On open-closed extension of boundary string field theory

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寺口俊介氏 (阪大 IFReC) **との共同研究**
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1. Introduction

Motivation

- open string field theory: tachyon **を含む** D-brane **系の解析に有用**
- tachyon condensation $\rightarrow$ closed string vacuum
- closed string **も含めた** formulation

(Covariant) open-closed string field theories

- oriented open-closed SFT [Zwiebach]
  (midpoint interaction)
- unorieneted open-closed SFT [Asakawa-Kugo-Takahashi]
  (light-cone type interaction)

closed SFT **に** boundary state $\langle B|$ **を** source **として入れる方法もある**:

$$\langle B| e^{-\int d\sigma V(\sigma)} (c_0 - \tilde{c}_0)|\Psi\rangle$$

$V(\sigma)$: open string excitations (on-shell)

- Hashimoto-Hata ('97)
  (Gauge inv. $\rightarrow$ DBI action **の存在**)
- Asakawa-Kobayashi-Matsuura ('03)
- Baba-Ishibashi-Murakami ('06$\sim$)
  (D-brane soliton state)

研究の目的

off-shell open string excitation **まで含めた** off-shell boundary state **を考え、** open-closed SFT **を構成する**

Boundary string field theory (BSFT)

- disk worldsheet **上の全** boundary **相互作用の空間上で構成された** open string field theory [Witten(1992)]
- boundary operator → string field: $O \equiv \int_{-\pi}^{\pi} \frac{d\sigma}{2\pi} \mathcal{O}(\sigma)$
  $\mathcal{O} = c(\sigma)(T(X) + A_\mu(X)\partial_\sigma X^\mu + \cdots)$
- off-shell boundary state $\langle B; \lambda| \equiv \langle B| e^{2i\kappa[b_0^-, O]}$ **が重要な役割を果たす** (κ: open string coupling constant, $b_0^- \equiv (b_0 - \tilde{b}_0)/2$)

BSFT action: [Teraguchi ('06)]

$$S = -\frac{1}{4\kappa^2}\langle B| e^{2i\kappa[b_0^-, O]} c_0^- Q_B c_0^- |0\rangle + \frac{i}{2\kappa}\langle B| \,\mathrm{Sym}\big[e^{2i\kappa[b_0^-, O]}; [Q_B, O]\big]\, c_0^- |0\rangle$$

open string sector **が** BSFT **であるような** open-closed SFT **を考える**

2. Batalin-Vilkovisky formalism and string field theory

guiding principle for SFTs $\rightarrow$ gauge invariance

string field:

$$|\Psi\rangle = \sum_I |\Psi_I\rangle \psi^I$$

fermionic 2-form ($d\omega = 0$):

$$\omega = -d\psi^I \wedge \omega_{IJ} d\psi^J$$

Anti-bracket:

$$\{A, B\} = \frac{\partial_r A}{\partial \psi^I} \omega^{IJ} \frac{\partial_l B}{\partial \psi^J} \qquad (\partial_r (\partial_l)\text{: } \textbf{右 (左) 微分})$$

Classical BV master equation:

$$\frac{1}{2}\{S, S\} + \cancel{\hbar \Delta S} = 0$$

$$0 = \frac{1}{2} \frac{\partial_r \{S, S\}}{\partial \psi^K} \epsilon^K = \frac{\partial_r S}{\partial \psi^I} \underbrace{\left(\omega^{IJ} \frac{\partial_r}{\partial \psi^K} \frac{\partial_l S}{\partial \psi^J} + \frac{1}{2} (-1)^{JK} \frac{\partial_r \omega^{IJ}}{\partial \psi^K} \frac{\partial_l S}{\partial \psi^J} \right) \epsilon^K}_{\delta \Psi^I}$$

Free closed SFT

closed string field: (ghost number=2)

$$|\Psi\rangle = \sum_I |\Psi_I\rangle\psi^I \qquad \left(b_0^-|\Psi\rangle = (L_0 - \tilde{L}_0)|\Psi\rangle = 0\right)$$

2-form:

$$\omega_c = \langle d\Psi|c_0^-|d\Psi\rangle \qquad (c_0^- \equiv c_0 - \tilde{c}_0)$$

Action:

$$S_c^{\text{free}} = \frac{1}{2}\langle\Psi|c_0^- Q_B|\Psi\rangle, \qquad (Q_B\text{: BRST operator})$$

$\Rightarrow \{S_c^{\text{free}}, S_c^{\text{free}}\} = 0$

$$\Rightarrow \delta_{\Lambda_c}|\Psi\rangle = |\Psi_I\rangle\delta\psi^I = Q_B|\Lambda_c\rangle \qquad (|\Lambda_c\rangle \equiv -|\Psi_K\rangle\epsilon^K)$$

Classical open-closed SFT

Open and closed string fields:

$$O = \sum_i O_i \lambda^i, \qquad |\Psi\rangle = \sum_I |\Psi_I\rangle \psi^I$$

fermionic 2-form: $\omega = \omega_c + \omega_o$

Anti-bracket:

$$\begin{aligned}\{A, B\} &= \{A, B\}_c + \{A, B\}_o \\ &= \frac{\partial_r A}{\partial \psi^I} \omega_c^{IJ} \frac{\partial_l B}{\partial \psi^J} + \frac{\partial_r A}{\partial \lambda^i} \omega_o^{ij} \frac{\partial_l B}{\partial \lambda^j}\end{aligned}$$

classical open-closed SFT action:

$$S_{\mathsf{oc}}(\psi, \lambda) = \underbrace{S_c(\psi)}_{\text{sphere}} + \underbrace{S_D(\psi, \lambda)}_{\text{disk}}$$

classical open-closed BV master equation: [Kajiura-Stasheff]

$$\begin{aligned}0 &= \{S_{\mathsf{oc}}, S_{\mathsf{oc}}\} \\ &= \underbrace{\{S_c, S_c\}_c}_{0} + \underbrace{2\{S_c, S_D\}_c + \{S_D, S_D\}_o}_{0} + \cancel{\{S_D, S_D\}_c}\end{aligned}$$

Gauge transformation:

$$\delta_c \psi^I = \left[\omega_c^{IJ} \frac{\partial_r}{\partial \psi^K} \frac{\partial_l S_c}{\partial \psi^J} + \frac{1}{2} (-1)^{JK} \frac{\partial_r \omega_c^{IJ}}{\partial \psi^K} \frac{\partial_l S_c}{\partial \psi^J} \right] \epsilon^K$$

$$\delta_c \lambda^i = \omega_o^{ij} \left(\frac{\partial_r}{\partial \psi^K} \frac{\partial_l S_D}{\partial \lambda^j} \right) \epsilon^K$$

$$\delta_o \psi^I = 0$$

$$\delta_o \lambda^i = \left[\omega_o^{ij} \frac{\partial_r}{\partial \lambda^k} \frac{\partial_l S_D}{\partial \lambda^j} + \frac{1}{2} (-1)^{jk} \frac{\partial_r \omega_o^{ij}}{\partial \lambda^k} \frac{\partial_l S_D}{\partial \lambda^j} \right] \epsilon^k$$

ゲージ変換のパラメーターには以下のような constraint **が付く**:

$$\left[\omega_c^{IJ} \frac{\partial_r}{\partial \psi^K} \frac{\partial_l S_D}{\partial \psi^J} + \frac{1}{2} (-1)^{JK} \frac{\partial_r \omega_c^{IJ}}{\partial \psi^K} \frac{\partial_l S_D}{\partial \psi^J} \right] \epsilon^K + \omega_c^{IJ} \left(\frac{\partial_r}{\partial \lambda^k} \frac{\partial_l S_D}{\partial \psi^J} \right) \epsilon^k = 0$$

3. Coupling BSFT to closed SFT

string field:

$$|\Psi\rangle = \sum_I |\Psi_I\rangle \psi^I, \qquad O = \int \frac{d\sigma}{2\pi} \mathcal{O}(\sigma) = \sum_i O_i \lambda^i$$

fermionic 2-form $\omega = \omega_c + \omega_o$:

$$\omega_c = \langle d\Psi | c_0^- | d\Psi \rangle, \quad \omega_o = \langle B | \, \mathrm{Sym}\big[e^{2i\kappa [b_0^-, O]}; dO, dO \big] |0\rangle$$

Ansatz:

$$S_c^{\text{free}} + S_D^{\text{int}} + S_D^{\text{BSFT}}$$

where

$$S_D^{\text{int}} = \frac{1}{2} \langle B | e^{2i\kappa [b_0^-, O]} c_0^- | \Psi \rangle$$

$$S_D^{\text{BSFT}} = -\frac{1}{4\kappa^2} \langle B | e^{2i\kappa [b_0^-, O]} c_0^- Q_B c_0^- |0\rangle + \frac{i}{2\kappa} \langle B | \, \mathrm{Sym}\big[e^{2i\kappa [b_0^-, O]}; [Q_B, O] \big] c_0^- |0\rangle \quad \text{(BSFT action)}$$

$$S_c^{\text{free}} = \frac{1}{2} \langle \Psi | c_0^- Q_B | \Psi \rangle$$

この ansatz **がどの程度** BV master equation **を満たすか確かめてみる**:

$$
\begin{aligned}
&\{S_c^{\mathsf{free}} + S_D^{\mathsf{int}} + S_D^{\mathsf{BSFT}}, S_c^{\mathsf{free}} + S_D^{\mathsf{int}} + S_D^{\mathsf{BSFT}}\} \\
&= \{S_c^{\mathsf{free}}, S_c^{\mathsf{free}}\}_c && (b=0) \\
&\quad + \{S_D^{\mathsf{BSFT}}, S_D^{\mathsf{BSFT}}\}_o && (b=1, n=0) \\
&\quad + 2\{S_D^{\mathsf{int}}, S_c^{\mathsf{free}}\}_c + 2\{S_D^{\mathsf{int}}, S_D^{\mathsf{BSFT}}\}_o && (b=1, n=1) \\
&\quad + \{S_D^{\mathsf{int}}, S_D^{\mathsf{int}}\}_o && (b=1, n=2) \\
&\quad + \xcancel{\{S_D^{\mathsf{int}}, S_D^{\mathsf{int}}\}_c} && (b=2, n=0)
\end{aligned}
$$

まず明らかに

$$\{S_c^{\mathsf{free}}, S_c^{\mathsf{free}}\}_c = \{S_D^{\mathsf{BSFT}}, S_D^{\mathsf{BSFT}}\}_o = 0$$

$$
\begin{aligned}
&2\{S_D^{\mathsf{int}}, S_c^{\mathsf{free}}\}_c + 2\{S_D^{\mathsf{int}}, S_D^{\mathsf{BSFT}}\}_o \\
&= \langle B|e^{2i\kappa[b_0^-,O]}c_0^-|\Psi_I\rangle\omega_c^{IJ}(-1)^J\langle\Psi_J|c_0^-Q_B|\Psi\rangle \\
&\quad + 2i\kappa\langle B|\,\mathrm{Sym}\big[e^{2i\kappa[b_0^-,O]};[O_i,b_0^-]\big]c_0^-|\Psi\rangle\omega_o^{ij}\omega_{jk}^o V^k \\
&= -\langle B|e^{2i\kappa[b_0^-,O]}c_0^-b_0^-c_0^-Q_B|\Psi\rangle + 2i\kappa\langle B|\,\mathrm{Sym}\big[e^{2i\kappa[b_0^-,O]};[O_i,b_0^-]\big]c_0^-|\Psi\rangle V^i \\
&= -\langle B|e^{2i\kappa[b_0^-,O]}c_0^-Q_B|\Psi\rangle + 2i\kappa\langle B|\,\mathrm{Sym}\big[e^{2i\kappa[b_0^-,O]};[[O,Q_B],b_0^-]\big]c_0^-|\Psi\rangle \\
&= -\langle B|e^{2i\kappa[b_0^-,O]}c_0^-Q_Bb_0^-c_0^-|\Psi\rangle - 2i\kappa\langle B|\,\mathrm{Sym}\big[e^{2i\kappa[b_0^-,O]};[[b_0^-,O],Q_B]\big]c_0^-|\Psi\rangle \\
&= \langle B|e^{2i\kappa[b_0^-,O]}Q_Bc_0^-|\Psi\rangle - \langle B|e^{2i\kappa[b_0^-,O]}Q_Bc_0^-|\Psi\rangle \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
\{S_D^{\mathsf{int}}, S_D^{\mathsf{int}}\}_o = \kappa^2\langle B|\,\mathrm{Sym}\big[e^{2i\kappa[b_0^-,O]};O_i\big]|\Psi\rangle \\
\times\,\omega_o^{ij}(-1)^{j+1}\langle B|\,\mathrm{Sym}\big[e^{2i\kappa[b_0^-,O]};O_j\big]|\Psi\rangle
\end{aligned}
$$

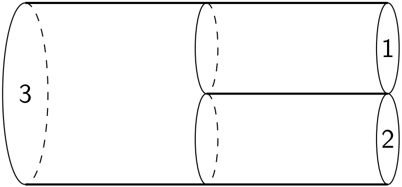
$\Rightarrow S_c^{\mathsf{int}} = \frac{\kappa^2}{3}\langle\Psi|c_0^-|\Psi*\Psi\rangle$ **を導入して消す。新たに現れる項は**

$$
2\{S_D^{\mathsf{int}}, S_c^{\mathsf{int}}\}_c = -\kappa^2\langle B|e^{2i\kappa[b_0^-,O]}c_0^-|\Psi*\Psi\rangle
$$

$$\langle B|e^{2i\kappa[b_0^-,O]}c_0^-|\Psi*\Psi\rangle$$
$$=(-1)^{j+1}\langle B|\,\mathrm{Sym}\big[e^{2i\kappa[b_0^-,O]};O_i\big]|\Psi\rangle\omega_o^{ij}\langle B|\,\mathrm{Sym}\big[e^{2i\kappa[b_0^-,O]};O_j\big]|\Psi\rangle$$

となるような star product (3-closed-string vertex) **を探す**

- factorization property in CFT?
- joining-splitting type vertex **で評価してみる**
- string length parameter α **を導入** ($\sum_i \alpha_i = 0$)



(3-closed string interaction for $|\alpha_1|+|\alpha_2|=|\alpha_3|$)

$$
\begin{aligned}
&\langle B| e^{2i\kappa[b_0^-, O]} c_0^- |\Psi * \Psi\rangle \\
&\quad = (-1)^{j+1} \langle B| \mathrm{Sym}\left[e^{2i\kappa[b_0^-, O]}; O_i\right] |\Psi\rangle \omega_o^{ij} \langle B| \mathrm{Sym}\left[e^{2i\kappa[b_0^-, O]}; O_j\right] |\Psi\rangle
\end{aligned}
$$

となるような star product (3-closed-string vertex) **を探す**

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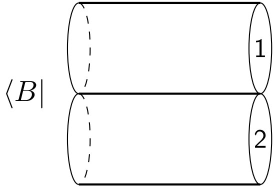
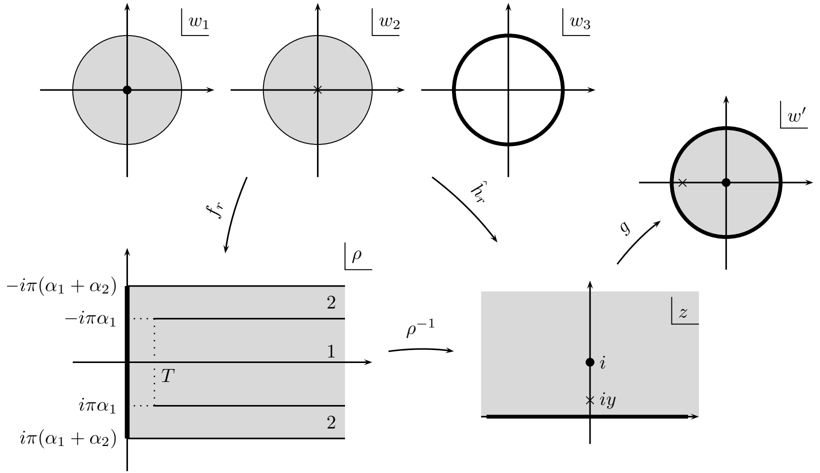


(3-closed string interaction for $|\alpha_1| + |\alpha_2| = |\alpha_3|$)

$\langle B|e^{2i\kappa[b_0^-,O]}c_0^-|\Psi*\Psi\rangle$ の計算 [cf. Baba-Ishibashi-Murakami '07]



$$g(\hat{h}_1(w_1)) \to w_1, \qquad g(\hat{h}_2(w_2)) \to -1 \qquad (T\to 0)$$

boundary deformation **が** on-shell **の場合**

$$\langle B|\,\mathrm{Sym}\big[e^{2i\kappa[b_0^-,O]};O_i\big]|\Psi(\alpha_1)\rangle\,F_2^i(\Psi(\alpha_2))$$

$1 \leftrightarrow 2$:

$$\langle B|\,\mathrm{Sym}\Big[e^{2i\kappa[b_0^-,O]};O_j\Big]|\Psi(\alpha_2)\rangle\,F_1^j(\Psi(\alpha_1))$$

よって factorized form **を得る**: [cf. Gaiotto-Rastelli-Sen-Zwiebach ('02)]

$$\langle B;\lambda|c_0^-|\Psi(\alpha_1)*\Psi(\alpha_2)\rangle = (-1)^{j+1}\langle B|\,\mathrm{Sym}\Big[e^{2i\kappa[b_0^-,O]};O_i\Big]|\Psi(\alpha_1)\rangle \times \omega_o^{ij}\langle B|\,\mathrm{Sym}\Big[e^{2i\kappa[b_0^-,O]};O_j\Big]|\Psi(\alpha_2)\rangle \qquad (\alpha_1\alpha_2>0)$$

- off-shell deformation **の場合も、上の式の右辺が** vertex **の性質を満たすことを確かめることができる**
- **したがってこの式は** off-shell deformation **でも成立すると仮定する**
- $\alpha_1\alpha_2<0$ **のときは** factorization **は起こらないので、この式は成り立たない**
- **以下では** BV master equation **が成り立つ領域** $(\alpha_1\alpha_2>0)$ **でゲージ変換を考える**

4. Gauge transformation

closed string sector: HIKKO closed SFT [Hata-Itoh-Kugo-Kunitomo-Ogawa]

$$S_c = \frac{1}{2}\langle\Psi|c_0^- Q_B|\Psi\rangle + \frac{\kappa^2}{3}\langle\Psi|c_0^-|\Psi * \Psi\rangle \qquad \left(|\Psi\rangle \equiv \int_{-\infty}^{\infty}\frac{d\alpha}{2\pi}|\Psi(\alpha)\rangle\right)$$

Action:

$$S = S_c + S_D^{\text{int}} + S_D^{\text{BSFT}}$$

BV master equation:

$$\begin{aligned}
\{S,S\} &= \{S_c + S_D^{\text{int}} + S_D^{\text{BSFT}}, S_c + S_D^{\text{int}} + S_D^{\text{BSFT}}\} \\
&= \underbrace{\{S_c, S_c\}_c}_{0} + \underbrace{\{S_D^{\text{BSFT}}, S_D^{\text{BSFT}}\}_o}_{0} \\
&\quad + \underbrace{2\{S_D^{\text{int}}, S_c^{\text{free}}\}_c + 2\{S_D^{\text{int}}, S_D^{\text{BSFT}}\}_o}_{0} \\
&\quad + \underbrace{\{S_D^{\text{int}}, S_D^{\text{int}}\}_o + 2\{S_D^{\text{int}}, S_c^{\text{int}}\}_c}_{0 \quad (\alpha_1\alpha_2>0)} \\
&\quad + (\text{terms with } b > 1)
\end{aligned}$$

ゲージ変換を読み取ると

$$\delta_{\Lambda_o} O = [Q_B, \Lambda_o]$$
$$\delta_{\Lambda_c} |\Psi\rangle = Q_B|\Lambda_c\rangle + 2\kappa^2 |\Psi * \Lambda_c\rangle$$
$$\delta_{\Lambda_c} O = i\kappa(-1)^{j+1} O_i\, \omega_o^{ij} \langle B|\, \mathrm{Sym}\left[e^{2i\kappa[b_0^-,O]}; O_j\right]|\Lambda_c\rangle$$

constraint: $[b_0^-, \Lambda_o] = 0$

open string gauge transformation

$$\delta_{\Lambda_o}\langle B|e^{2i\kappa[b_0^-,O]} = 2i\kappa\langle B|\,\mathrm{Sym}\left[e^{2i\kappa[b_0^-,O]}; [b_0^-, [Q_B, \Lambda_o]]\right] = 0$$

$$\Rightarrow \delta_{\Lambda_o} S_c = 0, \qquad \delta_{\Lambda_o} S_D^{\mathsf{int}} = 0, \qquad \delta_{\Lambda_o} S_D^{\mathsf{BSFT}} = 0$$

以下のように string length **を制限する**:

$$\langle B| \to \langle B^-| \equiv \int_0^\infty \frac{d\alpha}{2\pi}\langle B(\alpha)|\,, \qquad |\Lambda_c\rangle = |\Lambda_c^-\rangle + \cancel{|\Lambda_c^+\rangle}$$

closed string gauge transformation (by $|\Lambda_c^-\rangle$)

$$
\begin{aligned}
\delta_{\Lambda_c^-} S_c &= 0 \\
\delta_{\Lambda_c^-} S_D^{\mathsf{int}} &= \frac{1}{2}\big(\delta_{\Lambda_c}\langle B^-|e^{2i\kappa[b_0^-,O]}\big)c_0^-|\Psi\rangle + \frac{1}{2}\langle B^-;\lambda|c_0^-\left(\delta_{\Lambda_c}|\Psi\rangle\right) \\
&= i\kappa\langle B^-|\,\mathrm{Sym}\big[e^{2i\kappa[b_0^-,O]};\delta_{\Lambda_c}O\big]|\Psi^-\rangle \\
&\quad + \frac{1}{2}\langle B^-;\lambda|c_0^- Q_B|\Lambda_c^-\rangle + \kappa^2\langle B^-;\lambda|c_0^-|\Psi^- * \Lambda_c^-\rangle \\
&= \cancel{-\kappa^2\langle B^-;\lambda|c_0^-|\Psi^- * \Lambda_c^-\rangle} \\
&\quad + \frac{1}{2}\langle B^-;\lambda|c_0^- Q_B|\Lambda_c^-\rangle + \cancel{\kappa^2\langle B^-;\lambda|c_0^-|\Psi^- * \Lambda_c^-\rangle} \\
\delta_{\Lambda_c^-} S_D^{\mathsf{BSFT}} &= -\langle B|\,\mathrm{Sym}\big[e^{2i\kappa[b_0^-,O]};[Q_B,O],\delta_{\Lambda_c}O\big]|0\rangle \\
&= -\frac{1}{2}\langle B^-;\lambda|c_0^- Q_B|\Lambda_c^-\rangle
\end{aligned}
$$

$$\therefore\ \delta_{\Lambda_c^-}(S_c + S_D^{\mathsf{int}} + S_D^{\mathsf{BSFT}}) = 0$$

Action

$S = S_c + S_D^{\mathsf{int}} + S_D^{\mathsf{BSFT}}$

(S_c: HIKKO closed SFT, $S_D^{\mathsf{int}} = \frac{1}{2}\langle B^- | e^{2i\kappa[b_0^-, O]} c_0^- | \Psi\rangle$)

Gauge transformation

$$\delta_{\Lambda_o} O = [Q_B, \Lambda_o] \qquad ([b_0^-, \Lambda_o] = 0)$$

$$\delta_{\Lambda_c^-} |\Psi\rangle = Q_B |\Lambda_c^-\rangle + 2\kappa^2 |\Psi * \Lambda_c^-\rangle$$

$$\delta_{\Lambda_c^-} O = i\kappa(-1)^{j+1} O_i\, \omega_o^{ij} \langle B^- | \, \mathrm{Sym}\big[e^{2i\kappa[b_0^-, O]}; O_j\big] |\Lambda_c^-\rangle$$

$$\big(\delta_{\Lambda_c} \langle B^-; \lambda| = 2\kappa^2 \langle B^-; \lambda| * \langle \Lambda_c^-| \qquad \text{[cf. Hashimoto-Hata]}\big)$$

5. Conclusion and discussion

- open string sector が BSFT であるような open-closed SFT を考えた
- BV master equation から closed string part は HIKKO に決まる
- natural off-shell extension of Hashimoto-Hata (DBI→BSFT)
- ゲージパラメーターに対する 2 つの constraint:
  $[b_0^-, \Lambda_o] = 0$: classical truncation of full master equation
  $|\Lambda_c^+\rangle = 0$: master equation が成立しないパラメーター領域
- D-brane soliton state ($\sim \exp[-S_D^{\text{int}} + \cdots]|0\rangle\rangle$) [Baba et al.] との関係
  $|0\rangle\rangle$: second quantized vacuum; $\Psi(\alpha)|0\rangle\rangle = 0 \quad (\alpha > 0)$
- BSFT は D-brane soliton state の fluctuation を記述している??