HIGHER SPIN GAUGE THEORIES AND THEIR CFT DUALS

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1. INTRODUCTION

Higher spin gauge theories and their applications to AdS/CFT correspondence
Higher spin gauge theories

- Higher spin gauge fields
  - A totally symmetric spin-$s$ field
  - Yang-Mills ($s=1$), gravity ($s=2$), …

$$\phi_{\mu_1 \cdots \mu_s} \sim \phi_{\mu_1 \cdots \mu_s} + \partial_{(\mu_1} \xi_{\mu_2 \cdots \mu_s)}$$

- Vasiliev theory
  - Non-trivial interacting theories on AdS space
  - Only equations of motion are known

- Toy models of string theory in the tensionless limit
  - Singularity resolution
  - Simplified AdS/CFT correspondence
AdS/CFT correspondence

- Maldacena conjecture ’97

\[ \text{Superstring theory on AdS}_{d+1} \leftrightarrow d \text{ dim. conformal field theory (CFT)} \]

- Difficulties to proof the conjecture
  - Strong/weak duality
  - Superstrings on AdS have not been solved

- Simplified AdS/CFT

\[ \text{Higher spin gauge theory on AdS}_{d+1} \leftrightarrow d \text{ dim. CFT with higher spin currents} \]
Examples

- $\text{AdS}_4/\text{CFT}_3$ [Klebanov-Polyakov ’02]
  
  \hspace{2cm} 4d Vasiliev theory $\leftrightarrow$ 3d O($N$) vector model

- Evidences
  - Spectrum, RG-flow, correlation functions [Giombi-Yin ’09, ’10]

- $\text{AdS}_3/\text{CFT}_2$ [Gaberdiel-Gopakumar ’10]
  
  \hspace{2cm} 3d Vasiliev theory $\leftrightarrow$ Large $N$ minimal model

- Evidences
  - Symmetry, partition function, RG-flow, correlation functions
  - A supersymmetric extension [Creutzig-YH-Rønne ’11]
Plan of the talk

1. Introduction
2. Higher spin gauge theories
3. Higher spin holography
4. Conclusion
2. HIGHER SPIN GAUGE THEORIES

Higher spin gravity theories and Chern-Simons formulation
Field equation (free theory)

- A totally symmetric spin-$s$ field

\[ \phi_{\mu_1 \ldots \mu_s} \]

- Yang-Mills ($s=1$), Gravity ($s=2$), …

- **Field equations for free theory** [Fronsdal ’78]

\[ F_{\mu_1 \ldots \mu_s} \equiv \Box \phi_{\mu_1 \ldots \mu_s} - \partial_{(\mu_1} \partial^\lambda \phi_{\mid \mu_2 \ldots \mu_s)} \lambda + \partial_{\mu_1} \partial_{\mu_2} \phi_{\mu_3 \ldots \mu_s} \lambda^\lambda = 0 \]

- \( F_\mu = \partial^\nu F_{\nu \mu} \) ($s=1$), Linearized Ricci tensor ($s=2$)

- The higher spin gauge symmetry

\[ \delta \phi_{\mu_1 \ldots \mu_s} = \partial_{(\mu_1} \xi_{\mu_2 \ldots \mu_s)}, \quad \xi_\lambda^{\lambda}_{\mu_3 \ldots \mu_s} = 0 \]

- Abelian gauge tfm. ($s=1$), Linearized diffeomorphism ($s=2$)
Action (free theory)

- The action for free theory

\[ S = \frac{1}{2} \int d^D x \phi^{\mu_1 \ldots \mu_s} \left( F_{\mu_1 \ldots \mu_s} - \frac{1}{2} \eta_{(\mu_1 \mu_2} F_{\mu_3 \ldots \mu_s)\lambda^\lambda} \right) \]

- Uniquely fixed by the gauge symmetry
- Under the double-traceless constraint

\[ \phi^{\lambda \sigma}_{\lambda \sigma \mu_5 \ldots \mu_s} = 0 \]

- Free theory on dS or AdS space [Fronsdal '79]

\[ \partial_\mu \leftrightarrow \nabla_\mu, \ F_{\mu_1 \ldots \mu_s} \leftrightarrow \hat{F}_{\mu_1 \ldots \mu_s} \]

- Derivatives are replaced by covariant derivative
- The field strength receives corrections due to the curvature
Interacting theory

- Coleman-Mandula theorem
  - Any interacting theory is not possible with higher spin symmetry
  - Assumptions: mass gap, flat space, finitely many dof,…

- Vasiliev theory
  - Interacting theory by escaping assumptions
    - Defined on AdS space
    - With all higher spins (s=2,3,4,…)
  - Only equations of motion are known

- Higher spin AdS$_3$ gravity
  - Spin can be truncated (s=2,3,4,…N)
  - Chern-Simons description is possible
3d Einstein gravity

- **Chern-Simons description** [Achucarro-Townsend ’86, Witten ’88]
  - Action of SL(2) x SL(2) CS theory
    \[
    S = S_{CS}[A] - S_{CS}[\tilde{A}]
    \]
    \[
    S_{CS}[A] = \frac{k_{CS}}{4\pi} \int \text{tr} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right), \quad k_{CS} = \frac{\ell}{4G}
    \]
  - Gauge transformation
    \[
    \delta A = d\lambda + [A, \lambda], \quad \delta \tilde{A} = d\tilde{\lambda} + [\tilde{A}, \tilde{\lambda}]
    \]
    \[
    A = A_\alpha^\mu J_\alpha dx^\mu, \quad J_\alpha(a = 1, 2, 3) : \text{sl}(2) \text{ generator}
    \]

- **Einstein Gravity with } \Lambda < 0}
  - Dreibein: \( e_\mu^a = \frac{\ell}{2} (A_\mu^a - \tilde{A}_\mu^a) \)
  - Spin connection: \( \omega_{\mu,a,b} = \frac{1}{2} \epsilon_{abc} \omega_{\mu}^c, \quad \omega_{\mu}^c = \frac{1}{2} (A_\mu^c + \tilde{A}_\mu^c) \)
Higher spin AdS$_3$ gravity

- G x G Chern-Simons theory
  - Higher spin gravity can be obtained by replacing SL(2) by G
- Embed gravitational sl(2) into g

\[ \text{sl}(N) = \text{sl}(2) \oplus \left( \bigoplus_{s=3}^{N} g^{(s)} \right) \]  
\( (\text{c.f. } 8 = 3 + 5 \text{ for SL(3)}) \)

Gravitational sl(2)  Space-time spin s

- Examples

<table>
<thead>
<tr>
<th>Group G</th>
<th>Theory</th>
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</thead>
<tbody>
<tr>
<td>SL(N)</td>
<td>Higher spin gravity with s=2,3,…,N</td>
</tr>
<tr>
<td>SL(∞)</td>
<td>Bosonic Vasiliev theory</td>
</tr>
<tr>
<td>SL(N+1</td>
<td>N)</td>
</tr>
<tr>
<td>SL(∞+1</td>
<td>∞)</td>
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</tbody>
</table>
Asymptotic symmetry

- Chern-Simons theory with boundary
  - DOF exist only at the boundary and described by WZNW model

- Classical asymptotic symmetry
  - Boundary conditions
    - Asymptotically AdS condition has to be assigned for AdS/CFT
    - The condition is equivalent to Drinfeld-Sokolov Hamiltonian reduction
      [Campoleoni, Fredenhagen, Pfenninger, Theisen ’10, ’11]
  - Examples

<table>
<thead>
<tr>
<th>Group G</th>
<th>Symmetry</th>
<th>References</th>
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<tbody>
<tr>
<td>SL(2)</td>
<td>Virasoro</td>
<td>Brown-Henneaux ’86</td>
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<tr>
<td>SL(N)</td>
<td>$W_N$</td>
<td>Henneaux-Rey ’10, Campoleni-Fredenhagen-Pfenninger-Theisen ’10, Gaberdiel-Hartman ’11</td>
</tr>
<tr>
<td>SL(N+1</td>
<td>N)</td>
<td>$N=2 W_{N+1}$</td>
</tr>
</tbody>
</table>
Gauge fixings & conditions

• Coordinate system
  • $t$: time coordinate, $(\rho, \theta)$: coordinates of disk
  • Boundary at $\rho \to \infty$

• Solutions to the equations of motion
  • Gauge fixing & boundary condition ($A_\pm = A_\theta \pm A_t$)
    \[ A_+ = e^{-\rho V_0^{(2)}} a(t + \theta) e^{\rho V_0^{(2)}}, \quad A_- = 0, \quad A_\rho = e^{-\rho V_0^{(2)}} \partial_\rho e^{\rho V_0^{(2)}} \]

• The condition of asymptotically AdS space
  • Metric should decay properly near the boundary (e.g. Kerr/CFT)
    \[ a(t + \theta) = V_1^{(2)} + \sum_{s \geq 2} L_s(t + \theta) V^{(s)}_{-s+1}, \quad V^{(s)}_{n \neq -s+1} = 0 \]

• Same as the constraints for Drinfeld-Sokolov Hamiltonian reduction
  [Campoleoni, Fredenhagen, Pfenninger, Theisen ’10, ’11]
Asymptotic symmetry

- Residual gauge transformation \( (t=0) \)
  \[
  \Lambda(\theta) = e^{-\rho V_0^{(2)+}} \lambda(\theta) e^{\rho V_0^{(2)+}}, \quad \delta_\lambda a(\theta) = \partial_\theta \lambda(\theta) + [a(\theta), \lambda(\theta)]
  \]

- \( \lambda(\theta) \) not vanishing at the boundary generates physical symmetry

- Asymptotic symmetry
  - Generator
    \[
    Q(\lambda) = -\frac{k}{2\pi} \int d\theta \text{str} (\lambda(\theta) a(\theta))
    \]
  - Poisson brackets
    \[
    \{Q(\lambda), Q(\eta)\} = -\frac{k}{2\pi} \int d\theta \text{str} (\eta(\theta) \delta_\lambda a(\theta))
    \]

- Symmetry algebra
  - Same as the one from the Hamiltonian reduction
  - Virasoro symmetry with \( c = 3\ell/2G \) as subalgebra
3. HIGHER SPIN HOLOGRAPHY

Proposals of simplified AdS/CFT correspondence and their evidences
AdS$_4$/CFT$_3$

- Klebanov-Polyakov conjecture ’02

4d Vasiliev theory $\leftrightarrow$ 3d O($N$) vector model

- A weak/weak duality
- State counting

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<th>Gauge invariant operator</th>
<th>Bulk fields</th>
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<tbody>
<tr>
<td>Vector-type model</td>
<td>$h^a \partial_{(\mu_1} \cdots \partial_{\mu_s)} h^a$</td>
<td>One higher spin field $\phi_{\mu_1 \cdots \mu_s}$</td>
</tr>
<tr>
<td>Matrix-type model</td>
<td>$\text{tr}[\Phi \nabla^{l_1} \Phi \nabla^{l_2} \cdots \Phi]$</td>
<td>Many string states with fixed total spin</td>
</tr>
</tbody>
</table>
Evidences

- RG flow by a relevant operator $\mathcal{O} = \frac{\lambda}{2N} (h^a h^a)^2$

<table>
<thead>
<tr>
<th>Flow</th>
<th>$O(N)$ model</th>
<th>B.C. for bulk scalar</th>
</tr>
</thead>
<tbody>
<tr>
<td>UV</td>
<td>Free theory ($\lambda = 0$)</td>
<td>Dirichlet (usual)</td>
</tr>
<tr>
<td>IR</td>
<td>Critical theory ($\lambda = \lambda^*$)</td>
<td>Neumann (alternative)</td>
</tr>
</tbody>
</table>

- Correlation functions
  - Some boundary correlation functions are computed explicitly from the bulk side [Giombi-Yin ’09, ’10]
  - A higher spin symmetry is enough to fix the CFT correlation functions [Maldacena-Zhiboedov ’12, ’12]
AdS$_3$/CFT$_2$

- Gaberdiel-Gopakumar conjecture ’10

3d Vasiliev theory ↔ Large $N$ minimal model

- Gravity side
  - A bosonic truncation of higher spin supergravity by Prokushkin-Vasiliev ’98
  - It includes massive scalar fields

- CFT side
  - Minimal model with respect to $W_N$-algebra (higher spin extension of Virasoro algebra)
  - Exactly solvable in principle
Minimal model holography

Higher spin gravity

- Massless sector
  - Higher spin gauge fields \((s = 2, 3, \ldots)\)
  - Asymptotic \(W_\infty\) symmetry

- Massive sector
  - Complex scalars

\[ M^2 = -1 + \lambda^2 \]

Large \(N\) minimal model

- \(W_N\) minimal model
  - Coset description
    \[
    \frac{SU(N)_k \otimes SU(N)_1}{SU(N)_{k+1}}
    \]

- 't Hooft limit
  - Large \(N\) limit
    \[ k, N \to \infty \]
  - Fix the ratio
    \[ 0 < \lambda = \frac{N}{k+N} < 1 \]
Evidences

• Symmetry
  • Asymptotic symmetry of the gravity theory is $W$ algebra, while the dual CFT is $W_N$ minimal model

• RG flow
  • RG flow pattern is reproduced from the bulk

• Spectrum
  • One loop partition functions of the dual theories match [Gaberdiel-Gopakumar-Hartman-Raju ’11]

• Interactions
  • Some three point functions are studied [Chang-Yin ’11, Ammon-Kraus-Perlmutter ’11]
Generalization

- **A truncation version** [Ahn ’11, Gaberdiel-Vollenweider ’11]
  - Gravity side: Gauge fields with only spins $s=2, 4, 6, \ldots$
  - CFT side: $WD_N$ minimal model
    \[
    \frac{\text{SO}(N)_k \otimes \text{SO}(N)_1}{\text{SO}(N)_{k+1}}
    \]

- **A supersymmetric version** [Creutzig-YH-Rønne ’11]
  - Gravity side: Full sector of higher spin supergravity by Prokushkin-Vasiliev ’98
  - CFT side: $N=(2,2)$ CP$^N$ Kazama-Suzuki model
    \[
    \frac{\text{SU}(N+1)_k \otimes \text{SO}(2N)_1}{\text{SU}(N)_{k+1} \otimes \text{U}(1)^N(N+1)(k+N+1)}
    \]
Our proposal

Prokushkin-Vasiliev theory

- Higher spin gauge fields
  - Bosons ($s = 1,2,\ldots$) and fermions ($s = 3/2, 5/2,\ldots$)
  - $\mathcal{N}=(2,2)$ $\mathcal{W}_\infty$ symmetry near the boundary of AdS$_3$

- Massive matter fields
  - Complex scalars
    \[ (M_B^\pm)^2 = -1 + \frac{1}{4}(1 \mp 1 - 2\lambda)^2 \]
  - Dirac spin 1/2 spinors
    \[ (M_F^\pm)^2 = \left(\frac{1}{2} - \lambda\right)^2 \]

CP$^N$ Kazama-Suzuki model

- $\mathcal{N}=(2,2)$ $\mathcal{W}_N$ minimal model
  - Coset description
    \[ \frac{\text{SU}(N+1)_k \otimes \text{SO}(2N)}{\text{SU}(N)_{k+1} \otimes \text{U}(1)^N(N+1)(k+N+1)} \]

- ’t Hooft limit
  - Large $N$ limit
    \[ k, N \to \infty \]
  - Fix the ratio
    \[ 0 < \lambda = \frac{N}{k+N} < 1 \]
Evidences

• Symmetry
  • Asymptotic symmetry is $N=(2,2)$ $W$ algebra [Creutzig-YH-Rønne ’11, Henneaux-Gómez-Park-Rey ’12, Hanaki-Peng ’12]
  • The Kazama-Suzuki model has the same symmetry [Ito ’91]

• Spectrum
  • One-loop partition function is obtained from gravity one-loop determinants [Creutzig-YH-Rønne ’11]
  • One loop partition function is computed at the ’t Hooft limit and the agreement is found with the gravity result [Candu-Gaberdiel ’12]

• Interactions
  • Three point functions with one higher spin current are studied [Creutzig-YH-Rønne, to appear]
Agreement of the spectrum

• Gravity partition function
  • Bosonic sector
    • Massive scalars [Giombi-Maloney-Yin ’08, David-Gaberdiel-Gopakumar ’09]
    • Bosonic higher spin [Gaberdiel-Gopakumar-Saha ’10]
  • Fermionic sector
    • Massive fermions, fermionic higher spin [Creutzig-YH-Rønne ’11]
• CFT partition function at the ’t Hooft limit
  • It is obtained by the sum of characters over all states and it was found to reproduce the gravity results
  \[ Z^{N,k}(q) = \sum_{\Lambda} |b^{N,k}_{\Lambda}(q)|^2 \]
  • Bosonic case [Gaberdiel-Gopakumar-Hartman-Raju ’11]
  • Supersymmetric case [Candu-Gaberdiel ’12]
Partition function at 1-loop level

• Total contribution
  • Higher spin sector + Matter sector
    \[ Z^{\text{Bulk}} = Z^{\text{HS}} Z^{\text{matter}} \]
  • Higher spin sector
    • Two series of bosons and fermions
      \[ Z^{\text{HS}} = \prod_{s=2}^{\infty} Z_B^{(s)} (Z_F^{(s-1)})^2 Z_B^{(s-1)} \]
      \[ Z_B^{(s)} = \prod_{n=s}^{\infty} |1 - q^n|^{-2}, \quad Z_F^{(s)} = \prod_{n=s}^{\infty} |1 + q^{n+\frac{1}{2}}|^2 \]
• Matter part sector
  • 4 massive complex scalars and 4 massive Dirac fermions
    \[ Z^{\text{matter}} = Z^{\frac{1}{2}}_{\text{susy}} Z^{\frac{1}{2}}_{\text{susy}}, \quad Z^{h}_{\text{susy}} = Z^{h}_{\text{scalar}} (Z^{h+\frac{1}{2}}_{\text{spinor}})^2 Z^{h+\frac{1}{2}}_{\text{scalar}} \]
    \[ Z^{h}_{\text{scalar}} = \prod_{l,l'=0}^{\infty} (1 - q^{h+l} q^{h+l'})^{-2} \]
    \[ Z^{h}_{\text{spinor}} = \prod_{l,l'=0}^{\infty} (1 + q^{h+l} q^{h-\frac{1}{2}+l'}) (1 + q^{h-\frac{1}{2}+l} q^{h+l'}) \]
Prokushkin-Vasiliev theory

• Master fields
  • $W_\mu$: gauge fields, $B$: matter fields, $S_\alpha$: auxiliary fields
  • Parameters: $z_\alpha, y_\alpha, \psi_1, \psi_2, k, \rho$
    \begin{align*}
    k^2 = \rho^2 &= 1, \quad \{k, \rho\} = \{k, y_\alpha\} = \{k, z_\alpha\} = 0, \quad [\rho, y_\alpha] = [\rho, z_\alpha] = 0
    \end{align*}
  
• Field equations
    \begin{align*}
    dW &= W \ast \wedge W, \quad dB = W \ast B - B \ast W, \ldots
    \end{align*}

• Gauge transformations
    \begin{align*}
    \delta W &= d\varepsilon - W \ast \varepsilon + \varepsilon \ast W, \quad \delta B = \varepsilon \ast B - B \ast \varepsilon, \ldots
    \end{align*}

• Vacuum solutions & perturbations around $B = \nu$
    \begin{align*}
    dA + A \ast \wedge A &= 0, \quad dC + A \ast C - C \ast \bar{A} = 0
    \end{align*}

• Chern-Simons gauge theory on a large $N$ limit of SL($N+1|N$)
• On AdS matter fields with mass depends on $\nu$
Boundary 3-pt functions

• Scalar field in the bulk ↔ Scalar operator at the boundary
  \( \phi_{\lambda}, \; m^2 = -1 + \lambda^2 \quad \mathcal{O}_B^h, \; h = \frac{1+\lambda}{2} \)

• Boundary 3-pt functions from the bulk theory [Chang-Yin ’11, Ammon-Kraus-Perlmutter ’11]

\[
\left\langle \mathcal{O}_B^h(z_1) \tilde{\mathcal{O}}_B^h(z_2) J^{(s)}(z_3) \right\rangle = N_s(h) \left( \frac{z_{12}}{z_{13} z_{23}} \right)^s \left\langle \mathcal{O}_B^h(z_1) \tilde{\mathcal{O}}_B^h(z_2) \right\rangle
\]

\[
N_s(h) = \frac{(-1)^{s-1} \Gamma(s)^2 \Gamma(s - 1 + 2h)}{2\pi \Gamma(2s - 1) \Gamma(2h)}
\]

• Comparison to the boundary CFT
  • Direct computation for \( s=3 \) (for \( s=4 \) [Ahn ’11])
  • Consistence to the large \( N \) limit of \( W_N \) for \( s=4,5,.. \)

• Analysis is extended to the supersymmetric case
  • 3-pt functions with fermionic operator [Creutzig-YH-Rønne, to apper]
4. CONCLUSION

Summary and other related works
Summary

• Higher spin gauge theories
  • Gravity theory with spin 2 gauge field can be extended to theory with spin $s > 2$ gauge fields.
  • Vasiliev develops higher spin gravity theories on AdS with non-trivial interactions, though only equations of motion are known.
  • Chern-Simons formulation is possible in 3 dimensions.

• Higher spin holography
  • 4d Vasiliev theory is dual to 3d O($N$) vector model
  • 3d Vasiliev theory is dual to 2d $W_N$ minimal model
  • Lots of evidences are already given
    • Symmetry, RG-flow, spectrum, correlation functions
Other related works (I)

- **Resolution of black hole singularity** [Ammon-Gutperle-Kraus-Perlmutter ’11,…]
  - The higher spin gravity has a large gauge symmetry. The notion of singularity, horizon,… is not gauge invariant
  - Generalized BTZ black hole can be changed into a warm hole solution by gauge transformation.

- **1/N corrections** [Castro, Lepage-Jutier, Maloney ’10,…]
  - Dual CFT is defined at the finite $N$, and the finite $N$ effects should be related to the quantum effects of Vasiliev theory
  - Missing states in the CFT correspond to geometries with conical deficits
Other related works (II)

  - One parameter family of correspondence can be considered in the ’t Hooft limit with large $N,k$
  - Dual to a generalization of Vasiliev theory
  - Related to ABJ theory with $U(N)_k \times U(M)_{-k}$ gauge symmetry. Take large $N$ but finite $M$. Dual to superstring on $\text{AdS}_4 \times \text{CP}^3$