Effects of incoming (reflected) matters

When the shock wave is replaced by the flat space, we need to modify the Vaidya metric.

Inside of the shock wave is replaced by the flat space. Then, the singularity and horizon no longer exist.

**Junction condition**

- Shock wave is null in both side.
- Time(null) in flat space
- Time(null) in Vaidya

**Hawking radiation without the horizon.**

The energy density inside the BH is

\[ E = C a \]  

outside of the BH.

In the flat side, \( \hat{u} = -2r \) which is connected to Vaidya metric at the locus of the shock wave. So, we need to calculate relation \( r \) and \( u \) in Vaidya.

In the Vaidya side, \( \hat{u}(u) \) is given by

\[ \hat{u}(u) = a(u) + r_0 - \int \frac{du}{\hat{u}(u)} \exp \left[ - \int \frac{1}{2[a(u)]^2} du \right] \]

Approaches to “past horizon” exponentially.

Mass of BH changes very slowly.

\( a(u) \) can be neglected,

\[ \hat{a}(u) \Delta u \ll a(u) \]

\[ a(u) \sim a \]

\[ a(u) \approx a \]

**Hawking temperature**

\[ T = \frac{1}{4\pi a(u)} \]

**Effects of incoming (reflected) matters**

The Hawking radiation is partially reflected by the potential barrier.

\[ \phi(r) \sim (r - r_0)^{-2} \]

\[ \hat{u} \]

Mass and angular momentum are effectively zero.

We introduce a cutoff of angular momentum (it must be finite).

If there are only outgoing radiations, the geometry is given by Vaidya metric.

**Interaction**

- Near the horizon, the Hawking radiation behaves as
  \[ \phi(r) \sim (r - r_0)^{-2} \]
  Mass and angular momentum are effectively zero.
  
  We introduce a cutoff of angular momentum (it must be finite).

  If there are only outgoing radiations, the geometry is given by Vaidya metric.

  \[ ds^2 = \left( 1 - \frac{a(u)}{r} \right) du^2 - 2dudr + r^2d\Omega^2 \]

  For the collapse of an shell, the geometry is given by connecting the Vaidya metric and flat space by the shell (shock wave).

**Conservation law is given by**

\[ 0 = \partial_r T_{uu}(u, r) + \frac{1}{r} \frac{\partial}{\partial u} \left( T_{uu}(u, r) - \frac{\partial \phi(u, r)}{\partial r} \right) \]

\[ \frac{\partial \phi(u, r)}{\partial r} = 0 \]

The second equation is equivalent to EOM for \( p(u, r) \).

Evaporation is very slowly. We neglect \( u \) dependence.

\[ p^2(u, r) \frac{\partial \phi(u, r)}{\partial r} = C(u) \]

Where \( C \) can be estimated as

\[ T_{rr} \sim \frac{r^2}{p} T_{uu} \]

The solution can be approximated as

\[ p + aC \log \left( \frac{p - aC}{a} \right) = r - a_{\text{eff}} \]

Numerical solution is plotted as

\[ p(r) \]

For \( r > a \)

\[ p \sim r - a_{\text{eff}} \]

For \( r < a \)

\[ p \sim Ca = \text{const.} \]

This implies that shock wave is smeared around \( r \sim a \).

Outside is approximated by Vaidya.

**Static Geometry**

We put BH in a heat bath. incoming and outgoing of radiations are balanced.

- Finally goes some stationary state.

Stationary geometry with

\[ g_{\mu\nu} = 0 \quad \text{and} \quad g_{\phi\phi} = G_{\phi\phi} = 0 \]

\[ ds^2 = p(r) \left( -\frac{dr}{r} + \frac{r}{p(r)} dr^2 \right) + r^2d\Omega^2 \]

\[ S(r) \sim C \]

We assume that this metric can be used for energy density of \( p(r) \) as

\[ p(r) = Ca + C_1 \int dr r^2 e^{2/r(C_2)} \]

Instead of approaching to horizon, the second term gives explicit behavior:

\[ \dot{a} \sim -\sqrt{Ca(a_0 - u)} + C_2e^{2/r(C_2)} \]

The shock wave finally reaches \( r = 0 \).

- No Hawking radiation but incoming energy directly goes out.

Energy density inside the BH is

\[ 8\pi T_{\phi\phi} = G_{\phi\phi} = \frac{p^2(r)p^\prime(r)}{r^4p(r)} = \frac{C}{r^4} \]

\[ T = \frac{T_{\phi\phi}}{\sqrt{-g_{\phi\phi}}} = T_h \frac{r}{p(r)p(r)} \]

Local temperature inside the BH is

\[ T = T_h \frac{r}{p(r)p(r)} \]

Pressure is not isotropic.

We assume the following thermodynamic relation

\[ p = T_h \]

Then, the total entropy inside the BH is

\[ S = \int d^3x \sqrt{-g} s = \int_0^\infty dr \frac{2\pi aCr}{p(r)} = \pi a^2 \]

which reproduces the area law:

\[ S = \frac{1}{4} a \]