin extension of Virasoro algebra)
  - Exactly solvable in principle

Minimal model holography

Higher spin gravity

- Massless sector
  - Higher spin gauge fields $(s = 2, 3, \dots)$
  - Asymptotic W_∞ symmetry
- Massive sector
  - Complex scalars

$$M^2 = -1 + \lambda^2$$

Large *N* minimal model

- W_N minimal model
  - Coset description

$$\frac{\mathrm{SU}(N)_k \otimes \mathrm{SU}(N)_1}{\mathrm{SU}(N)_{k+1}}$$

- 't Hooft limit
  - Large *N* limit

$$k, N \to \infty$$

  - Fix the ratio

$$0 < \lambda = \frac{N}{k+N} < 1$$

Evidences

- Symmetry
  - Asymptotic symmetry of the gravity theory is W algebra, while the dual CFT is W_N minimal model
- RG flow
  - RG flow pattern is reproduced from the bulk
- Spectrum
  - One loop partition functions of the dual theories match [Gaberdiel-Gopakumar-Hartman-Raju '11]
- Interactions
  - Some three point functions are studied [Chang-Yin '11, Ammon-Kraus-Perlmutter '11]

Generalization

- A truncation version [Ahn ’11, Gaberdiel-Vollenweider ’11]
  - Gravity side: Gauge fields with only spins *s*=2,4,6,…
  - CFT side: WD_N minimal model

$$\frac{\mathrm{SO}(N)_k \otimes \mathrm{SO}(N)_1}{\mathrm{SO}(N)_{k+1}}$$

- A supersymmetric version [Creutzig-YH-Rønne ’11]
  - Gravity side: Full sector of higher spin supergravity by Prokushkin-Vasiliev ’98
  - CFT side: N=(2,2) CP^N Kazama-Suzuki model

$$\frac{\mathrm{SU}(N+1)_k \otimes \mathrm{SO}(2N)_1}{\mathrm{SU}(N)_{k+1} \otimes \mathrm{U}(1)_{N(N+1)(k+N+1)}}$$

Our proposal

Prokushkin-Vasiliev theory

- Higher spin gauge fields
  - Bosons ($s = 1,2,\ldots$) and fermions ($s = 3/2, 5/2, \ldots$)
  - $\mathcal{N}$=(2,2) W_∞ symmetry near the boundary of AdS_3
- Massive matter fields
  - Complex scalars

    $$(M_B^\pm)^2 = -1 + \frac{1}{4}(1 \mp 1 - 2\lambda)^2$$

  - Dirac spin 1/2 spinors

    $$(M_F^\pm)^2 = (\frac{1}{2} - \lambda)^2$$

CP^N Kazama-Suzuki model

- $\mathcal{N}$=(2,2) W_N minimal model
  - Coset description

    $$\frac{SU(N+1)_k \otimes SO(2N)_1}{SU(N)_{k+1} \otimes U(1)_{N(N+1)(k+N+1)}}$$

- 't Hooft limit
  - Large N limit

    $$k, N \to \infty$$

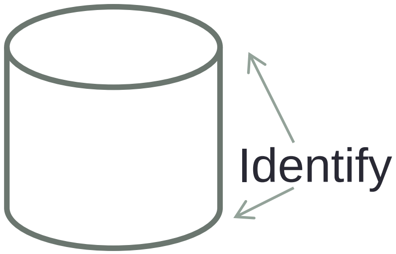
  - Fix the ratio

    $$0 < \lambda = \frac{N}{k+N} < 1$$

Evidences

- Symmetry
  - Asymptotic symmetry is $\mathcal{N}=(2,2)$ W algebra [Creutzig-YH-Rønne '11, Henneaux-Gómez-Park-Rey '12, Hanaki-Peng '12]
  - The Kazama-Suzuki model has the same symmetry [Ito '91]
- Spectrum
  - One-loop partition function is obtained from gravity one-loop determinants [Creutzig-YH-Rønne '11]
  - One loop partition function is computed at the 't Hooft limit and the agreement is found with the gravity result [Candu-Gaberdiel '12]
- Interactions
  - Three point functions with one higher spin current are studied [Creutzig-YH-Rønne, to appear]

Agreement of the spectrum

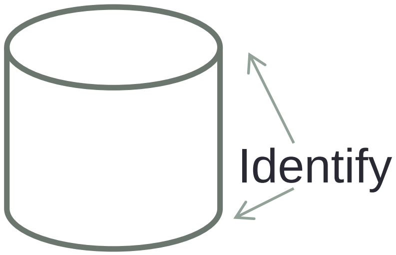


- Gravity partition function
  - Bosonic sector
    - Massive scalars [Giombi-Maloney-Yin ’08, David-Gaberdiel-Gopakumar ’09]
    - Bosonic higher spin [Gaberdiel-Gopakumar-Saha ’10]
  - Fermionic sector
    - Massive fermions, fermionic higher spin [Creutzig-YH-Rønne ’11]
- CFT partition function at the ’t Hooft limit
  - It is obtained by the sum of characters over all states and it was found to reproduce the gravity results

$$Z^{N,k}(q) = \sum_\Lambda |b_\Lambda^{N,k}(q)|^2$$

- - Bosonic case [Gaberdiel-Gopakumar-Hartman-Raju ’11]
  - Supersymmetric case [Candu-Gaberdiel ’12]

Partition function at 1-loop level



- Total contribution
  - Higher spin sector + Matter sector

$$Z^{\text{Bulk}} = Z^{\text{HS}} Z^{\text{matter}}$$

- Higher spin sector
  - Two series of bosons and fermions

$$Z^{\text{HS}} = \prod_{s=2}^{\infty} Z_B^{(s)} (Z_F^{(s-1)})^2 Z_B^{(s-1)}$$

$$Z_B^{(s)} = \prod_{n=s}^{\infty} |1 - q^n|^{-2}, \quad Z_F^{(s)} = \prod_{n=s}^{\infty} |1 + q^{n+\frac{1}{2}}|^2$$

- Matter part sector
  - 4 massive complex scalars and 4 massive Dirac fermions

$$Z^{\text{matter}} = Z_{\text{susy}}^{\frac{\lambda}{2}} Z_{\text{susy}}^{\frac{1-\lambda}{2}}, \quad Z_{\text{susy}}^{h} = Z_{\text{scalar}}^{h} (Z_{\text{spinor}}^{h+\frac{1}{2}})^2 Z_{\text{scalar}}^{h+\frac{1}{2}}$$

$$Z_{\text{scalar}}^{h} = \prod_{l,l'=0}^{\infty} (1 - q^{h+l} \bar{q}^{h+l'})^{-2}$$

$$Z_{\text{spinor}}^{h} = \prod_{l,l'=0}^{\infty} (1 + q^{h+l} \bar{q}^{h-\frac{1}{2}+l'})(1 + q^{h-\frac{1}{2}+l} \bar{q}^{h+l'})$$

Prokushkin-Vasiliev theory

- Master fields
  - W_μ: gauge fields, B: matter fields, S_α: auxiliary fields
  - Parameters: $z_\alpha, y_\alpha, \psi_1, \psi_2, k, \rho$

$$k^2 = \rho^2 = 1, \ \{k, \rho\} = \{k, y_\alpha\} = \{k, z_\alpha\} = 0, \ [\rho, y_\alpha] = [\rho, z_\alpha] = 0$$

- Field equations

$$dW = W * \wedge W, \ dB = W * B - B * W, \dots$$

  - Gauge transformations

$$\delta W = d\varepsilon - W * \varepsilon + \varepsilon * W, \ \delta B = \varepsilon * B - B * \varepsilon, \dots$$

- Vacuum solutions & perturbations around $B = \nu$

$$dA + A * \wedge A = 0, \ dC + A * C - C * \bar{A} = 0$$

  - Chern-Simons gauge theory on a large *N* limit of SL(*N*+1|*N*)
  - On AdS matter fields with mass depends on ν

Boundary 3-pt functions

- Scalar field in the bulk ⇔ Scalar operator at the boundary

  $\phi_\lambda, \; m^2 = -1 + \lambda^2$ $\qquad\qquad$ $\mathcal{O}_B^h, \; h = \frac{1 \pm \lambda}{2}$

- Boundary 3-pt functions from the bulk theory [Chang-Yin ’11, Ammon-Kraus-Perlmutter ’11]

$$\left\langle \mathcal{O}_B^h(z_1) \bar{\mathcal{O}}_B^h(z_2) J^{(s)}(z_3) \right\rangle = N_s(h) \left(\frac{z_{12}}{z_{13} z_{23}} \right)^s \langle \mathcal{O}_B^h(z_1) \bar{\mathcal{O}}_B^h(z_2) \rangle$$

$$N_s(h) = \frac{(-1)^{s-1}}{2\pi} \frac{\Gamma(s)^2 \Gamma(s-1+2h)}{\Gamma(2s-1)\Gamma(2h)}$$

- Comparison to the boundary CFT
  - Direct computation for *s*=3 (for *s*=4 [Ahn ’11])
  - Consistence to the large *N* limit of W_N for *s*=4,5,..
- Analysis is extended to the supersymmetric case
  - 3-pt functions with fermionic operator [Creutzig-YH-Rønne, to apper]

4. CONCLUSION

Summary and other related works

Summary

- Higher spin gauge theories
  - Gravity theory with spin 2 gauge field can be extended to theory with spin $s > 2$ gauge fields.
  - Vasiliev develops higher spin gravity theories on AdS with non-trivial interactions, though only equations of motion are known.
  - Chern-Simons formulation is possible in 3 dimensions.
- Higher spin holography
  - 4d Vasiliev theory is dual to 3d O(N) vector model
  - 3d Vasiliev theory is dual to 2d W_N minimal model
  - Lots of evidences are already given
    - Symmetry, RG-flow, spectrum, correlation functions

Other related works (I)

- Resolution of black hole singularity [Ammon-Gutperle-Kraus-Perlmutter ’11,…]
  - The higher spin gravity has a large gauge symmetry. The notion of singularity, horizon,… is not gauge invariant
  - Generalized BTZ black hole can be changed into a warm hole solution by gauge transformation.
- 1/*N* corrections [Castro, Lepage-Jutier, Maloney ’10,…]
  - Dual CFT is defined at the finite *N*, and the finite *N* effects should be related to the quantum effects of Vasiliev theory
  - Missing states in the CFT correspond to geometries with conical deficits

Other related works (II)

- 3d CS theory with vector matters [Aharony, Gur-Ari, Yacoby ’11, Giombi-Minwalla-Prakash-Trivedi-Wadia-Yin’11, Chang-Minwalla-Sharma-Yin ’12,…]
  - One parameter family of correspondence can be considered in the ’t Hooft limit with large N,k
  - Dual to a generalization of Vasiliev theory
  - Related to ABJ theory with $U(N)_k \times U(M)_{-k}$ gauge symmetry. Take large N but finite M. Dual to superstring on $AdS_4 \times CP^3$