Holographic Wilsonian Renormalisation Group

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IR Effective actions of strongly coupled systems are as important as microscopic actions. Wilsonian RG: useful to find effective action, difficult to carry out in the field theory

Holographic realisation, geometrisation of coarse graining of Wilsonian RG

concrete motivation: toy model of singular Fermi liquids in the context of AdS/condensed matter system

probe fermion on an extremal RN AdS black hole near horizon limit: \( \text{AdS}_2 \times \mathbb{R}^2 \)

This NH (IR) geometry is universal for (near-)extremal BH

expectation: the IR physics is governed by some strongly-coupled dual “CFT” to this AdS: (IR quantum criticality)

write down an IR eff. action in terms of the IR CFT with something more

proposition: semi-holographic action [Faulkner-Polchinski 2010]

\[
\int [dM] \int dx_+ d\tau_+ \exp \left( S[M] + \int x_+ O_++ \mathcal{O}_- \chi_- + \int \mathcal{F} O_+^{-1}(k-k_F)\chi_- \right)
\]

propagator of the new field = IR behavior of the full retarded Green’s function

what we did: derive or verify this conjecture explicitly in a holographic manner

we need some logic which can holographically connect IR and UV ---> consider holographic Wilsonian RG

**idea**

\[
\text{PI(whole aAdS)} \xrightarrow{\text{GKP-W}} \int [d\phi] \text{PI(left : IR)} \xrightarrow{\text{Hoemskerk-Polchinski}} \text{PI(right : UV)}
\]

relating two equivalent theories

**complete relations**

convenient choices of basis

boon: \( 1 = \int \phi d\phi \)

fermion: \( 1 = \int \psi \bar{\psi} \)

non-normalisable modes : normalisable modes

**example : bulk is free massive**

effective generating functional

flow equation of double-trace coupling

boson: \( \exp \left[ -\frac{1}{2} \epsilon \int F(\phi) \bar{\phi}^2 + \ldots \right] \)

fermion: \( \exp \left[ \int \tau_+ F(\chi_+) \right] \)

\( \epsilon \partial_+ F = -F^2 + dF + \epsilon^2 k^2 + m^2 \)

\( \partial_+ F = F i(2\gamma^\mu k_\mu) + i(2\gamma^\mu k_\mu - 2mF) \)

**double-trace coupling**

two fixed points alternative to standard quantisation


bulk fermion: Elander-III-Mandal (our paper: 2011)
let's see the low energy effective action at AdS2 × R2

Fact: When the system has a Fermi surface, F develops a pole. Thus, this becomes non-local. [Our paper (arXiv:1109.3366)]

Application to condensed matter physics

Consider the following toy model which can exhibit singular Fermi liquid behaviour:

probe fermion on an extremal RN AdS black hole
near horizon limit: e⁻¹ − e⁻¹ = 0, t = e⁻¹, e → 0 (ζ, τ finite)

AdS2 × R²

Let's see the low energy effective action at AdS2 × R²

Pl(IR:AdS2 × R²) = \langle \exp \int \overline{\tau}_+ O_+ + \overline{\tau}_- \rangle

| boundary |
| zero temp. + finite chemical pot. |
| low energy limit \( \omega \ll \mu \) |

How to make this into a (quasi-)local form?

Answer: skip the integration with momenta around \( k_F \)

undo the integration of z around \( k = k_F \)

focus on modes around \( k = k_F \)

S[M]: dual CFT to AdS2 × R²

\( F \sim F_0(k - k_F)^{-1} \)

\[ \int [dM] \int d\chi_+ d\overline{\tau}_+ \exp \left( S[M] + \int_k \overline{\tau}_+ O_+ + \overline{\tau}_- + \int_k \overline{\tau}_+ F_0^{-1}(k - k_F) \right) \]

dual CFT + dynamical fermions around the Fermi surface

coupling to composite operators from the CFT

Faulkner-Polchinski's semi-holographic action (2010)

open problems

1. application to non-relativistic background
   --- This is actually straightforward.

2. generalisation to dynamical gravity
   --- membrane paradigm: relate horizon dynamics to boundary dynamics
   --- to see flows of a-function, c-function etc.

3. done: bulk PI  boundary RG
   Q: boundary WRG  bulk PI ??

[Lee], [Radicevic]: WRG in d-dim to some PI in (d+1)-dim

problem: no bulk expressions on AdS have been found.

We are trying to find hWRG bulk relations from exact renormalisation group equation.

We can “derive” GKP-W or AdS geometry from field theory !?