Holographic Wilsonian Renormalisation Group

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based on arXiv [hep-th] 1109.3366 [JHEP 1111 (2011) 155] in collaboration with D. Elander, G. Mandal (TIFR)

IR Effective actions of strongly coupled systems are as important as microscopic actions.
Wilsonian RG : useful to find effective action, difficult to carry out in the field theory
⟶ Holographic realisation, geometrisation of coarse graning of Wilsonian RG

concrete motivation : toy model of singular Fermi liquids in the context of AdS/condensed matter system

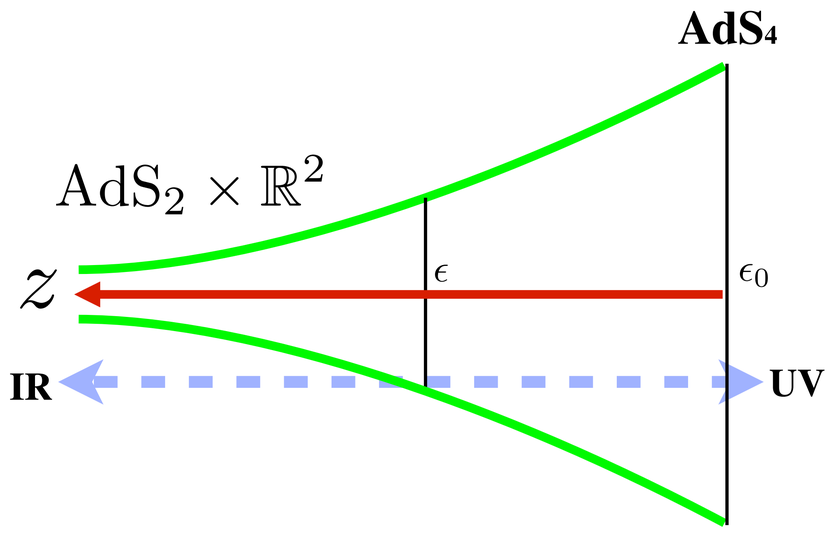
bulk
probe fermion on an extremal RN AdS black hole
near horizon limit : $\mathrm{AdS}_2 \times \mathbb{R}^2$

$z^{-1} - z_H^{-1} = \epsilon\zeta^{-1}, \quad t = \epsilon^{-1}\tau, \quad \epsilon \to 0 \ (\zeta, \tau \text{ finite})$

boundary
zero temperature + finite chemical potential
low energy limit : $\omega \ll \mu$

This NH (IR) geometry is universal for (near-)extremal BH

expectation : the IR physics is governed by
some strongly-coupled dual "CFT" to this AdS₂
(IR quantum criticality)



write down an IR eff. action in terms of the IR CFT with something more

proposal : semi-holographic action [Faulkner-Polchinski 2010]

$$\int [dM] \int d\chi_- d\overline{\chi}_+ \ \exp\left(S[M] + \int_k \overline{\chi}_+ O_+ + \overline{O}_- \chi_- + \int_k \overline{\chi}_+ F_0^{-1}(k - k_F)\chi_- \right)$$

propagator of the new field = IR behavior of the full retarded Green's function

what we did : derive or verify this conjecture explicitly in a holographic manner

we need some logic which can *holographically* connect IR and UV ---> consider holographic Wilsonian RG

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idea

PI = path integral (bulk side)

$$\begin{array}{ccccc}
\text{PI(whole aAdS)} & \overset{\text{GKP-W}}{\longleftarrow} & \text{GKP-W} & \overset{\text{Heemskerk-Polchinski}}{\longrightarrow} & \int[d\widetilde{\phi}]\,\text{PI(left : IR)}\,\text{PI(right : UV)} \\
\| & & & & \uparrow \text{coarse graning} \downarrow \\
\left\langle \exp\int \phi_0 O_s \right\rangle^{\text{std}}_{\Lambda_0(\epsilon_0)} & = & & & \int[d\widetilde{\phi}] \left\langle \exp\int \widetilde{\phi} O_s \right\rangle^{\text{std}}_{\Lambda(\epsilon)} \ \text{PI(right : UV)}
\end{array}$$

relating two equivalent theories

completeness relations

convenient choices of basis

boson

$$1 = \int |\phi\rangle d\phi \langle\phi| \qquad 1 = \int |\psi_-, \overline{\psi}_+\rangle d\psi_- d\overline{\psi}_+ \langle\psi_-, \overline{\psi}_+|$$

$$1 = \int |\pi\rangle d\pi \langle\pi| \qquad 1 = \int |\psi_+, \overline{\psi}_-\rangle d\psi_+ d\overline{\psi}_- \langle\psi_+, \overline{\psi}_-|$$

$$[\phi, \pi] = i$$

fermion

$$\begin{cases} \psi_-, \ \overline{\psi}_+ & : \text{ non-normalisable modes} \\ \psi_+, \ \overline{\psi}_- & : \text{ normalisable modes} \end{cases}$$

to solve 2nd class constraint, divide a Dirac field into two

$$\{\psi_\pm, \overline{\psi}_\pm\} = \pm 1$$

example : bulk is free massive

	PI(UV)	effective generating functional	flow equation of double-trace coupling
boson:	$\exp\left[-\frac{1}{2}\epsilon^{-d}\int_k F(\epsilon)\widetilde{\phi}^2 + \cdots\right]$	$\left\langle \exp\int_k \frac{\epsilon^d}{2F(\epsilon)} O_s^2 + \cdots \right\rangle^{\text{std}}_{\Lambda(\epsilon)}$	$\epsilon\partial_\epsilon F = -F^2 + dF + \epsilon^2 k^2 + m^2$
fermion:	$\exp\left[\int_k \overline{\chi}_- F(\epsilon)\chi_+ + \cdots\right]$	$\left\langle \exp\int_k \overline{O}_- F(\epsilon) O_+ + \cdots \right\rangle^{\text{std}}_{\Lambda(\epsilon)}$	$\epsilon\partial_\epsilon F = F(i\epsilon\gamma^{\widehat{\mu}}k_\mu)F + i\epsilon\gamma^{\widehat{\mu}}k_\mu - 2mF$

double-trace coupling

two fixed points alternative to standard quantisation

refs. bulk scalar, U(1) gauge boson : Heemskerk-Polchinski, Faulkner-Liu-Rangamani (2010)
bulk fermion : Elander-HI-Mandal (our paper; 2011)

Application to condensed matter physics

[Our paper (arXiv:1109.3366)]

consider the following toy model which can exhibit singular Fermi liquid behaviour

bulk

probe fermion on an extremal RN AdS black hole
near horizon limit : $z^{-1} - z_H^{-1} = \epsilon\zeta^{-1}\,, \quad t = \epsilon^{-1}\tau\,, \quad \epsilon \to 0 \;\; (\zeta, \tau \text{ finite})$

boundary

zero temp. + finite chemical pot.
low energy limit $\omega \ll \mu$

$\mathrm{AdS}_2 \times \mathbb{R}^2$

Let's see the low energy effective action at $\mathrm{AdS}_2 \times \mathbb{R}^2$

$$\mathrm{PI}(\mathrm{IR}{:}\mathrm{AdS}_2 \times \mathbb{R}^2) = \left\langle \exp \int \overline{\chi}_+ O_+ + \overline{O}_- \chi_- \right\rangle$$

expectation value with dual CFT to $\mathrm{AdS}_2 \times \mathbb{R}^2$

$$\mathrm{PI}(\mathrm{UV}; \chi_+, \overline{\chi}_-) = \exp \int_k \overline{\chi}_- F(\epsilon) \chi_+ + \cdots$$

integration

$$\left\langle \exp \int_k \overline{O}_- F(\epsilon) O_+ + \cdots \right\rangle$$

Fact: When the system has a Fermi surface, F develops a pole.
Thus, this becomes non-local. [Our paper (arXiv:1109.3366)]

How to make this into a (quasi-)local form ?

Answer : skip the integration with momenta around k_F

$$\left\langle \exp \int_k \overline{O}_- F(\epsilon) O_+ + \cdots \right\rangle = \int d\chi_- d\overline{\chi}_+ \;\; \mathrm{PI}(\mathrm{IR} : \chi_-, \overline{\chi}_+)$$

$$\mathrm{PI}(\mathrm{UV} : \chi_-, \overline{\chi}_+) = \exp \int_k \overline{\chi}_+ F(\epsilon)^{-1} \chi_- + \cdots$$

$$\mathrm{PI}(\mathrm{UV}; \chi_+, \overline{\chi}_-) = \exp \int_k \overline{\chi}_- F(\epsilon) \chi_+ + \cdots$$

undo the integration of χ around $k = k_F$
focus on modes around $k = k_F$
S[M] : dual CFT to $\mathrm{AdS}_2 \times \mathbb{R}^2$
$F \sim F_0 (k - k_F)^{-1}$

$$\int [dM] \int d\chi_- d\overline{\chi}_+ \;\; \exp\left(S[M] + \int_k \overline{\chi}_+ O_+ + \overline{O}_- \chi_- + \int_k \overline{\chi}_+ F_0^{-1}(k - k_F) \chi_- \right)$$

dual CFT + dynamical fermions around the Fermi surface
coupling to composite operators from the CFT

Faulkner-Polchinski's semi-holographic action (2010)

open problems

1. **application to non-relativistic background**
   --- **This is actually straightforward.**

2. **generalisation to dynamical gravity**
   --- **membrane paradigm : relate horizon dynamics to boundary dynamics**
   --- **to see flows of a-function, c-function etc.**

3. **done : bulk PI** $\longrightarrow$ **boundary RG**
   Q : boundary WRG $\longrightarrow$ **bulk PI ??**

[Lee], [Radicevic] : WRG in d-dim to some PI in (d+1)-dim
problem : NO bulk expressions on AdS have been found.

We are trying to find hWRG bulk relations from exact renormalisation group equation.

We can "derive" GKP-W or AdS geometry from field theory !?