Spacetime structure of Black Hole Evaporation

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Back reaction from the Hawking radiation

Near the horizon, the Hawking radiation behaves as

$$\phi(r) \sim (r - r_H)^{\pm i\omega}$$

⇒ Mass and angular momentum are effectively zero.

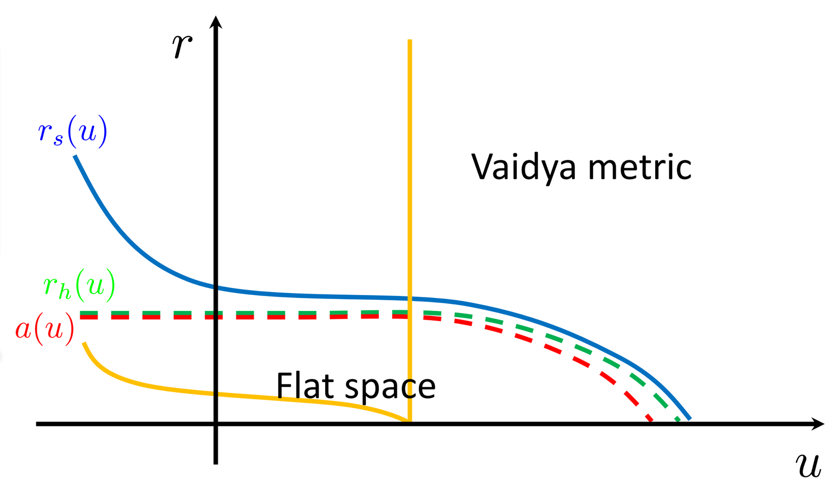
We introduce a cutoff of angular momentum (it must be finite).

If there are only outgoing radiations the geometry is given by Vaidya metric.

$$ds^2 = -\left(1 - \frac{a(u)}{r}\right) du^2 - 2du dr + r^2 d\Omega^2$$

For the collapse of an shell, the geometry is given by connecting the Vaidya metric and flat space by the shell (shock wave).

Inside of the shock wave is replaced by the flat space. Then, the singularity and horizon no longer exist.



Junction condition = Shock wave is null in both side.

$$-2dr = d\tilde{u} = \left(1 - \frac{a(u)}{r(u)}\right) du$$

Time(null) in flat space Time(null) in Vaidya

Hawking radiation without the horizon.

Change of the wave profile is calculated by using the Eikonal approximation.

$$A_{\omega,\omega'} = \int du e^{-i\omega' u} e^{i\omega \tilde{u}(u)}$$

In the flat space side, $\tilde{u} = -2r$, which is connected to Vaidya metric at the locus of the shock wave. So, we need to calculate relation *r* and *u* in Vaidya.

In the Vaidya side, locus is given by

$$r_s(u) = a(u) + r_0 - \int^u du' \, \dot{a}(u') \exp\left[-\int_{u'}^{u} \frac{1}{2a(u'')} du''\right]$$

Approaches to "past horizon" exponentially ⇒ Hawking radiation

Mass of BH changes very slowly. ⇐ $\dot{a}(u)\Delta u \ll a(u)$

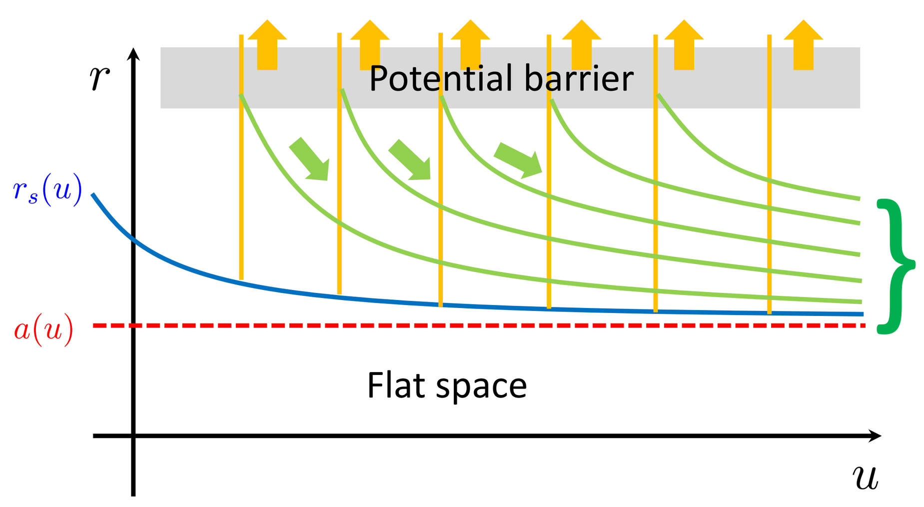
⇒ *u* dependence can be neglected: $a(u) \sim a$

$\tilde{u}(u) = a + C_0 u + C e^{-\frac{u}{2a}}$ ⇒ same exp behavior to ordinary BH

Hawking temperature is $T = \dfrac{1}{4\pi a(u)}$

Effects of incoming (reflected) matters

The Hawking radiation is partially reflected by the potential barrier



⇒ We need to modify the Vaidya metric.

$$ds^2 = -\partial_r p(u,r)\left[\frac{p(u,r)}{r} du^2 + 2du\, dr\right] + r^2 d\Omega^2$$

p(u,r) satisfies the following EOM:

$$2r\partial_u\left(\frac{\partial_r^2 p(u,r)}{r\partial_r p(u,r)}\right) - \frac{1}{p(u,r)}\partial_r\left(\frac{p^2(u,r)\partial_r^2 p(u,r)}{r\partial_r p(u,r)}\right) = 0$$

The ingoing energy flow can be expressed as

$$S(u,r) = p^2(u,r) T_{rr}(u,r) = \frac{p^2(u,r)\partial_r^2 p(u,r)}{r\partial_r p(u,r)}$$

Conservation law is given by

$$0 = \partial_r T_{uu}(u,r) + \frac{2}{r} T_{uu}(u,r) - \frac{1}{rp(u,r)}\left(-\frac{\partial_u p(u,r)}{p(u,r)} S(u,r) + \partial_u S(u,r)\right)$$

$$0 = \partial_u S(u,r) - \frac{p(u,r)}{2r}\partial_r S(u,r) - 2\frac{\partial_u p(u,r)}{p(u,r)} S(u,r)$$

The second equation is equivalent to EOM for *p(u,r)*.

Evaporation is very slow ⇒ We neglect *u* dependence.

$$\frac{p^2(u,r)\partial_r^2 p(u,r)}{r\partial_r p(u,r)} = C(u)$$

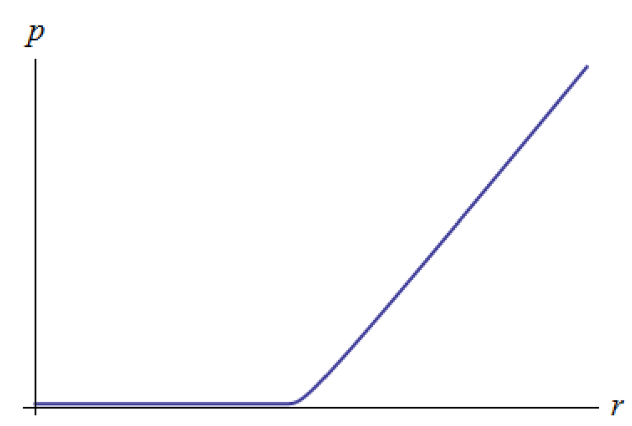
Where *C* can be estimated as

$$T_{rr} \sim \frac{r^2}{p^2} T_{uu} \Rightarrow C = S = p^2 T_{rr} \sim \dot{a} \sim \left(\frac{l_p}{a}\right)^2$$

The solution can be approximated as

$$p + aC\log\left(\frac{p - aC}{a}\right) = r - a_{\text{eff}}$$

Numerical solution is plotted as



For $r > a$

$p \sim r - a_{\text{eff}}$

For $r < a$

$p \sim Ca = \text{const.}$

This implies that shock wave is smeared around $r \sim a$.

Outside is approximated by Vaidya.

Static Geometry

We put BH in a heat bath. ⇒ incoming and outgoing of radiations are balanced

⇒ finally goes some stationary state.

Stationary geometry with $G_{\mu\nu}g^{\mu\nu} = 0$ and $G_{\theta\theta} = G_{\varphi\varphi} = 0$

$$ds^2 = p'(r)\left(-\frac{p(r)}{r}dt^2 + \frac{r}{p(r)}dr^2\right) + r^2 d\Omega^2$$

⇒ *S*(*r*) = *C* becomes exact.

We assume that this metric can be used for *r* < *a*, where *p*(*r*) behaves as

$$p(r) = Ca + C_1 \int dr e^{r^2/(Ca^2)}$$

Instead of approaching to horizon, the second term gives exp behavior:

$$\tilde{u} \simeq -\sqrt{Ca(u_0 - u)} + C_2 e^{-\frac{u}{2a}}$$

The shock wave finally reaches *r* = 0.

⇒ No Hawking radiation but incoming energy directly goes out.

Energy density inside the BH is

$$8\pi T_{tt} = G_{tt} = \frac{p^2(r)p''(r)}{r^3 p'(r)} = \frac{C}{r^2} \Rightarrow \rho = -T^t_t = \frac{1}{8\pi}\frac{C}{rp(r)p'(r)}$$

Local temperature inside the BH is

$$T = \frac{T_H}{\sqrt{-g_{tt}}} = T_H\sqrt{\frac{r}{p(r)p'(r)}}$$

where T_H is the Hawking temperature $\sim \dfrac{1}{4\pi a}$

Pressure is not isotropic

We assume the following thermodynamic relation

$$\rho = Ts$$

Then, the total entropy inside the BH is

$$S = \int_\Sigma d^3x\sqrt{g_\Sigma}\, s = \int_0^a dr \frac{2\pi a C r}{p(r)} = \pi a^2$$

which reproduces the area law: $S = \dfrac{1}{4}A$