On the double-brane solution in open SFT

Toru Masuda

The university of tokyo, Komaba
Motivation

• To justify the Murata-Schnabl solutions (multiple-brane solutions)

• Regularize the double-brane solution to satisfy the EOM
Witten’s open SFT

• String field
  \[ \Psi = c_1 |0\rangle t(x) + \alpha^\mu_{-1} c_1 |0\rangle A_\mu(x) + \ldots \]

• Action
  \[ S = \frac{1}{2} \langle \Psi, Q \Psi \rangle + \frac{1}{3} \langle \Psi, \Psi \star \Psi \rangle \]

• EOM
  \[ Q \Psi + \Psi \star \Psi = 0 \]
Witten’s open SFT

\[ \Psi \ast \Phi \]

\[ \Psi \]

\[ \Phi \]
Witten’s open SFT

Application to Tachyon condensation

• Numerical studies (2000~~)

• Analytic solutions
  – Schnabl (2005)
Witten’s open SFT

Double D25-branes?

\[ \Psi = 0 \]

Single D25-brane

\[ \Psi = \frac{1}{\sqrt{1 - K}} c(K - 1) Bc \frac{1}{\sqrt{1 - K}} \]

No D-brane

(Erler-Schnabl 2009)
K\text{Bc} \text{ subalgebra}

- Okawa (2006)

\[ [K, B] = 0, \quad \{B, c\} = 1, \quad B^2 = 0, \quad c^2 = 0 \]

\[ QB = K, \quad QK = 0, \quad Qc = cKc \]
KBc subalgebra

Correlation functions

$$\text{tr}[cc^xK \, cc^yK \, cc^zK] = - \left(\frac{x + y + z}{\pi}\right)^3 \sin\frac{\pi x}{x + y + z} \sin\frac{\pi y}{x + y + z} \sin\frac{\pi z}{x + y + z}$$

$$= -\frac{1}{4} \left(\frac{x + y + z}{\pi}\right)^3 \left(\sin\frac{2\pi x}{x + y + z} + \sin\frac{2\pi y}{x + y + z} + \sin\frac{2\pi z}{x + y + z}\right)$$
KBc subalgebra

Correlation functions

\[ \text{tr}\left[ cB e^{xK} c e^{yK} c e^{zK} c e^{wK} \right] \]
KBc subalgebra

Formal solution to the eom (Okawa 2006)

\[ \Psi = F(K)c \frac{KB}{1 - F^2(K)} cF(K) \]

① Algebraic construction (choose F(K) )
② Check the regularity
History of multiple-brane solutions

• Proposed (Murata-Schnabl, 2010)
  – Energy calculation using some analytic continuation

• $\epsilon$-regularization
  (Murata-Schnabl, 2011; Hata-Kojita 2011)
  – Energy OK
  – Gauge invariant observable OK

• Winding number picture (Hata-Kojita, 2011)
History of multiple-brane solutions

- Boundary state not reproduced (Takahashi, 2011) (c.f. Noumi’s Talk, Masuda-Noumi-Takahashi)

- Anomaly of the eom for $\varepsilon$-regularization (Murata-Schnabl, 2011)
Double-brane solution

\[ \Psi = \frac{1}{K} C \frac{K^2 B}{K - 1} C \]

Why we expect double-brane?

• Murata-Schnabl's energy formula

\[ \hat{V}(\Psi) = \frac{1}{2\pi^2} \lim_{z \to 0} z \frac{G''(-z)}{G(-z)}, \]

\[ G(z) = 1 - F^2(z). \]
Double-brane solution

Why $\frac{1}{K}$ is dangerous?

$$\frac{1}{K} = - \int_{0}^{\infty} e^{Kx} \, dx$$

No suppression factor

– Energy divergent (?)
ε-regularization

- Hata-Kojita (2011)
- Murata-Schnabl (2011)
- For lump solution, Erler-Maccaferri (2011)

\[
K \rightarrow K - \epsilon
\]

\[
\Psi = \frac{1}{K} c \frac{K^2 B}{K - 1} c \rightarrow \frac{1}{K - \epsilon} c \frac{(K - \epsilon)^2 B}{K - \epsilon - 1} c
\]
ε-regularization

✓ Desired energy
✓ Desired gauge invariant observable

◆ eom violated (Murata-Schnabl)

\[ Q\Psi + \Psi\Psi \sim \frac{\pi}{2} c_1 c_0 |0\rangle + \ldots \]

But the eom contracted to \( \Psi \) vanishes:

\[ \langle (Q\Psi + \Psi\Psi), \Psi \rangle = 0 \]
multiple-brane solutions exist in OSFT.

Yes 2/6
(one is me.)

No 4/6
Our regularization

Two steps to regularize

① Choose convergence factor for 1/K integral

- Energy OK.
- Anomalies of the EOM (only two terms)

\[ Q\Psi + \Psi\Psi \sim p c_0 c_1 |0\rangle + p c_{-1} c_1 |0\rangle \]

② Add a phantom piece

- Energy OK.
- No anomaly on the Fock space.
Our regularization

Example of the convergence factor

\[
R_0(\Lambda; x) = \begin{cases} 
1 - \log(x + 1)/\log(\Lambda + 1) & (0 \leq x \leq \Lambda) \\
0 & (\Lambda < x),
\end{cases}
\]

Phantom piece: slightly complicated
Our regularization

That is,

\[ \phi_p = - \lim_{\Lambda \to \infty} \frac{1}{\log(\Lambda + 1)} \int_{0}^{\Lambda} d\lambda \frac{1}{\lambda + 1} \int_{0}^{\lambda} e^{Kx} dx \, cK^2 e^{\lambda K} \, Bc \]

\[ q = \frac{2 - \gamma + \text{Ci}(2\pi) - \log(2\pi)}{-1 - 2\text{Ci}(\pi) + 2\text{Ci}(2\pi) - 2 \log 2 + 2\pi \text{Si}(2\pi)} \cong 0.0691204. \]

\[ \Psi_{\text{double}} = \Psi_{R_0} - q\phi_p \]
Boundary states

(c.f. Noumi’s talk)

• Fail to reproduce the desired boundary state;

\[ |B_*(\Psi)\rangle = 2 |B\rangle \]

• No sign of breaking the eom

(good news ?)
Towards No-go theorem wrt boundary states

Some loose ends;
• Ellwood correspondence?
• Schnabl gauge calculation?

>> Murata-Schnabl solution not totally killed.
Masuda, Noumi and Takahashi

``Constraints on a class of analytic solutions in open string field theory’’
Summary and Discussion

- Regularization of the double-brane solution respecting the eom

- Desired boundary states not obtained
  - a) the eom is broken in unspecified manner (?)
  - b) Ellwood conjecture subject to some change (?)