On the double-brane solution in open SFT

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Motivation

- To justify the Murata-Schnabl solutions (multiple-brane solutions)

- Regularize the double-brane solution to satisfy the EOM

Witten's open SFT

- String field

$$\Psi = c_1|0\rangle t(x) + \alpha^{\mu}_{-1} c_1|0\rangle A_{\mu}(x) + \ldots$$

- Action

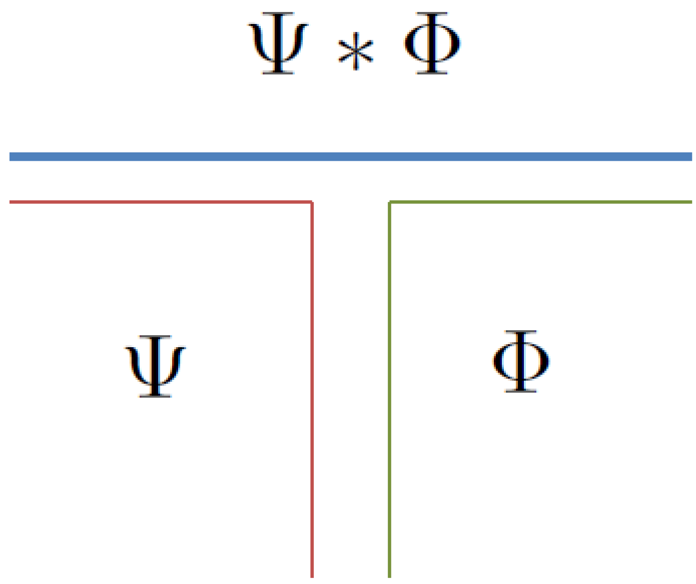
$$S = \frac{1}{2}\langle\Psi,\, Q\Psi\rangle + \frac{1}{3}\langle\Psi,\, \Psi * \Psi\rangle$$

- EOM

$$Q\Psi + \Psi * \Psi = 0$$

Witten's open SFT

$* -$ product



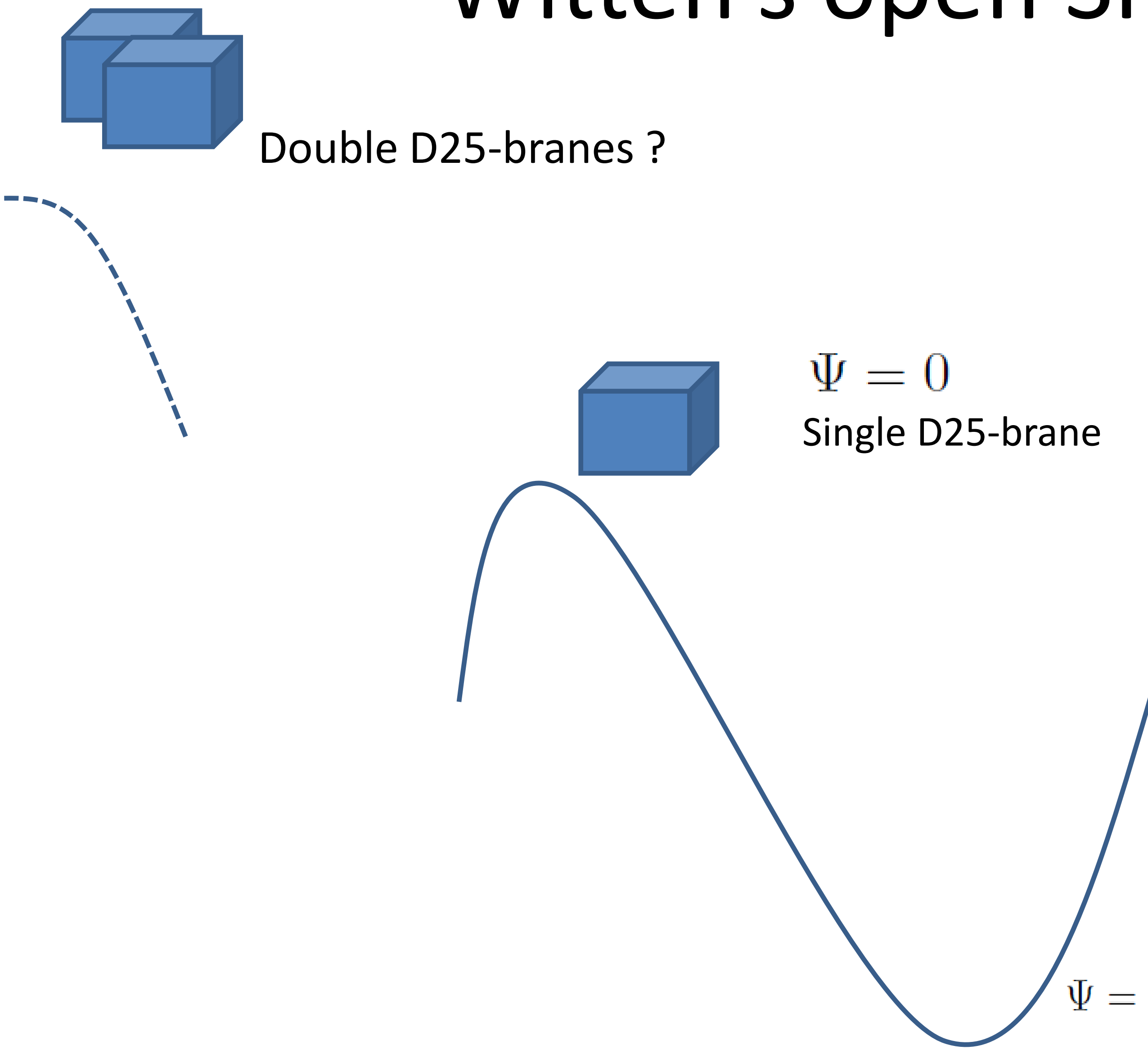
Witten's open SFT

Application to Tachyon condensation

- Numerical studies (2000~~)

- Analytic solutions
  - Takahashi-Tanimoto (2002)
  - Schnabl (2005)

Witten's open SFT



$$\Psi = \frac{1}{\sqrt{1-K}} c(K-1)Bc \frac{1}{\sqrt{1-K}}$$

(Erler-Schnabl 2009)

KBc subalgebra

- Okawa (2006)

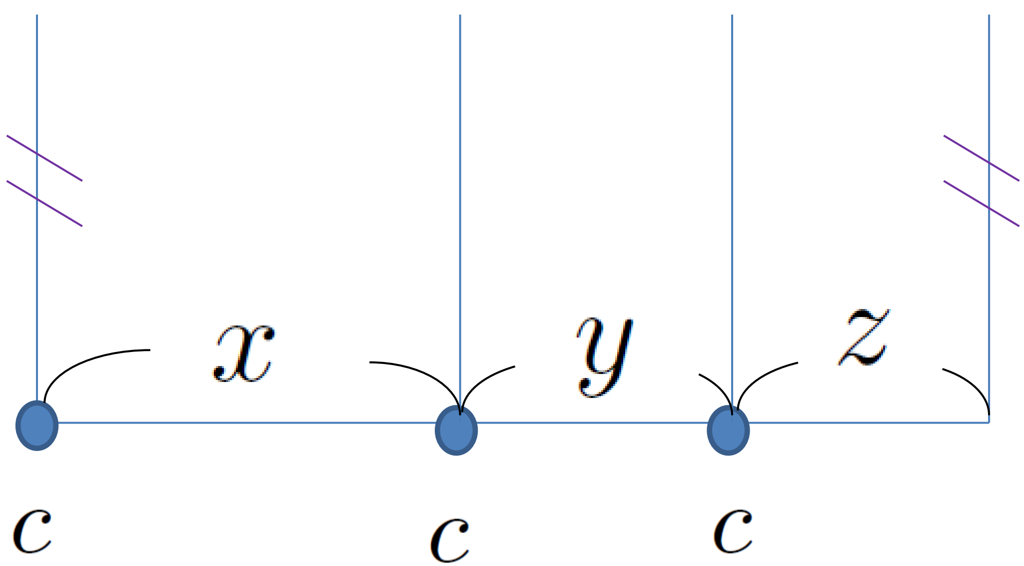
$$[\,K,\,B\,]=0\,,\qquad \{\,B,\,c\,\}=1\,,\qquad B^2=0\,,\qquad c^2=0$$

$$QB=K,\quad QK=0,\quad Qc=cKc$$

KBc subalgebra

Correlation functions

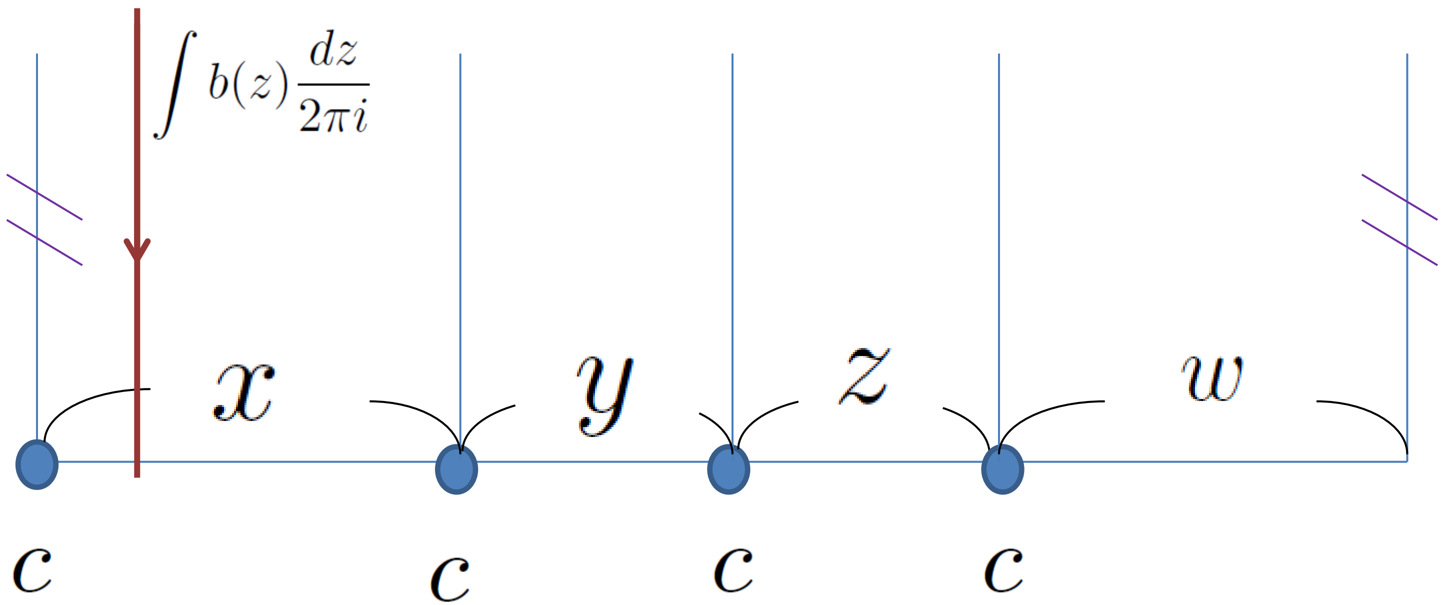
$$\mathrm{tr}[ce^{xK}ce^{yK}ce^{zK}] = -\left(\frac{x+y+z}{\pi}\right)^3 \sin\frac{\pi x}{x+y+z}\sin\frac{\pi y}{x+y+z}\sin\frac{\pi z}{x+y+z}$$
$$= -\frac{1}{4}\left(\frac{x+y+z}{\pi}\right)^3\left(\sin\frac{2\pi x}{x+y+z}+\sin\frac{2\pi y}{x+y+z}+\sin\frac{2\pi z}{x+y+z}\right)$$



KBc subalgebra

Correlation functions

$$\mathrm{tr}[cBe^{xK}ce^{yK}ce^{zK}ce^{wK}]$$



KBc subalgebra

Formal solution to the eom (Okawa 2006)

$$\Psi = F(K)c\frac{KB}{1-F^2(K)}cF(K)$$

① Algebraic construction (choose F(K))

② Check the regularity

History of multiple-brane solutions

- Proposed (Murata-Schnabl, 2010)
  - Energy calculation using some analytic continuation
- ε-regularization
  (Murata-Schnabl, 2011; Hata-Kojita 2011)
  - Energy OK
  - Gauge invariant observable OK
- Winding number picture (Hata-Kojita, 2011)

History of multiple-brane solutions

- Boundary state not reproduced   (Takahashi, 2011)
  (c.f. Noumi's Talk, Masuda-Noumi-Takahashi)

- Anomaly of the eom for ε-regularization
  (Murata-Schnabl, 2011)

Double-brane solution

$$\Psi = \frac{1}{K} c \frac{K^2 B}{K-1} c$$

Why we expect double-brane ?

- Murata-Schnabl's energy formula

$$\widehat{V}(\Psi) = \frac{1}{2\pi^2} \lim_{z\to 0} z \frac{G'(-z)}{G(-z)},$$

$$G(z) = 1 - F^2(z).$$

Double-brane solution

Why $\frac{1}{K}$ is dangerous ?

$$\frac{1}{K} = -\int_0^\infty e^{Kx} dx \quad (?)$$

No suppression factor

– Energy divergent (?)

ε-regularization

- Hata-Kojita (2011)
- Murata-Schnabl (2011)
- For lump solution, Erler-Maccaferri (2011)

$$K \to K - \epsilon$$

$$\Psi = \frac{1}{K} c \frac{K^2 B}{K-1} c \to \frac{1}{K-\epsilon} c \frac{(K-\epsilon)^2 B}{K-\epsilon-1} c$$

ε-regularization

✓ Desired energy

✓ Desired gauge invariant observable

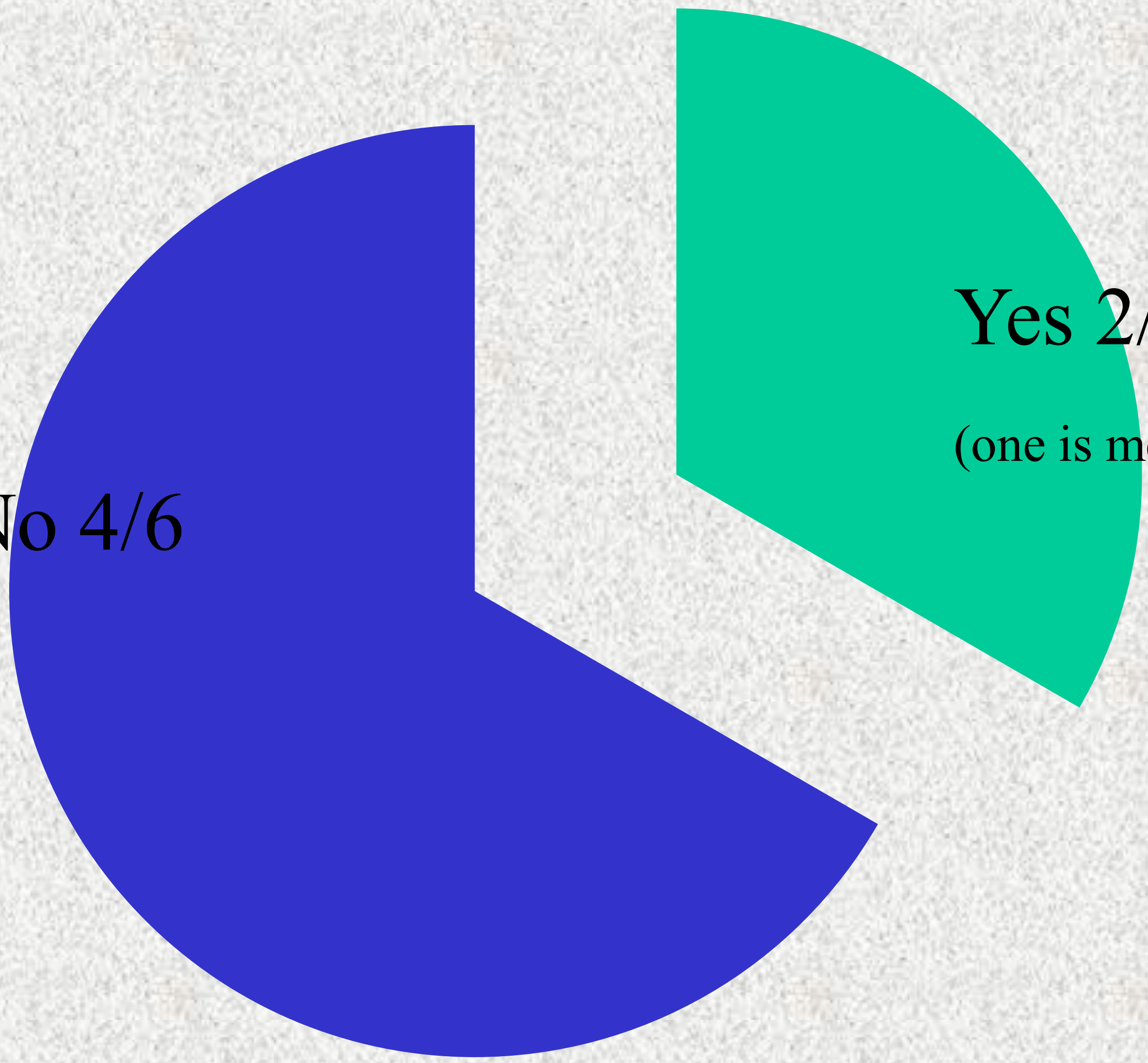
◆ eom violated (Murata-Schnabl)

$$Q\Psi + \Psi\Psi \sim \frac{\pi}{2} c_1 c_0 |0\rangle + \ldots$$

But the eom contracted to Ψ vanishes:

$$\langle (Q\Psi + \Psi\Psi),\, \Psi \rangle = 0$$

multiple-brane solutions exist in OSFT.



Our regularization

Two steps to regularize

① Choose convergence factor for 1/K integral

- ✓ Energy OK.
- ◆Anomalies of the EOM (only two terms)

$$Q\Psi + \Psi\Psi \sim p\, c_0 c_1 |0\rangle + p\, c_{-1} c_1 |0\rangle$$

② Add a phantom piece

- ✓ Energy OK.
- ✓ No anomaly on the Fock space.

Our regularization

Example of the convergence factor

$$R_0(\Lambda; x) = \begin{cases} 1 - \log(x+1)/\log(\Lambda+1) & (0 \le x \le \Lambda) \\ 0 & (\Lambda < x), \end{cases}$$

Phantom piece: slightly complicated

Our regularization

That is,

$$\phi_p = -\lim_{\Lambda\to\infty} \frac{1}{\log(\Lambda+1)} \int_0^{\Lambda} d\lambda \frac{1}{\lambda+1} \int_0^{\lambda} e^{Kx} dx\, cK^2 e^{\lambda K} Bc$$

$$q = \frac{2-\gamma+\mathrm{Ci}(2\pi)-\log(2\pi)}{-1-2\mathrm{Ci}(\pi)+2\mathrm{Ci}(2\pi)-2\log 2+2\pi\mathrm{Si}(2\pi)} \cong 0.0691204.$$

$$\Psi_{double} = \Psi_{R_0} - q\phi_p$$

Boundary states

(c.f. Noumi's talk)

- Fail to reproduce the desired boundary state;

$$\cancel{|B_*(\Psi)\rangle = 2|B\rangle}$$

$$|B_*(\Psi)\rangle = |B\rangle$$

- No sign of breaking the eom

  (good news ?)

Boundary states

Towards No-go theorem wrt boundary states

Some loose ends;

- Ellwood correspondence ?
- Schnabl gauge calculation ?

>> Murata-Schnabl solution not totally killed.

Illustration by Machiko Kyo, http://juicyfruit.exblog.jp/

Masuda, Noumi and Takahashi

``Constraints on a class of analytic solutions in open string field theory''

Summary and Discussion

- Regularization of the double-brane solution respecting the eom

- Desired boundary states not obtained
  - a) the eom is broken in unspecified manner (?)
  - b) Ellwood conjecture subject to some change (?)