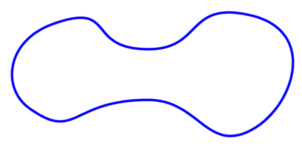
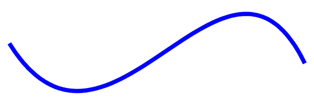
Correspondence between Cylinder Amplitude and Sphere Amplitude at the Hagedorn Temperature

Hagedorn Temparature





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maximum temperature for perturbative string

The oscillation mode of a single energetic string captures most of the energy.

degeneracy of oscillation mode $d_n \sim e^{2\pi\sqrt{2n}}$

density of state $\Omega(E) \sim e^{\beta_H E}$

partition function $Z(\beta) = \int_0^\infty dE\ \Omega(E)\ e^{-\beta E} = \mathrm{Tr}\ e^{-\beta H}$

Hagedorn temperature $\mathcal{T}_H$

$$\beta_H \equiv \frac{1}{\mathcal{T}_H} = 2\pi\sqrt{2\alpha'}$$

The partition function diverges above $\mathcal{T}_H$

$$Z(\beta) \to \infty \quad \text{for} \quad \beta < \beta_H$$

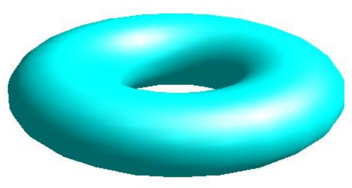
Hagedorn Transition of Closed Strings

- Matsubara Method

compactified Euclidean time with period $\beta = \frac{1}{\mathcal{T}}$

ideal closed string gas (type II)

→ 1-loop (torus world sheet)



- 1-loop Free Energy of Closed Strings (dual-rep.)

winding mode in the Euclidean time direction

\`mass' $M^2 = \frac{2}{\alpha'}\frac{\beta^2 - {\beta_H}^2}{{\beta_H}^2}$,

This mode becomes tachyonic for $\beta < \beta_H$. → winding tachyon

World sheet wraps around the Euclidean time once.

- Hagedorn Transition (Sathiapalan, Kogan, Atick-Witten)

A phase transition occurs

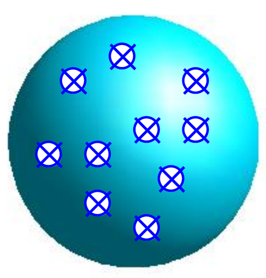
due to the condensation of winding tachyon.

$\mathcal{T} < \mathcal{T}_H$ → Sphere world sheet does not cotribute to free energy.

It cannot wrap around the compactified Euclidean time.

$\mathcal{T} > \mathcal{T}_H$ → Sphere world sheet contribute to free energy.

It is no longer simply connected.



insertion of winding tachyon vertex

creation of a tiny hole in the world sheet

which wraps around Euclidean time

Coupling of dilaton ϕ and winding tachyon w, w^*

$$\langle V_\phi V_w V_{w^*}\rangle \neq 0$$

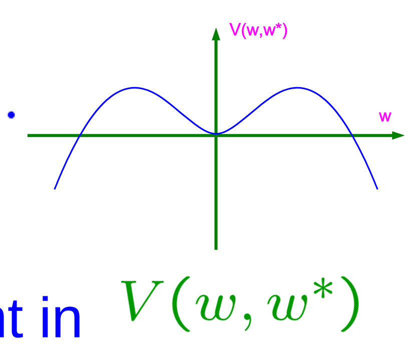
effective potential

$$V(\phi, w, w^*) = M^2 w^* w + g_s \phi w^* w + \cdots$$

$$M^2 = \frac{2}{\alpha'}\frac{\beta^2 - {\beta_H}^2}{{\beta_H}^2},$$

Lagrangian

$$L = \frac{1}{2}\phi\nabla^2\phi - M^2 w^* w - g_s \phi w^* w + \cdots$$



integration out of dilaton ϕ

→ quartic term $(w^* w)^2$ of negative coefficient in $V(w, w^*)$

→ first order phase transition (large latent heat)

cf) QCD confinement / deconfinement transition

- Problem

We have not known the stable minimum

of winding tachyon potential.

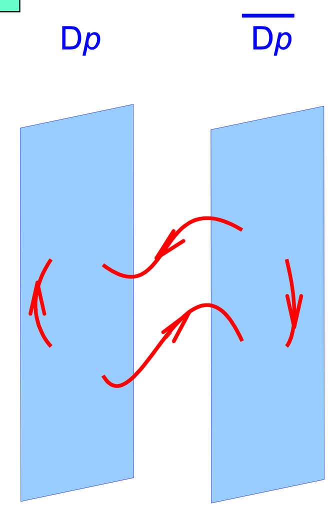
It is difficult to compute the potential of closed string tachyon.

closed string field theory has not been well-established.

We cannot identify which mode condensate in Lorentzian time.

Brane—antibrane Pair Creation Transition

(Hotta)



- Dp-$\overline{\mathrm{Dp}}$ pair

unstable at zero temperature

open string tachyon → tachyon potential

Sen's Conjecture potential hight = brane tention

- BSFT (Boudary String Field Theory)

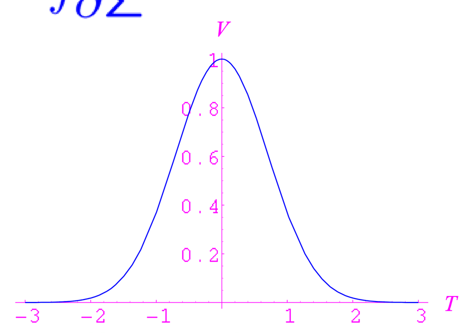
solution of classical master eq.

$$S_{eff} = Z$$

S_{eff} : 10-dim. effective action Z : 2-dim. partition function

2-dim. action : $S_2 = \frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma\ \partial_a X_\mu \partial^a X^\mu + \int_{\partial\Sigma} d\tau |T|^2 + \cdots$

- Tachyon potential of Dp-$\overline{\mathrm{Dp}}$ based on BSFT



tree level (disk world sheet)

$$V(T) = 2\tau_p \mathcal{V}_p \exp(-8|T|^2),$$

T : complex scalar field, τ_p: brane tension, $\mathcal{V}_p$: p-dim. volume

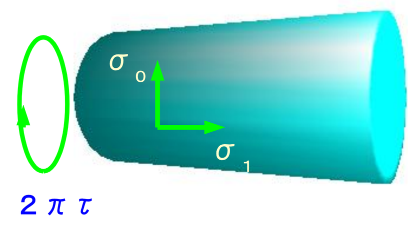
finite temperature case

Matsubara method → 1-loop (cylinder world sheet)

Conformal invariance is broken by the boundary terms.

→ ambiguity in the choice of the Weyl factors of two boundaries

- Cylinder Boundary Action (Andreev-Oft)



$$S_b = \int_0^{2\pi\tau} d\sigma_0 \int_0^\pi d\sigma_1 [|T|^2 \delta(\sigma_1) + |T|^2 \delta(\pi - \sigma_1)].$$

Both sides of cylinder world sheet are treated on an equal footing.

- 1-loop Free Energy of Open Strings

$$F_o(T,\beta) = -\frac{16\pi^4 \mathcal{V}_p}{{\beta_H}^{p+1}} \int_0^\infty \frac{d\tau}{\tau} \tau^{-\frac{p+1}{2}} e^{-4\pi|T|^2\tau} \times \left[\left(\frac{\vartheta_3(0|i\tau)}{\vartheta_1'(0|i\tau)}\right)^4 \left\{ \vartheta_3\left(0\left|\frac{i\beta^2}{8\pi^2\alpha'\tau}\right.\right) - 1 \right\} - \left(\frac{\vartheta_2(0|i\tau)}{\vartheta_1'(0|i\tau)}\right)^4 \left\{ \vartheta_4\left(0\left|\frac{i\beta^2}{8\pi^2\alpha'\tau}\right.\right) - 1 \right\} \right].$$

finite temperature effective potential

$$V(T,\beta) = V(T) + F_o(T,\beta)$$

- Brane-antibrane Pair Creation Transition
- N D9-$\overline{\mathrm{D9}}$ pairs

$|T|^2$ term of $V(T,E)$ $\left[-16N\tau_9\mathcal{V}_9 + \frac{8\pi N^2 \mathcal{V}_9}{{\beta_H}^{10}} \ln\left(\frac{\pi {\beta_H}^{10} E}{2N^2\mathcal{V}_9}\right)\right]|T|^2.$

critical temperature $\mathcal{T}_c \simeq {\beta_H}^{-1}\left[1 + \exp\left(-\frac{{\beta_H}^{10}\tau_9}{\pi N}\right)\right]^{-1}.$

Above $\mathcal{T}_c$, $T = 0$ becomes the potential minimum.

→ A phase transition occurs and D9-$\overline{\mathrm{D9}}$ pairs become stable.

$\mathcal{T}_c$ is a decreasing function of N

→ Multiple D9-$\overline{\mathrm{D9}}$ pairs are created simultaneously.

- N Dp-$\overline{\mathrm{Dp}}$ pairs with $p \leq 8$

No phase transition occurs.

Thermodynamic Balance on D9-$\overline{\text{D9}}$ Pairs

(Hotta)

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closed strings

We can reach $\mathcal{T}_H$ by supplying a finite energy.

open strings

We need an infinite energy to reach $\mathcal{T}_H$.

open strings $\longleftrightarrow$ closed strings

balancd condition: $\mathcal{T}_{open} = \mathcal{T}_{closed}$

Energy flows from closed strings to open strings.

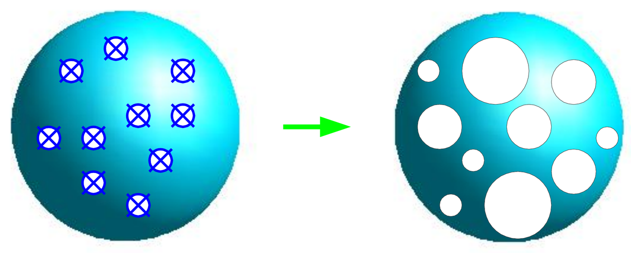
$\rightarrow$ Open strings dominate the total energy.

Relation between Two Phase Transitions

Let us identify boundary of a hole by insertion of winding tachyon $\longleftrightarrow$ boundary of an open string on a D9-$\overline{\text{D9}}$ pair

We propose following conjecture:

Conjecture: D9-$\overline{\text{D9}}$ Pairs are created by the Hagedorn transition of closed strings.



In other words, the stable minimum of the Hagedorn transition is the open string vacuum on D9-$\overline{\text{D9}}$ pairs.

phase transition from the closed string vacuum to the open string vacuum

We describe some circumstantial evidences for this conjecture.

Correspondence in the Closed String Vacuum Limit

- Cylinder Amplitude

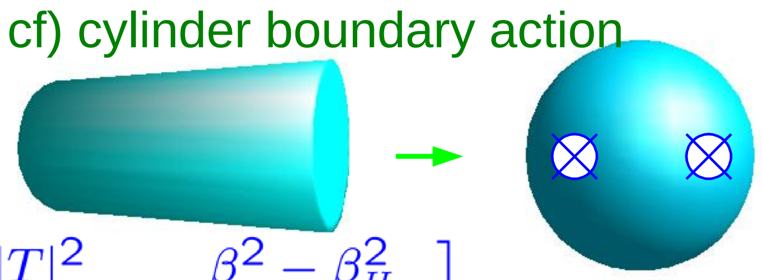
1-loop free energy of open string on D9-$\overline{\text{D9}}$ pair

$$F_o(T,\beta) = -\frac{16\pi^4\mathcal{V}_9}{\beta_H^{10}}\int_0^\infty \frac{d\tau}{\tau^6}\, e^{-4\pi|T|^2\tau} \times\left[\left(\frac{\vartheta_3(0|i\tau)}{\vartheta_1{}'(0|i\tau)}\right)^4\left\{\vartheta_3\left(0\left|\frac{i\beta^2}{\beta_H^2\tau}\right.\right)-1\right\}-\left(\frac{\vartheta_2(0|i\tau)}{\vartheta_1{}'(0|i\tau)}\right)^4\left\{\vartheta_4\left(0\left|\frac{i\beta^2}{\beta_H^2\tau}\right.\right)-1\right\}\right].$$

large $|T|^2$ $\rightarrow$ small τ $\rightarrow$ small hole

$e^{-4\pi|T|^2\tau}$ term

cf) cylinder boundary action



$\tau = \frac{1}{t}$

leading term for large t (small τ)

$$F(T,\beta) \simeq -\frac{4\mathcal{V}_9}{\beta_H{}^{10}}\int_0^\infty dt\ \exp\left[-\frac{4\pi|T|^2}{t}-\pi\,\frac{\beta^2-\beta_H^2}{\beta_H{}^2}\,t\right]$$

We can ignore the $|T|^2$ term for large t.

This is the propagator of zero momentum winding tachyon.

The world sheet wraps around Euclidean time once.

momentum conservation for Neumann direction

$\rightarrow$ zero momentum

$$M^2 = \frac{2}{\alpha'}\,\frac{\beta^2-\beta_H{}^2}{\beta_H{}^2},$$

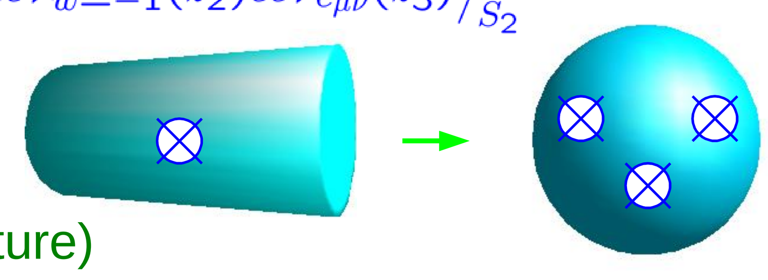
- Cylinder Amplitude with a Massless Closed String Vertex

$\mathcal{T} \to \mathcal{T}_H,\ |T| \to \infty$

$$\int_0^\infty \frac{dt}{2t}\left\langle cb\mathcal{V}_{e_{\mu\nu}}\right\rangle_{C_2} \to -\frac{8\pi(2\pi)^{10}g_s}{\alpha'}e_{00}$$
$$= \left\langle c\tilde{c}\mathcal{V}_{w=+1}(z_1)c\tilde{c}\mathcal{V}_{w=-1}(z_2)c\tilde{c}\mathcal{V}_{e_{\mu\nu}}(z_3)\right\rangle_{S_2}$$

closed string massless boson

$$\mathcal{V}_{e_{\mu\nu}} = \frac{2g_s}{\alpha'}\,e_{\mu\nu} : \partial X_L^\mu\bar{\partial}X_R^\nu :$$



winding tachyon vertex ((-1,-1) picture)

$$\mathcal{V}^{-1,-1}_{w=+1} = g_s e^{-\phi-\tilde{\phi}} : e^{i\sqrt{\frac{2}{\alpha'}}X_L^0(z)-i\sqrt{\frac{2}{\alpha'}}X_R^0(\bar{z})}$$
$$\mathcal{V}^{-1,-1}_{w=-1} = g_s e^{-\phi-\tilde{\phi}} : e^{-i\sqrt{\frac{2}{\alpha'}}X_L^0(z)+i\sqrt{\frac{2}{\alpha'}}X_R^0(\bar{z})}$$

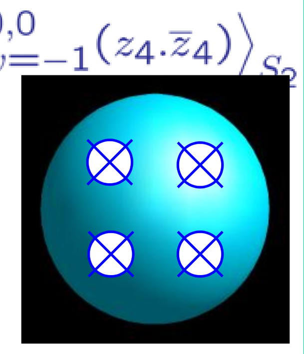
Sphere with 4 Winding Tachyon Insertion

We can calculate sphere amplitude

with 4 winding tachyon vertex insertion by using CFT.

The winding tachyon becomes massless at $\mathcal{T}_H$.

$$S^4_{S_2} \doteq g_s{}^{-2}\int dz_4\left\langle \tilde{c}c\mathcal{V}^{-1,-1}_{w=+1}(z_1.\bar{z}_1)\tilde{c}c\mathcal{V}^{-1,-1}_{w=-1}(z_2.\bar{z}_2)\tilde{c}c\mathcal{V}^{0,0}_{w=+1}(z_3.\bar{z}_3)\mathcal{V}^{0,0}_{w=-1}(z_4.\bar{z}_4)\right\rangle_{S_2}$$
$$= g_s^4 C_{S_2}\int d^2z_4\,\frac{|1-z_4|^2}{|z_4|^2}$$
$$= \frac{1}{2\pi}\,g_s^4 C_{S_2}\,\frac{\Gamma(0)\Gamma(2)\Gamma(-1)}{\Gamma(2)\Gamma(-1)\Gamma(1)}$$
$$= \frac{1}{2\pi}\,g_s^4 C_{S_2}\Gamma(0)$$



$z_1 = 0,\ z_2 = 1,\ z_3 = \infty$

$C_{S_2} \propto g_s{}^{-2}$

Virasoro-Shapiro amplitude

This amplitude diverges.

winding tachyon vertex ((0,0) picture)

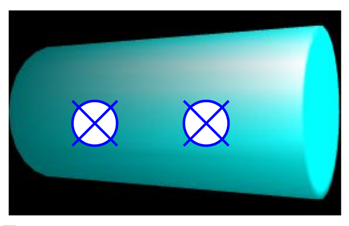
$$\mathcal{V}^{0,0}_{w=+1} = g_s\psi_L^0(z)\psi_R^0(\bar{z}) : e^{i\sqrt{\frac{2}{\alpha'}}X_L^0(z)-i\sqrt{\frac{2}{\alpha'}}X_R^0(\bar{z})}$$
$$\mathcal{V}^{0,0}_{w=-1} = g_s\psi_L^0(z)\psi_R^0(\bar{z}) : e^{-i\sqrt{\frac{2}{\alpha'}}X_L^0(z)+i\sqrt{\frac{2}{\alpha'}}X_R^0(\bar{z})}$$

Cylinder with 2 winding tachyon insertion

- Boundary State for D9-$\overline{\text{D9}}$ Pair

cf) Asakawa-Sugimoto-Terashima

$$\left|D9-\overline{D9}\right\rangle = \exp(-S_b)\left(|B9,+\rangle_{NSNS}-|B9,-\rangle_{NSNS}\right)$$



$$S_b = \oint d\sigma\ |T|^2$$

$$|B9_{mat},\eta\rangle_{NSNS} = \exp\left[-\sum_{n=1}^\infty \frac{1}{n}\,\alpha_{-n}\cdot\tilde{\alpha}_{-n}+i\eta\sum_{u>0}\psi_{-u}\cdot\tilde{\psi}_{-u}\right]|B9_{mat},\eta\rangle^{(0)}_{NSNS}$$

thermal GSO projection

$$P = \frac{1}{2}\left\{1+(-1)^{F+w}\right\}$$

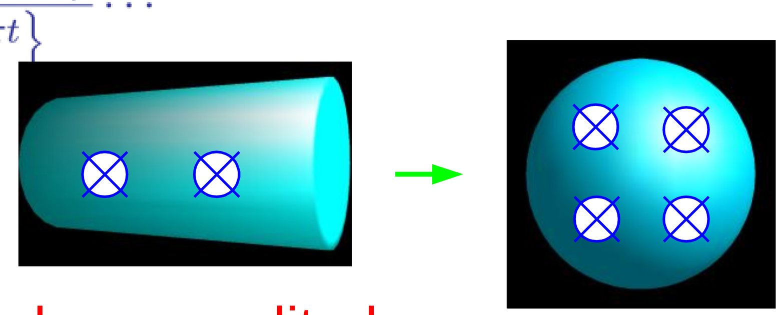
- Cylinder Amplitude

$$S^2_{C_2} = \left\langle D9-\overline{D9}\right|\Delta(t_1)\mathcal{V}^{0,0}_{w=+1}\Delta(t_2,\phi)\mathcal{V}^{0,0}_{w=-1}\Delta(t_3)\left|D9-\overline{D9}\right\rangle$$

$$\Delta(t,\phi) = \frac{1}{4\pi}\int_0^\infty dt\int_0^{2\pi}d\phi\ e^{-t(L_0+\tilde{L}_0-2)}e^{i\phi(L_0-\tilde{L}_0)}$$

$$S^2_{C_2} = -\frac{16\pi^4 v_9}{\beta_H{}^{10}}\,g_s^2\int_0^\infty dt\ \exp\left(-\pi\,\frac{|\mathbf{T}|^2}{t}\right)$$
$$\times\left[\left\{\frac{\vartheta_3(0|it)}{\vartheta_1'(0|it)}\right\}^{\frac{7}{2}}\prod_{u=1}^\infty\left(-e^{-2\pi t_2 u}-e^{-2\pi(t_1+t_3)u}\right)\left\{\vartheta_3\left(0\left|\frac{i\beta^2 t}{\beta_H{}^2}\right.\right)-1\right\}\right.$$
$$\left.-\left\{\frac{\vartheta_4(0|it)}{\vartheta_1'(0|it)}\right\}^{\frac{7}{2}}\prod_{u=1}^\infty\left(-e^{-2\pi t_2 u}+e^{-2\pi(t_1+t_3)u}\right)\left\{\vartheta_4\left(0\left|\frac{i\beta^2 t}{\beta_H{}^2}\right.\right)-1\right\}\right]$$
$$\times\frac{\left\{1-e^{-pi(t_2-i\phi)}\right\}\left\{1-e^{-pi(t_2+i\phi)}\right\}}{\left\{1-e^{-2\pi(t_2+t_3)}\right\}\left\{1-e^{-2\pi t}\right\}}\cdots$$

$$\xrightarrow[\mathcal{T}\to\mathcal{T}_H,\ |T|\to\infty]{} \propto g_s^2\int d^2z_4\,\frac{|1-z_4|^2}{|z_4|^2}$$



This Cylinder amplitude corresponds to sphere amplitude

with two more winding tachyon insertion in this limit.

Conclusion

We have proposed the conjecture that *D9-$\overline{\text{D9}}$ pairs are created by the Hagedorn transition of closed strings.*

In other words, the stable minimum of the Hagedorn transition is the open string vacuum.

Then we have shown some circumstantial evidences.

Some types of amplitude of open string in closed string vacuum limit agree with closed string ones.

We have shown that the potential energy at the open string vacuum decreases limitlessly as $\beta \to \beta_H$.