Thermal Skyrmion configurations in Yang-Mills theory

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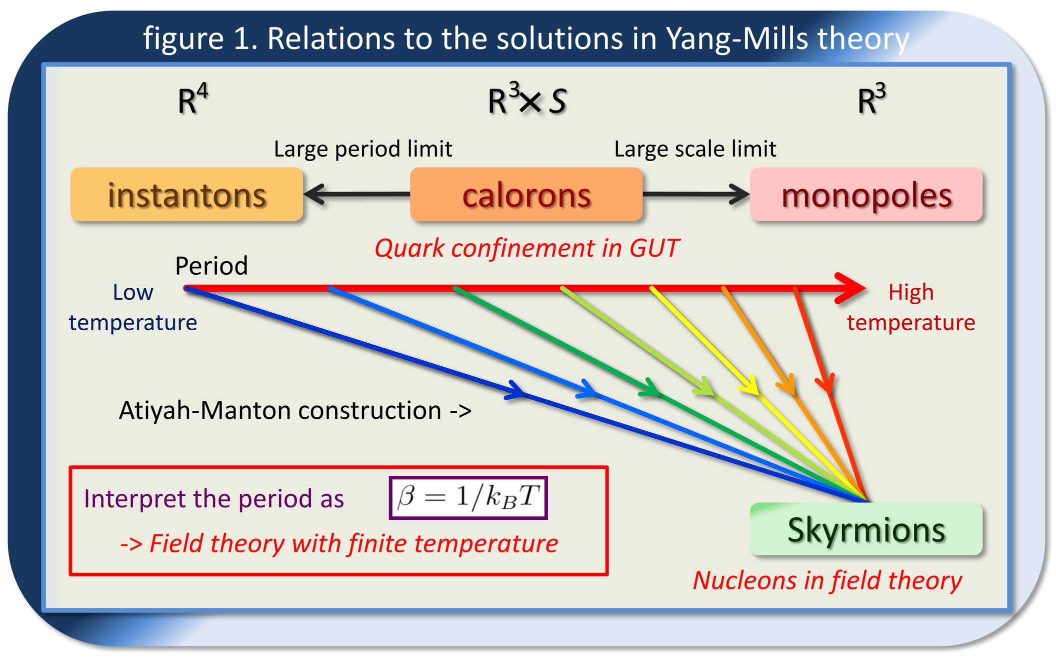
Introduction

Calorons are periodic instantons in Yang-Mills theory which interpolate between instantons and monopoles in some limits. And also thermal Skyrmions can be constructed from the calorons in terms of the Atiyah-Manton construction with interpretation that the period of the calorons relates to temperature (figure 1). Since the physical character of the Skyrmions is a model of nucleons, the study of the thermal Skyrmions gives us some knowledges of the field theory with finite temperature.

Recently some significant features of the calorons are revealed. Particularly some complicated calorons, with higher charge or non-trivial holonomy, can be obtained in terms of the Nahm construction which constructs the gauge fields of the calorons in terms of zeromodes in the moduli space of the calorons. And we also established the method of the Nahm transform in a simple case. In this poster we explain the way to obtain the thermal Skyrmions from our calorons.

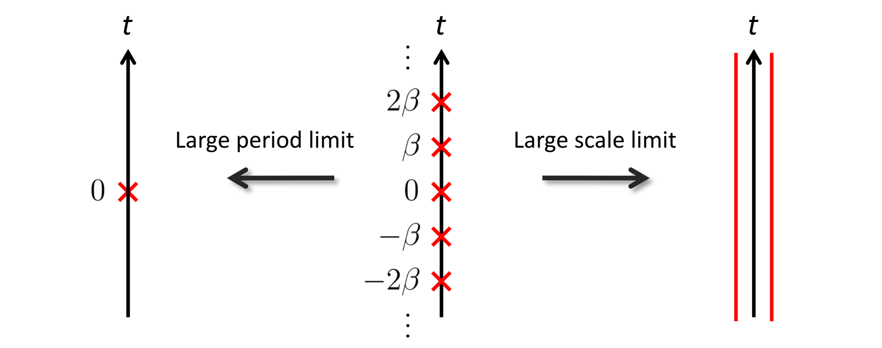
figure 1. Relations to the solutions in Yang-Mills theory



Calorons

The simplest case of the calorons is Harrington-Shepard caloron with charge 1 which includes the instanton and the monopole in each limit.

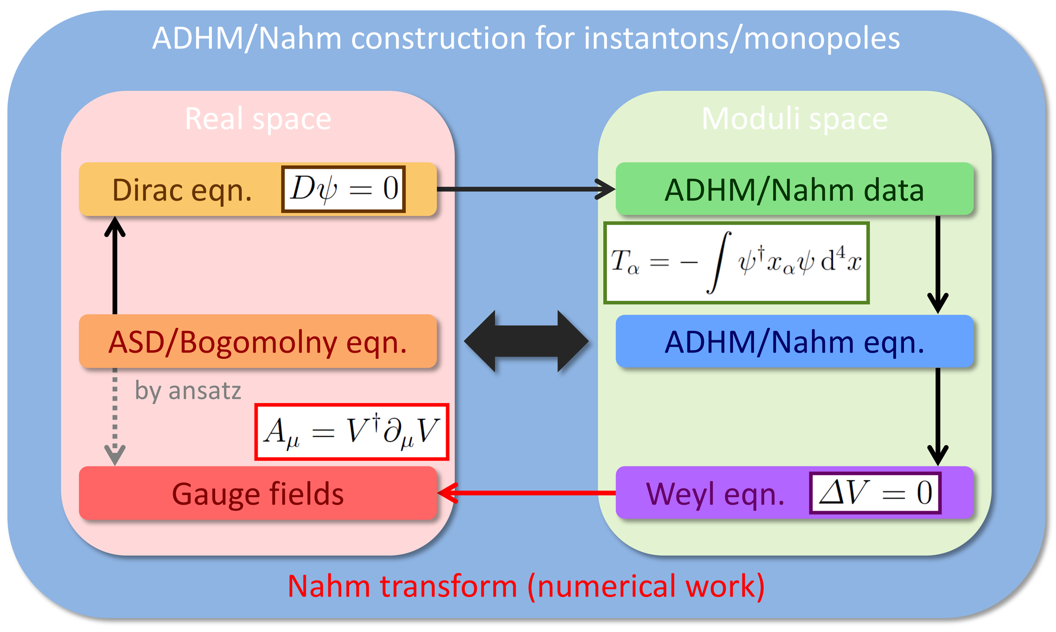
$$A_\mu(x) = \frac{\mathrm{i}}{2}\eta^{(+)}_{\mu\nu}\partial_\nu \ln\phi \qquad \phi(x) = \frac{\lambda^2}{\mu_0}\sum_{i=-\infty}^{\infty}\frac{1}{|x-a_i|^2} = \frac{\lambda^2}{2r}\frac{\sinh\mu_0 r}{\cosh\mu_0 r - \cos\mu_0 t}$$



Nahm construction

The Nahm construction is a systematic method to obtain the gauge field of the monopoles and the calorons which is an application of the ADHM construction for the instantons. Since the duality between the real and the moduli space was proved, the Nahm data of the calorons which is the gauge fields in the moduli space can be obtained by solving the Nahm equation and the corresponding gauge fields can be constructed in terms of the zeromodes of the Weyl equation.

Since the forms of the Nahm data is complicated, the Nahm transform is usually performed numerically. There are some Nahm data of the calorons and also the Nahm transform was performed in some cases.



History of study of calorons

1978 Harrington-Shepard 1-caloron (*trivial*)

ADHM construction (Atiyah, Drinfeld, Hitchin, Manin, *for instantons*)

1984 Nahm construction (*for monopoles and calorons*)

2004 Nahm data of the 2,3,4-symmetric caloron (Ward, *trival*)

Nahm transform of the 2-caloron (Bruckmann, Nogradi, van Baal, *non-trivial*)

2007 Nahm data of the 1,2-caloron (Harland, *non-trivial*)

2010 General Nahm data of the 2-caloron (Nakamula, Sakaguchi, *non-trivial*)

2011 Nahm transform of the 2-caloron (Muranaka, Nakamula, Sawado, Toda, *trivial*)

Nahm transform for SU(2) *k*-caloron

In the case of SU(2) calorons with charge *k* the Nahm equation is following bulk equation and the matching condition

$$\frac{\mathrm{d}}{\mathrm{d}s}T_a - \mathrm{i}[T_0, T_a] - \frac{\mathrm{i}}{2}\varepsilon_{abc}[T_b, T_c] = 0 \qquad T_j(-\mu_0/2) - T_j(\mu_0/2) = \frac{1}{2}\mathrm{Tr}_2(\sigma_j W^\dagger W)$$

where *T* and *W* are Nahm data which are *k*-matrices and *k*-row vector with quaternion entry. If the Nahm data are obtained, the Weyl equation is defined by

$$\left\{\mathbf{1}_{2k}\frac{\mathrm{d}}{\mathrm{d}s} - \mathrm{i}\big(T_\mu(s) + x_\mu \mathbf{1}_k\big)\otimes e_\mu\right\}U(s, x^\alpha) = \mathrm{i}W^\dagger V(x^\alpha)\delta(s - \mu_0/2)$$

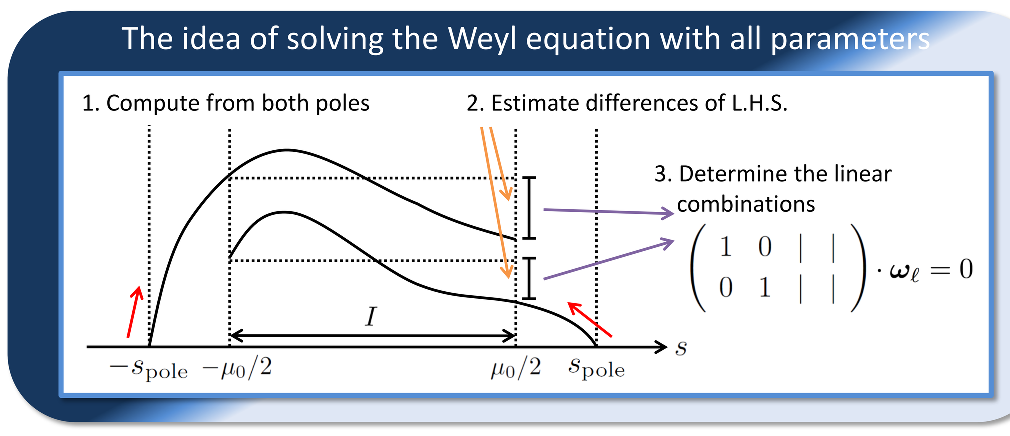
And then the gauge field of the calorons can be constructed in terms of normalized zeromodes of the Weyl equation.

$$\int_I U^\dagger U \mathrm{d}s + V^\dagger V = \mathbf{1}_2 \qquad A_\alpha(x) = \int_I U^\dagger \partial_\alpha U \mathrm{d}s + V^\dagger \partial_\alpha V$$

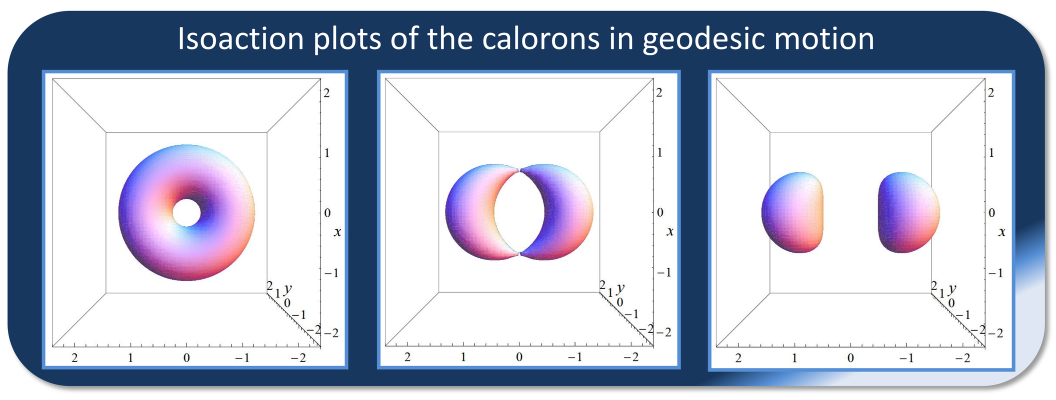
Nahm transform for 2-caloron

General Nahm data of the calorons with charge 2 were obtained and the Nahm transform in simple case was performed. Our method is simply regards the Weyl equation as a pair of bulk and boundary equations which can be solved numerically.

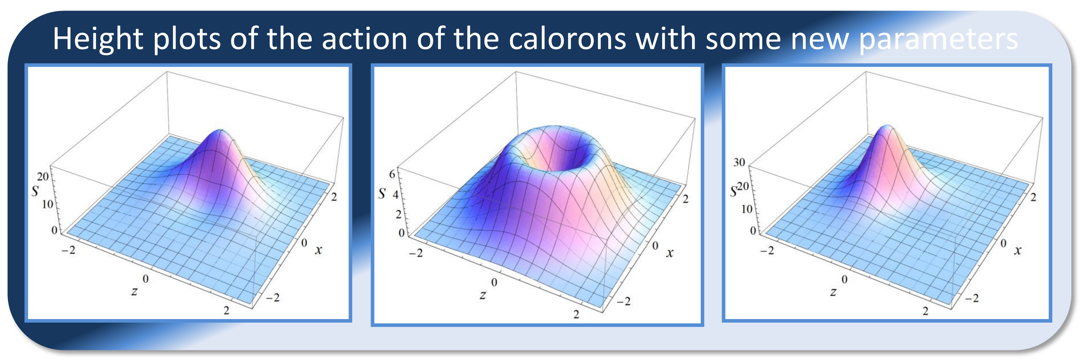
The idea of solving the Weyl equation with all parameters



Isoaction plots of the calorons in geodesic motion



Height plots of the action of the calorons with some new parameters



Skyrmions

The lagrangian of the Skyrmions are defined in this way.

$$S_{\mathrm{skrm}} = \int \mathrm{d}^4 x \left[-\frac{f_\pi^2}{4}\mathrm{Tr}[L^\mu L_\mu] + \frac{1}{32e^2}\mathrm{Tr}[L^\mu,\, L^\nu][L_\mu,\, L_\nu]\right] \qquad L_\mu = U^\dagger \partial_\mu U$$

The most simple case of the Skyrmion is hedgehog solution with charge 1 which can be solved numerically.

$$U = \exp[\mathrm{i}f(r)\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}] = \cos f(r) + \mathrm{i}\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\sin f(r)$$

	A.N.W.	Expt
f_π [MeV]	64.5	93
e	5.45	
M_{N} [MeV]	939	939
$M_\Delta - M_{\mathrm{N}}$ [MeV]	293	294
$g_{\mathrm{N}}^{\mathrm{A}}$	1.02	1.26
$\langle r_E^2\rangle_0^{1/2}$ [fm]	0.59	0.72
$\langle r_E^2\rangle_1^{1/2}$ [fm]	∞	0.88
$\langle r_M^2\rangle_0^{1/2}$ [fm]	0.92	0.81
$\langle r_M^2\rangle_1^{1/2}$ [fm]	∞	0.80
μ_{p} [n.m.]	1.87	2.79
μ_{n} [n.m.]	- 1.31	- 1.91

Also the physical variables can be calculated which are expected to the one of nucleon (cf. G.S. Adkins, C.R. Nappi, E. Witten, Nucl.Phys. B228 (1983) 552).

Holonomy and Atiyah-Manton construction

The holonomy of the instantons is defined by

$$U(\boldsymbol{x}, \infty) = \pm\mathcal{P}\exp\left[-\int_{-\infty}^{\infty} A_t(\boldsymbol{x}, t)\mathrm{d}t\right]$$

which is path ordered exponential of the gauge field toward the time direction. Then the ansatz of the Atiyah-Manton construction is defined by

$$\mathcal{L} = \int\left(\frac{F_\pi^2}{16}\mathrm{Tr}\,\partial_\mu U\partial^\mu U^\dagger + \frac{1}{32e^2}\mathrm{Tr}\,[\partial_\mu UU^\dagger, \partial_\nu UU^\dagger]^2\right)\mathrm{d}^3 x$$

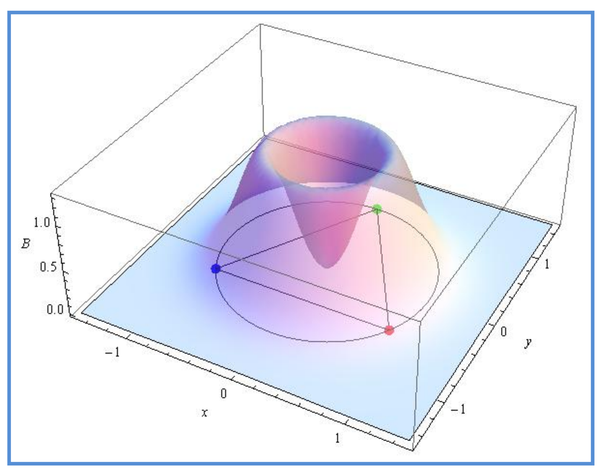
(cf. M.F. Atiyah, N.S. Manton, Phys.Lett. B222 (1989) 438-442), which can be calculated numerically.

There are some cases of the Atiyah-Manton construction with analytic solutions of the instanton.

2-Skyrmion from 2-JNR instanton

In the case of the 2-Skyrmion the energy and the baryon density can be obtained in terms of the 2-JNR instanton which have three poles in a plane.

$$A_t(\boldsymbol{x}, t) = \frac{\mathrm{i}}{2}\tau_j\partial_j \ln\rho\,, \quad \rho(\boldsymbol{x}, t) = \frac{1}{(\boldsymbol{x}-\boldsymbol{X}_1)^2 + t^2} + \frac{\lambda}{(\boldsymbol{x}-\boldsymbol{X}_2)^2 + t^2} + \frac{1}{(\boldsymbol{x}-\boldsymbol{X}_3)^2 + t^2}$$

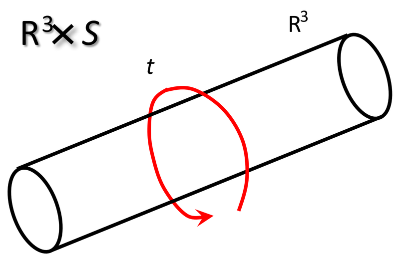


A. Hosaka, S.M. Griffies,
B. M. Oka, R.D. Amado,
C. Phys.Lett. B251 (1990) 1-5

Our works

The Atiyah-Manton construction can also be applied for the calorons. In this case the path ordered exponential of the holonomy is loop integral.

$$\Omega_\beta(x^j) = \mathcal{P}\exp\left[-\int_0^\beta A_t(x^j, t)\,\mathrm{d}t\right]$$



The integral seems to be zero, which is called trivial holonomy and the non-zero case is called non-trivial holonomy.

The ansatz of the Atiyah-Manton construction is defined by

$$\mathcal{L}_{\mathrm{Skrm}} = \int\left(\frac{F_\pi^2}{16}\mathrm{Tr}\,\partial_\mu\Omega_\beta\partial^\mu\Omega_\beta^\dagger + \frac{1}{32e^2}\mathrm{Tr}\left[\partial_\mu\Omega_\beta\Omega_\beta^\dagger, \partial_\nu\Omega_\beta\Omega_\beta^\dagger\right]^2\right)\mathrm{d}^3 x$$

Our work is to perform the Atiyah-Manton construction with gauge fields constructed with the Nahm construction.

Summary

- The Atiyah-Manton construction is a method to construct Skyrmions from the instantons, particularly the Skyrmions constructed from the calorons can be regarded as thermal Skyrmions.
- There are some examples of the Atiyah-Manton construction with analytic solutions of instantons.