Universality Crossover in Parametric (ch)RMT and Lattice Dirac Spectrum

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PLAN

1. Intro/Motivation
    - universality crossover
    - Dirac spectrum vs chRMT
    - NLσM and universality

review

2. Parametric (chiral) RMT
    - level spacing / smallest EV

··· analytical

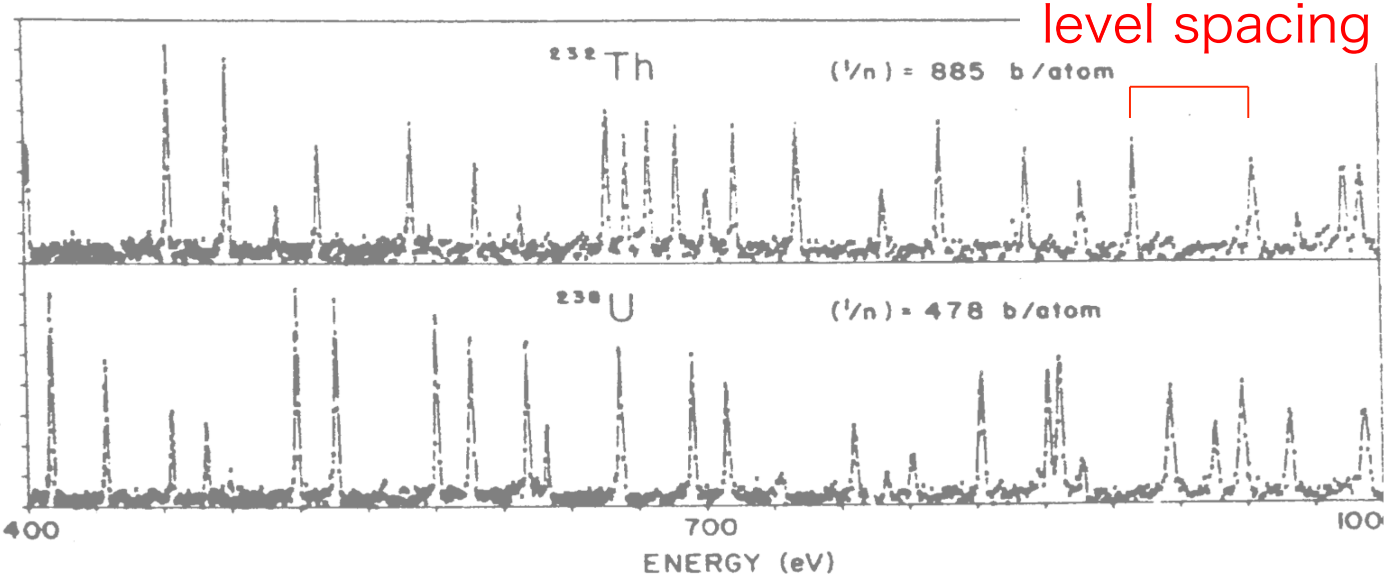
3. SU(2)×U(1) Dirac Spectrum
    - random flux × noise model (check)
    - pure LGT
    - scaling and low-energy constant

numerical

1. Universality of spectral fluctuation

energy levels of excited nuclei

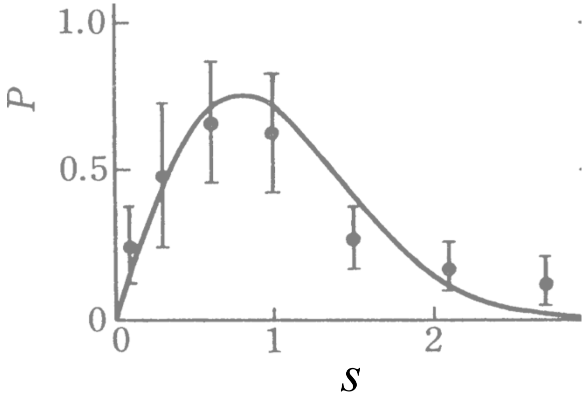
Wigner 1950s



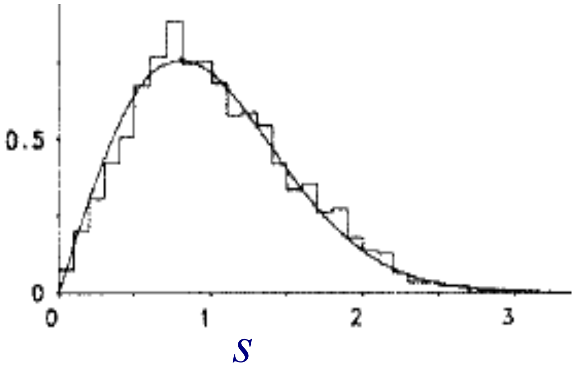
level spacing

Universality of spectral fluctuation

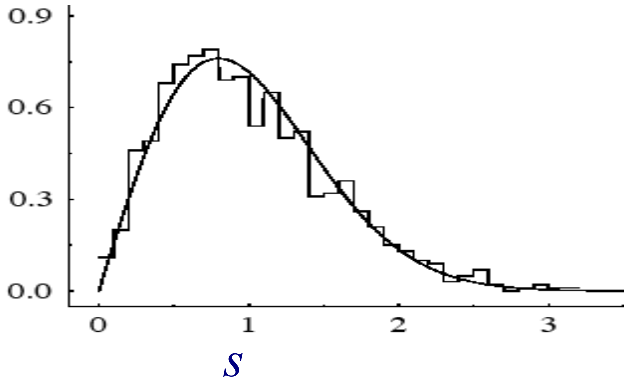
excited nuclei



H-atom in **E**-field



resonance of amethyst



chaotic billiard's H ··· quantum chaotic

Anderson tight-bind. H ··· disordered

stock correlation matrix , Mexico city bus , ···

$$P_{\mathrm{GOE}}(s) \cong \frac{\pi}{2}\, s\; \mathrm{e}^{-\frac{\pi}{4}s^2}$$

+ 9 more classes

Universality classes

$\sigma : \mathbf{C}\text{-herm.} \to \mathbf{C}\text{-herm.}, \quad \sigma^2 = 1$

Involutive symmetry $\quad H = \sigma(H)$

none	**C**-herm.	GUE
$\sigma(H) = H^*$	**R**-sym.	GOE
$\sigma(H) = \tau_2 H^* \tau_2^{-1}$	**H**-selfdual	GSE
$\sigma(H) = -\gamma_5 H \gamma_5^{-1}$		chG*E
$\cdots$	$\cdots$	

classification by Riemann symm. spaces

Zirnbauer 96

Universality classes

$\sigma : \mathbf{C}\text{-herm.} \rightarrow \mathbf{C}\text{-herm.}, \quad \sigma^2 = 1$

Involutive symmetry $\quad H = \sigma(H)$

$D_{\mathrm{U(1)}} = \gamma_\mu \left(i\partial_\mu + A_\mu \right) = -\gamma_5 D_{\mathrm{U(1)}} \gamma_5^{-1}$ same symm. as chGUE

$D_{\mathrm{SU(2)\ fnd}} = \gamma_\mu \left(i\partial_\mu + A_\mu^a \tau_a \right) = (\tau_2 C)\ D_{\mathrm{SU(2)\ fnd}}^{*} (\tau_2 C)^{-1}$ chGOE

$D_{\mathrm{SU(N)\ adj}} = \gamma_\mu \left(i\partial_\mu + A_\mu^a f_a \right) = C\ D_{\mathrm{SU(N)\ adj}}^{*}\ C^{-1}$ chGSE

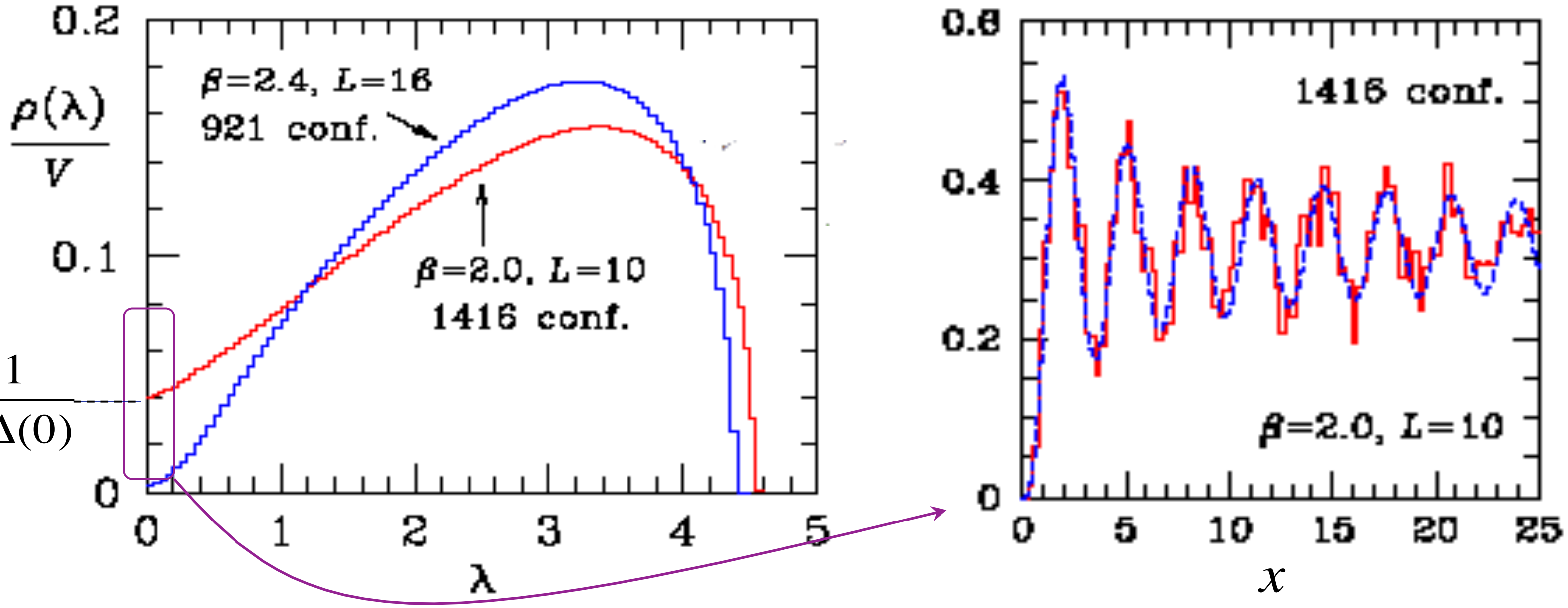
expect to agree w/ chG#E

Verbaarschot 94

Dirac spectrum

SU(2) fund KS

Berbenni-Bitsch et al 97



$$\frac{\langle\bar{\psi}\psi\rangle}{\pi} = \frac{1}{V\Delta(0)}$$

magnify by unfolding $x = \dfrac{\lambda}{\Delta(0)}$

match w/ chGSE ⇒

precise determin. $\langle\bar{\psi}\psi\rangle$

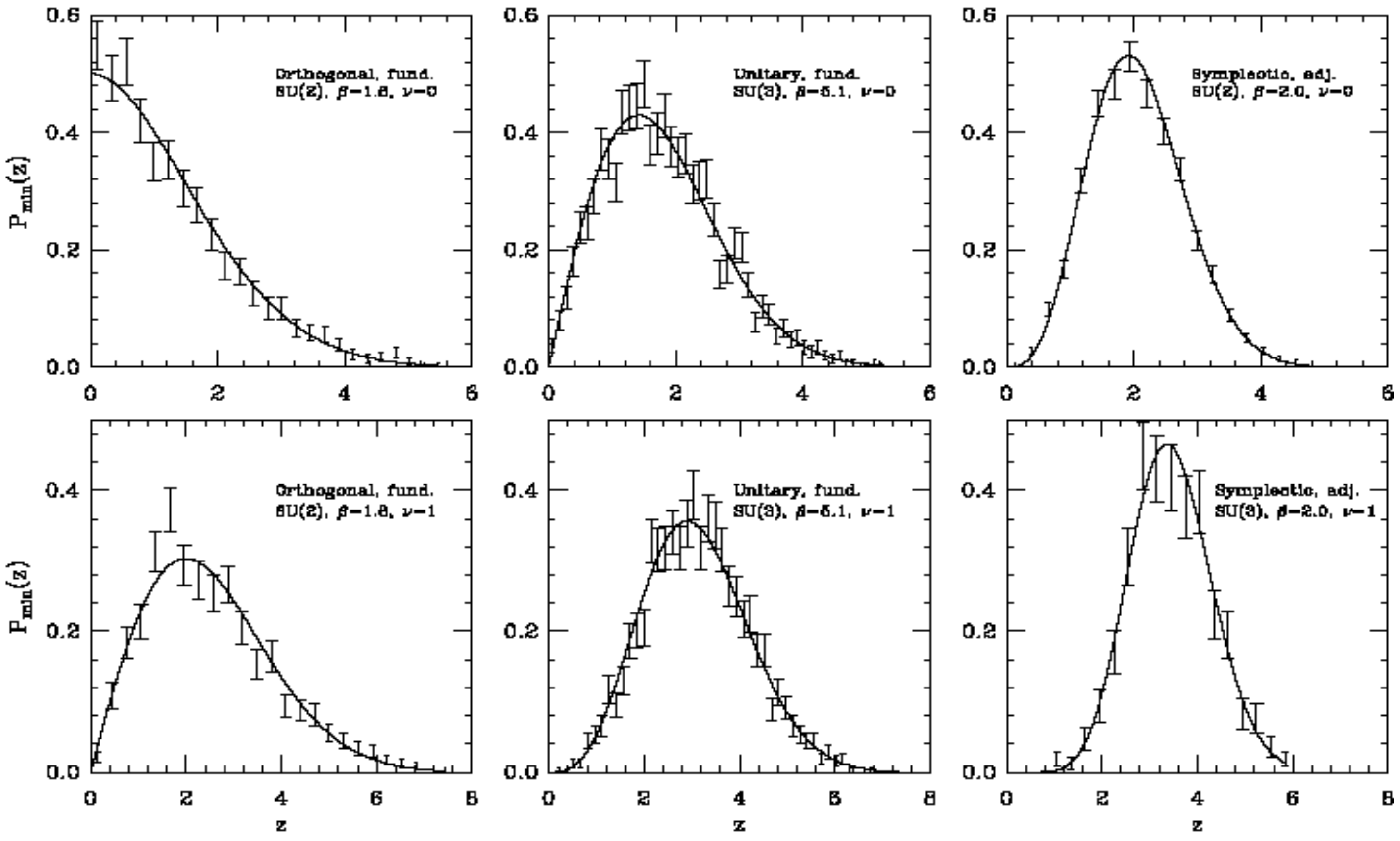
Smallest Dirac EV

SU(2 or 3) overlap
Edwards et al 99

SU(2) fund SU(3) fund SU(3) adj

ν=0

ν=1



match w/ chGOE, chGUE, chGSE

Origin of universality

spectral correlator NLσM

$$\begin{tikzcd}
\left\langle \det\left(\lambda - H\right)\cdots\right\rangle \arrow[r, dashed, shift left] \arrow[r, shift right] \arrow[dr, bend right] & \int_{\mathrm{RSS}} DQ\ \exp\left\{-\frac{\pi}{4\Delta V}\int d\mathbf{r}\left(\frac{D}{2}\mathrm{str}\ \left|\nabla Q\right|^{2} - i\ \mathrm{str}\ \Lambda Q\right)\right\} \arrow[d, "\varepsilon\text{ regime}"] \\
 & \int_{\mathrm{RSS}} dQ\ \exp\left\{i\frac{\pi}{4\Delta}\mathrm{str}\ \Lambda Q\right\}
\end{tikzcd}$$

⇢	QCD : flavor SSB	Weinberg 60s, Leutwyler-Smiga 92
→	AH : H-S transf, sdl pt	Wegner 81, Efetov 83
	QCD : strong coupling	Kawamoto-Smit 81, Nagao-Nishigaki 01
	gauge/gravity	?
→	RMT : H-S transf	Efetov 83
	QCh : sum over periodic orbit pairs	Muller et al 09

Origin of universality

spectral correlator NLσM

$$\begin{tikzcd}
\left\langle \det\left(\lambda - D\right)\cdots \right\rangle \arrow[r, dashed, shift left] \arrow[r, shift right] \arrow[dr, bend right] & \int_{\mathrm{SU}(N_{\mathrm{F}})} DU \ \exp\left\{-\int dx \left(\frac{f_\pi^2}{2} \ \mathrm{tr}\ \left|\partial_\mu U\right|^2 - i\ \Sigma\ \mathrm{Re\ tr}\ \Lambda U \right)\right\} \arrow[d, "\varepsilon \text{ regime}"] \\
 & \int_{\mathrm{SU}(N_{\mathrm{F}})} dU \ \exp\left\{ i\ V\Sigma\ \mathrm{Re\ tr}\ \Lambda U \right\}
\end{tikzcd}$$

⇢ QCD : flavor SSB Weinberg 60s, Leutwyler-Smiga 92

→ AH : H-S transf, sdl pt Wegner 81, Efetov 83

QCD : strong coupling Kawamoto-Smit 81, Nagao-Nishigaki 01

gauge/gravity ?

→ RMT technical advantage (orth. polyn.)

QCh : sum over periodic orbit pairs Muller et al 09

2. Parametric RMT

symmetry breaking perturb.

$$H = H_0 + \alpha H_1, \quad \sigma\left(H_0\right) = H_0, \ \ \sigma\left(H_1\right) \neq H_1$$

$$\alpha = O\left(\Delta\sqrt{V}\right)^{<1} \ \Rightarrow \ H_0 \text{ dominates}$$

$$\alpha = O\left(\Delta\sqrt{V}\right)^{>1} \ \Rightarrow \ H_1 \text{ dominates}$$

$$\alpha = O\left(\Delta\sqrt{V}\right)^{1} \ \ \Rightarrow \ \text{crossover, parametrized by } \rho \equiv \frac{\alpha}{\Delta\sqrt{V}}$$

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nonchiral	GOE, GSE→GUE	Pandey Mehta 83
chiral	chGOE, chGSE→chGUE	Nagao 03
supercond	GUE, CI→C	Koziy Skvortsov 11

Parametric RMT

spectral correlator NLσM

$$\left\langle \det(\lambda - H)\cdots \right\rangle \begin{array}{c}\dashrightarrow \\ \longrightarrow\end{array} \int_{\mathrm{RSS}} dU\, \mathrm{e}^{-S[U]}$$

$$S[U] = i\frac{\pi}{2}\,\mathrm{tr}\,\underbrace{X}_{\text{unf. EV}}\,U \;-\; \underbrace{\frac{\pi^2\rho^2}{4}}_{\text{Xover parameter}}\,\mathrm{tr}\,(U\tau_3)^2$$

- the same NLσM as 2C chiral L at finite density
- fit the spectrum to pRMT = measure the LEC ρ

$\dashrightarrow$ QCD : flavor SSB — Kogut-Stephanov-Toublan 00, Dunne-Nishigaki 03

$\longrightarrow$ AH・RMT : H-S transf — Altland-Iida-Efetov 93

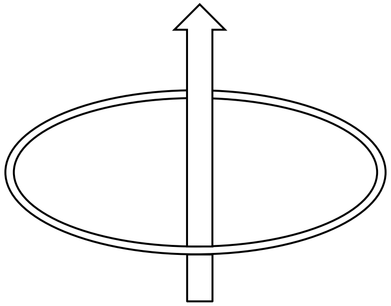
QCD [SC, gauge/gravity] — ?

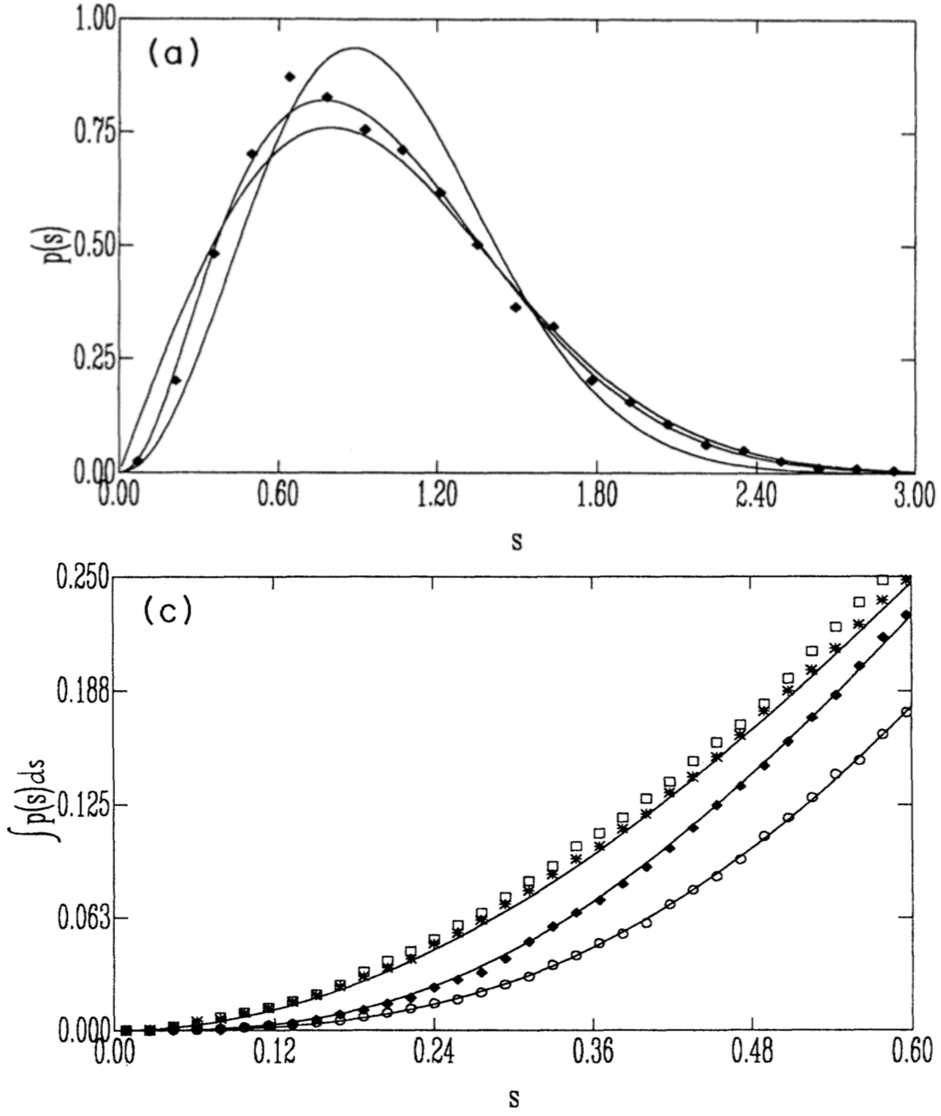
QCh : sum over p.o. pairs — Saito-Nagao-Muller-Braun 09

Fit to param. RMT

Dupuis-Montambaux 91

disordered ring
+ AB flux





caution: $P(s)$ curve is just
Wigner-like approximant

counterpart in
SU(2)×U(1) Dirac spectra?

Parametric chRMT : the results

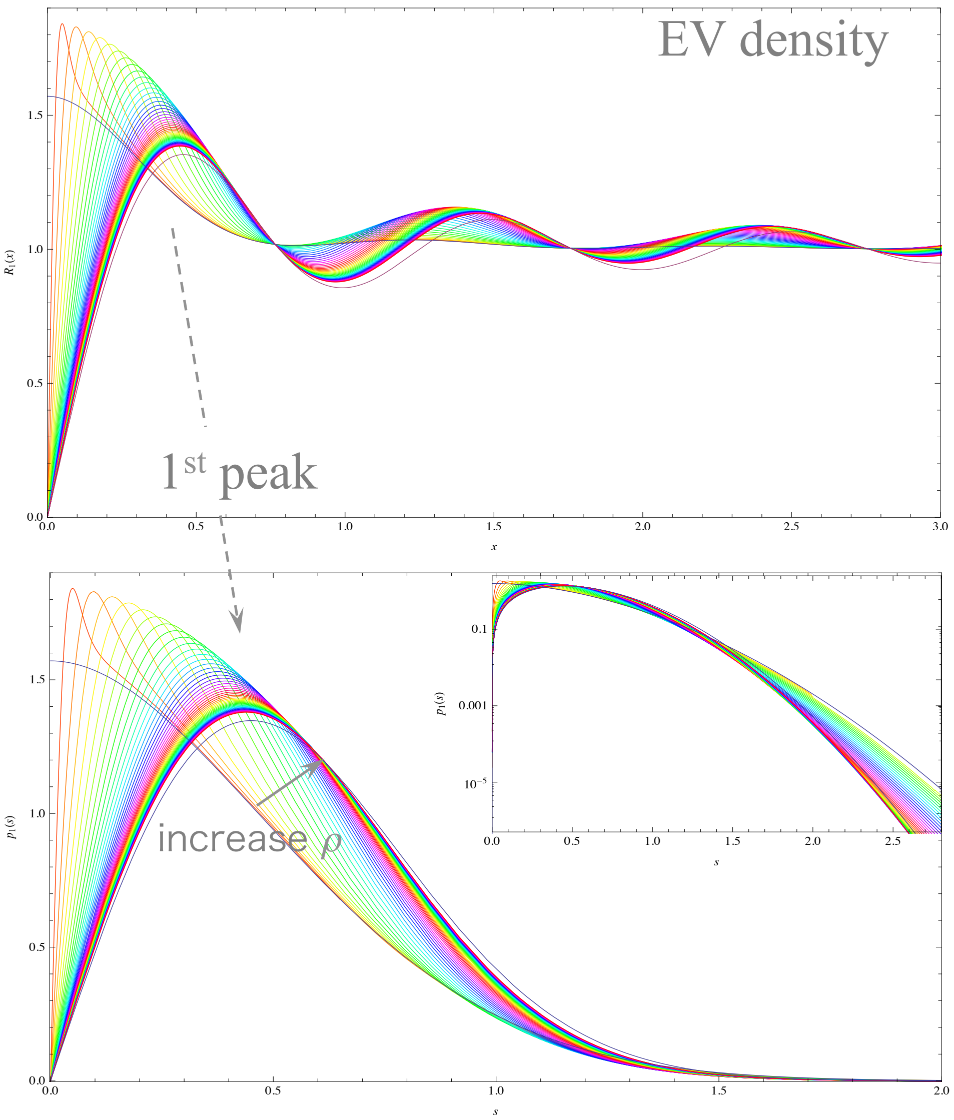
by using Nystrøm-type method with Gauss-Legendre quadratures of computing Fredholm Pfaffian of the matrix-valued dynamical Bessel kernels, I have computed and plotted

the 1st EV distributions for chGOE-chGUE, chGSE-chGUE xover

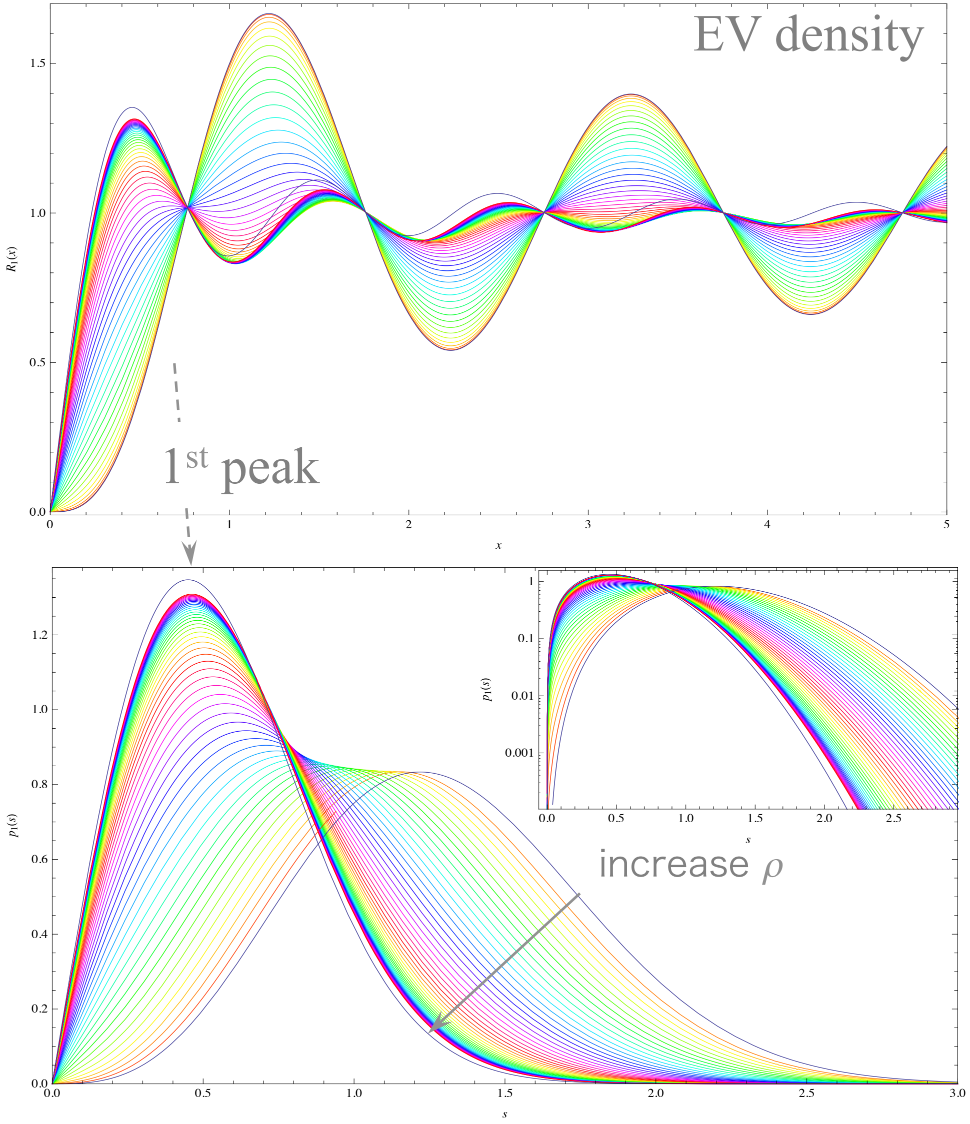
$$H = \begin{pmatrix} & \mathbf{R} \\ \mathbf{R}^T & \end{pmatrix} + \alpha \begin{pmatrix} & \mathbf{C} \\ \mathbf{C}^+ & \end{pmatrix}, \quad \begin{pmatrix} & \mathbf{H} \\ \mathbf{H}^D & \end{pmatrix} + \alpha \begin{pmatrix} & \mathbf{C} \\ \mathbf{C}^+ & \end{pmatrix}$$

$$\mathbf{R},\ \mathbf{C},\ \mathbf{H} : \text{Gauss random } N \times N$$

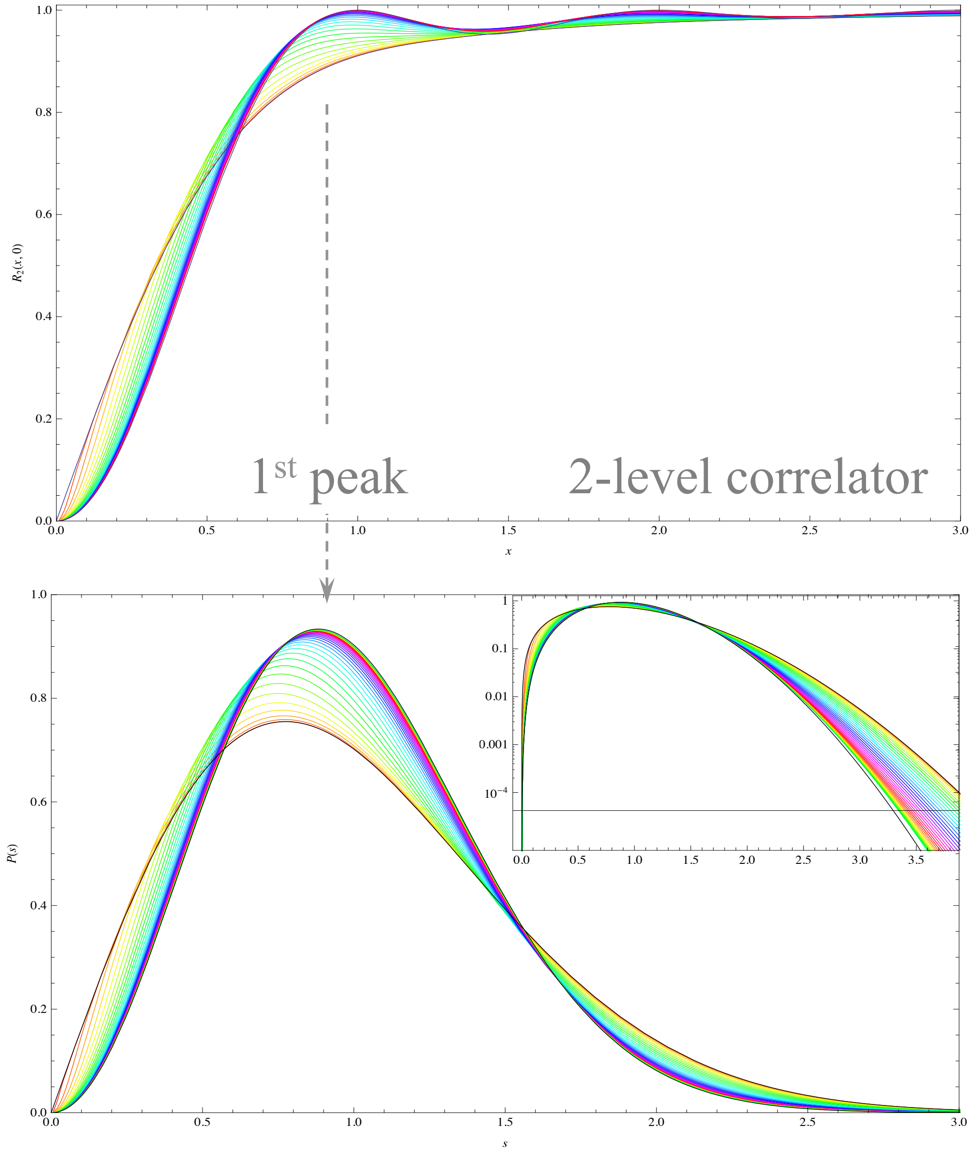
chGOE-chGUE



chGSE-chGUE



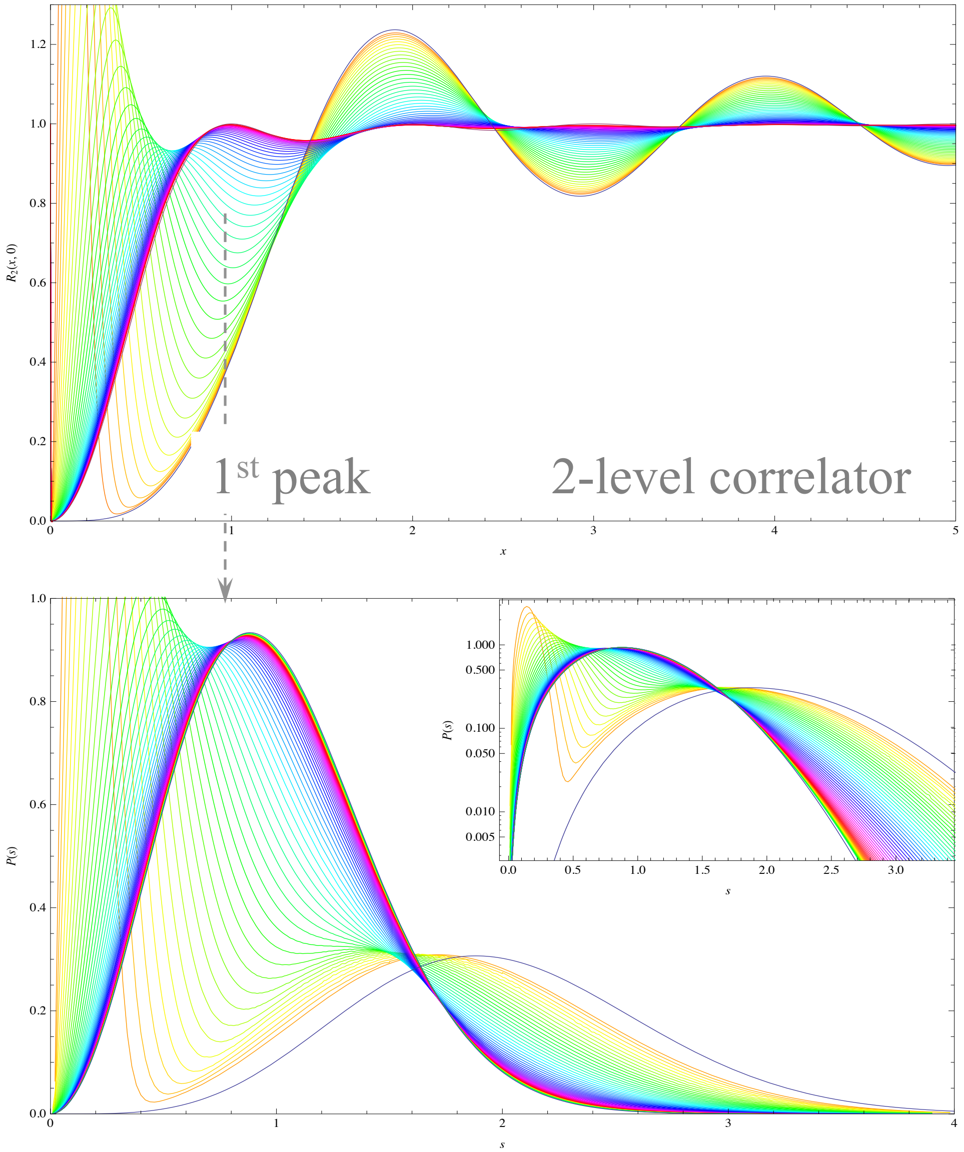
GOE-GUE



1st peak

2-level correlator

GSE-GUE



1st peak

2-level correlator

3. SU(2)×U(1) Lattice Dirac Spectrum

Open question : does the chG#E-chGUE xover occur in physics?

partly inspired by the recent trend on isospin-related physics :

QCD+QED simulation, measure Dirac spectra of 2C-QCD+QED

$$D = \gamma_\mu \left(i\partial_\mu + g\, A_\mu^a T_a + e\, A_\mu \right)$$

U(1) : symmetry breaker

- can be fitted to the parametric chRMT at all?
- can determine the symmetry-breaking LEC ρ accurately?
- if yes, does ρ scale as expected?

SU(2)×U(1) Lattice Dirac Spectrum

Strategy : smaller lattice $V=4^4\sim6^4$, more samples $O(10^4)$

- toy model for scaling check:

  SU(2) random x U(1) noise $\theta_{x,\mu}\in[-p,p]$ i.i.d.

  $$\beta_2=0,\quad p=0.01\sim10$$

- quenched SU(2) x U(1)

  $$\{\beta_2=0,0.5,\ 1\}\ \times\ \{\beta_1=1000\sim3000\}$$

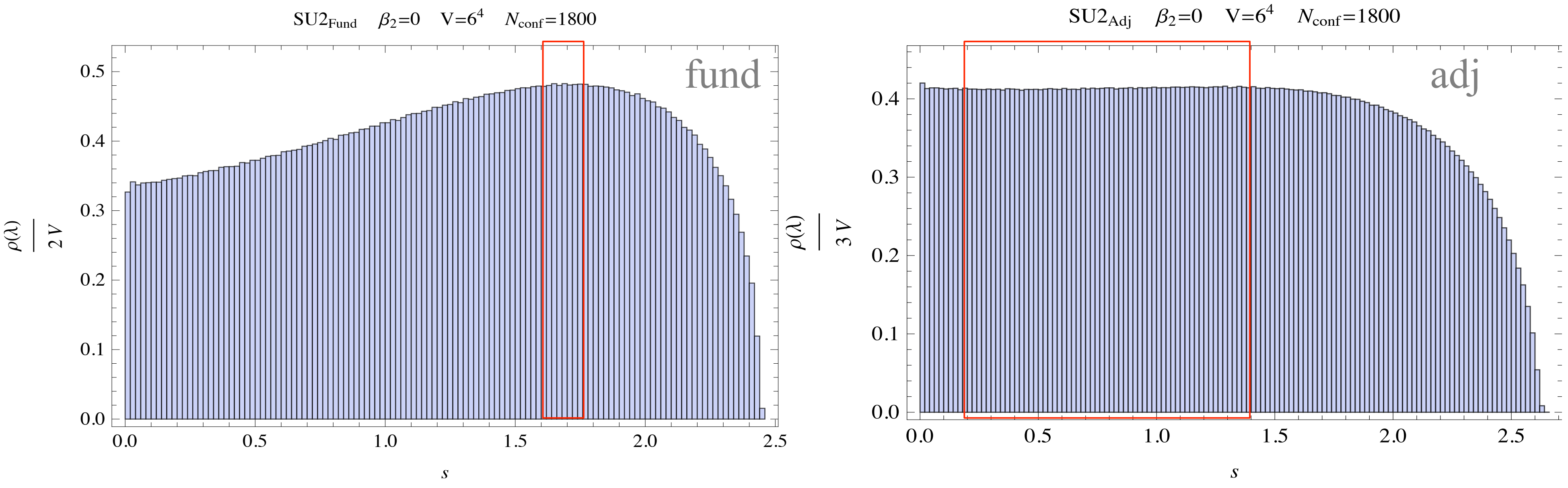
  strong cpl extremely weak cpl

fund & adj

KS Dirac

spectrum

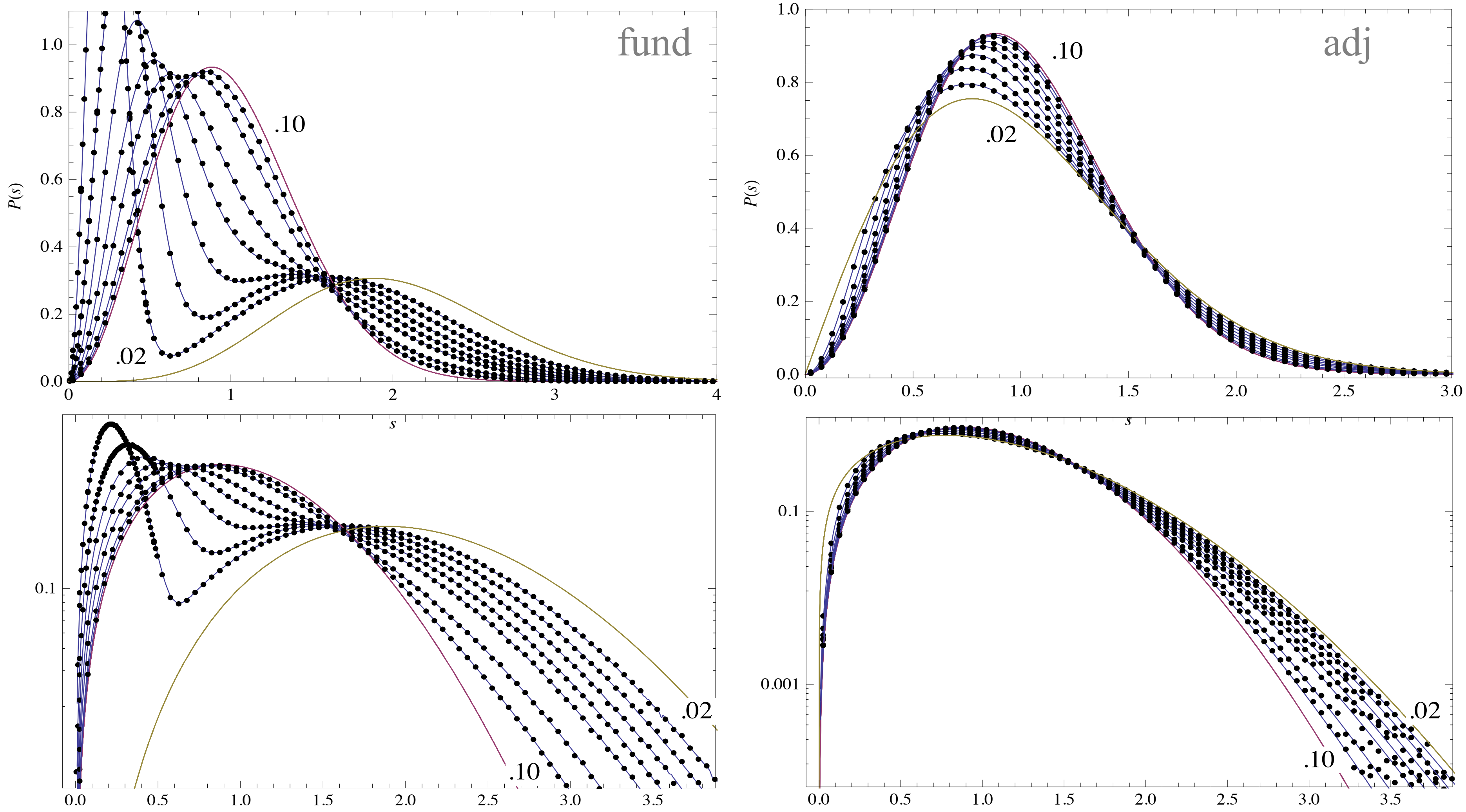
global SU(2) Dirac spectrum



- global spectrum of SU(2) intact w/ weak U(1)
- parameter ρ depend on mean spacing Δ ⇒ choose a plateau from the global spectrum for best bulk result

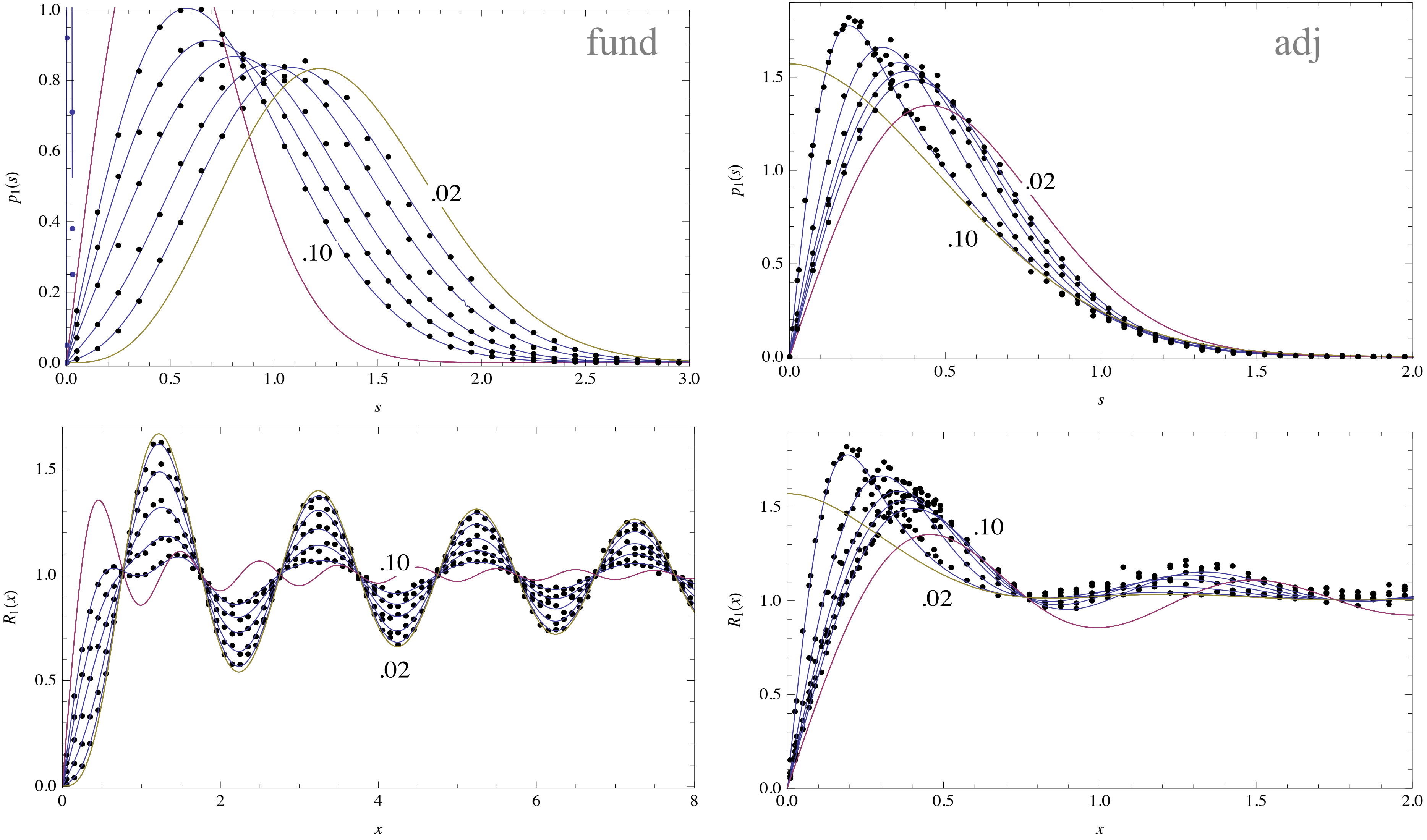
SU(2)×U(1) noise model - bulk

$V = 4^4,\ N_{\mathrm{conf}} = 4\mathrm{e}4,\ \beta_2 = 0,\ p = 0.02{\sim}10$



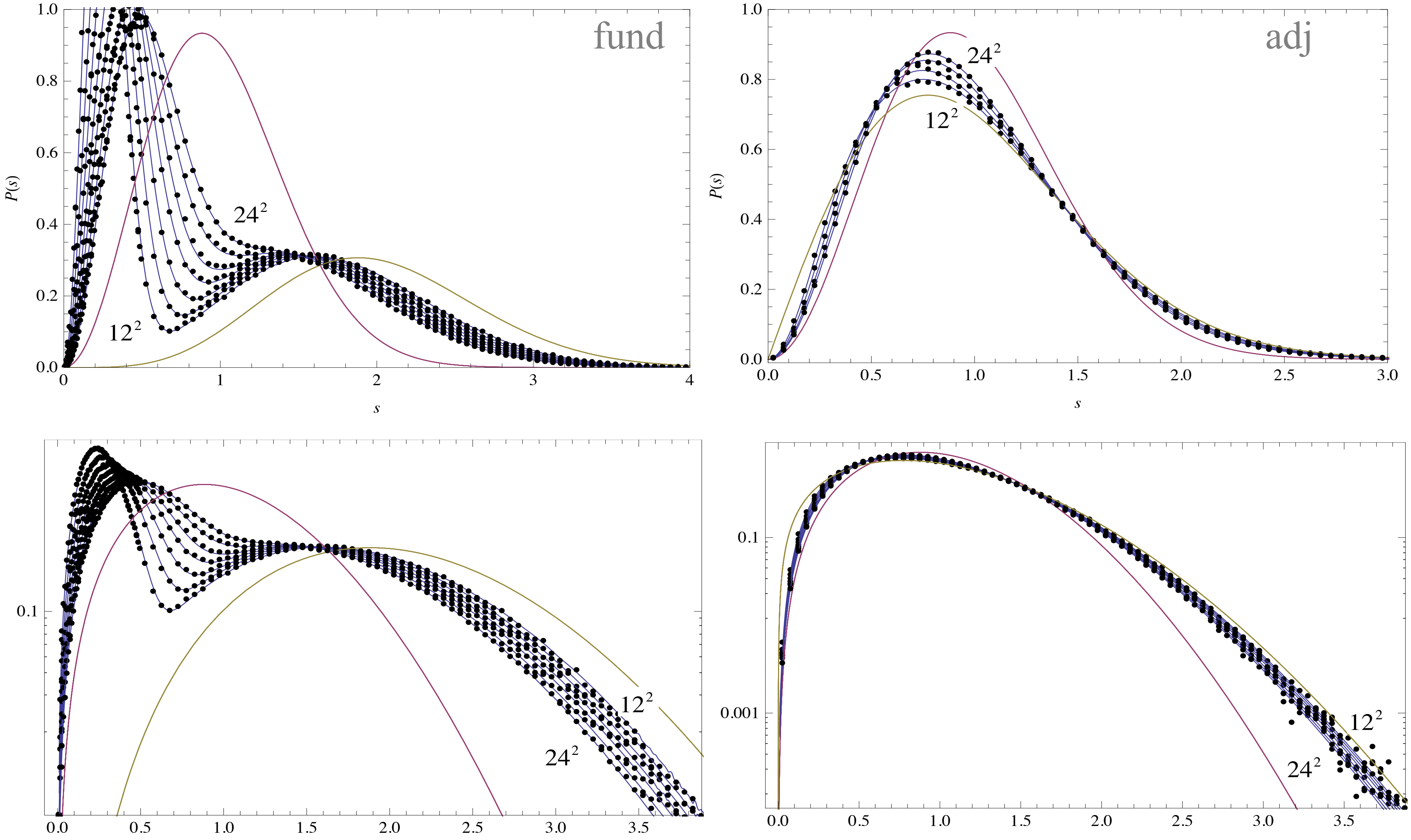
SU(2)×U(1) noise model - origin

$V = 4^4,\ N_{\text{conf}} = 4\text{e}4,\ \beta_2 = 0,\ p = 0.02\sim10$

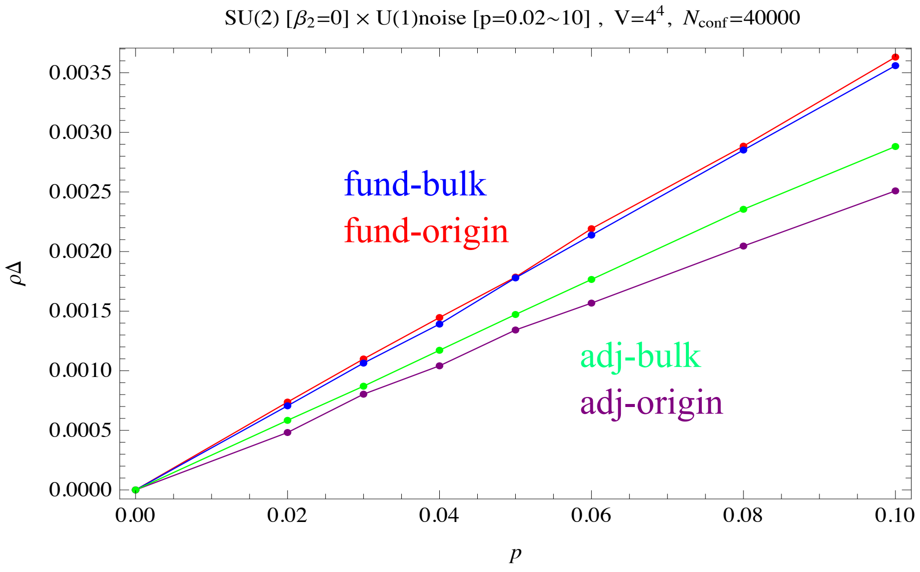


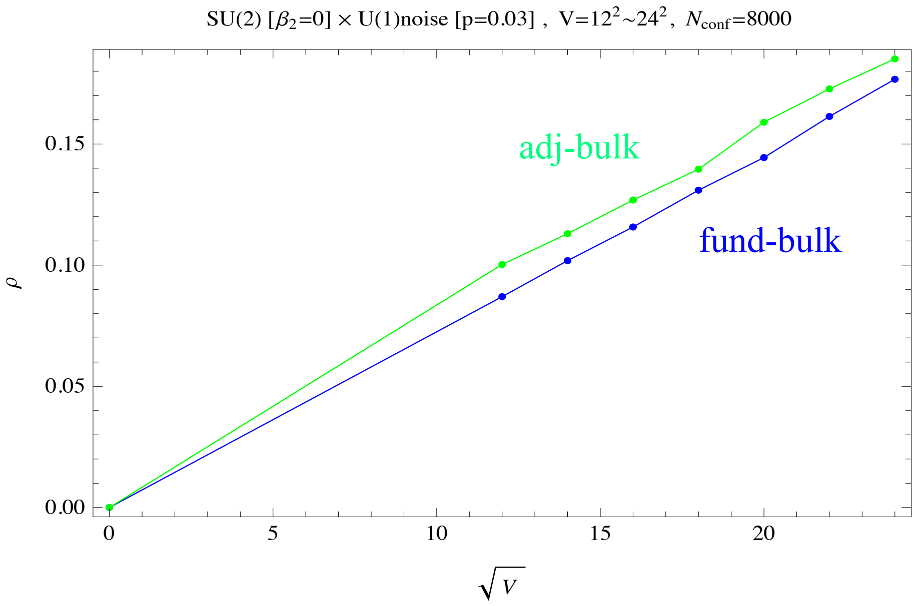
SU(2)×U(1) noise model - bulk

$V=12^2\sim24^2,\ N_{\text{conf}}=8\text{e}3,\ \beta_2=0,\ p=0.03$



SU(2)×U(1) noise model


SU(2) [β_2=0] × U(1)noise [p=0.02~10] , V=4^4, $N_{\rm conf}$=40000

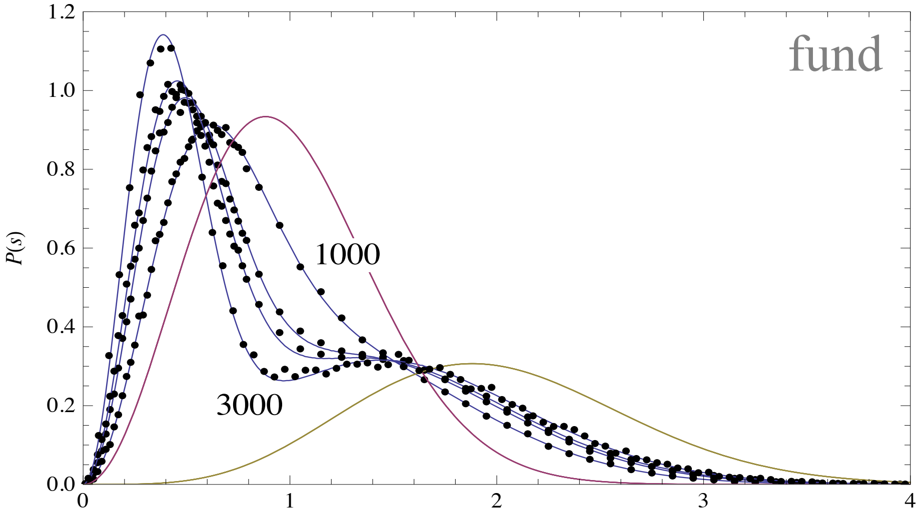

SU(2) [β_2=0] × U(1)noise [p=0.03] , V=12^2~24^2, $N_{\rm conf}$=8000

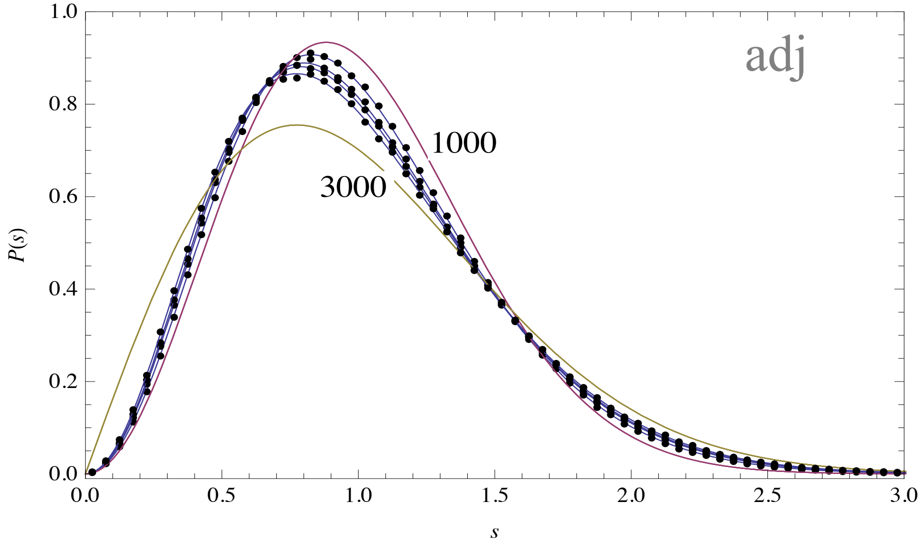
- perfectly fittable, accurate determine ρ
- Xover parameter ρ scales as expected :

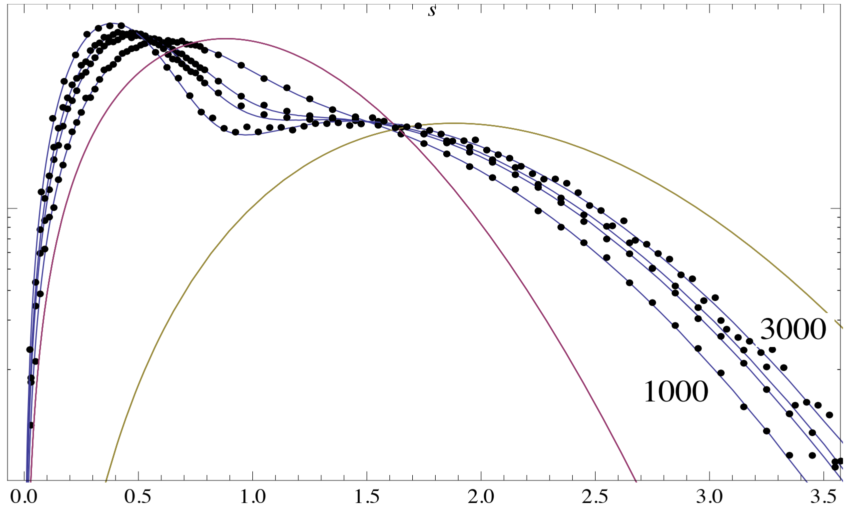
$$\rho = \underset{\substack{\uparrow \\ \text{LEC}}}{\text{const}}\,\frac{p}{\Delta\sqrt{V}}$$

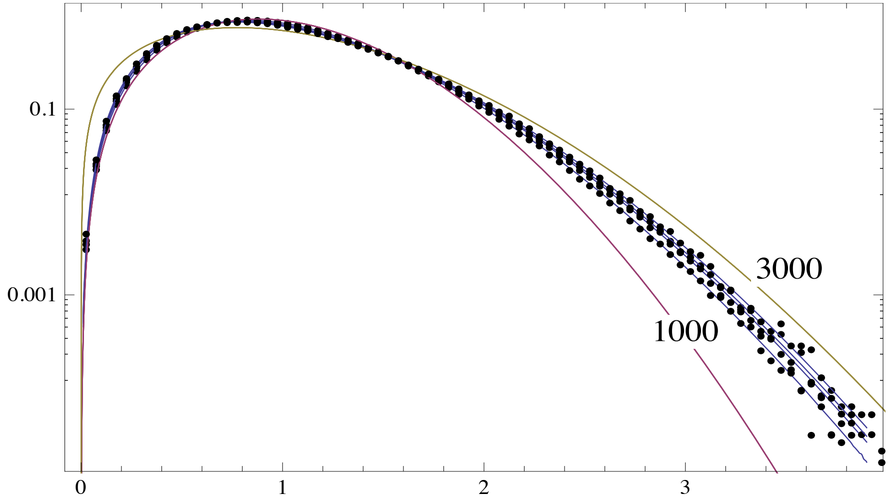
SU(2)×U(1) LGT - bulk

$V = 6^4,\ N_{\mathrm{conf}} = 1800,\ \beta_2 = 1,\ \beta_1 = 1000 \sim 3000$



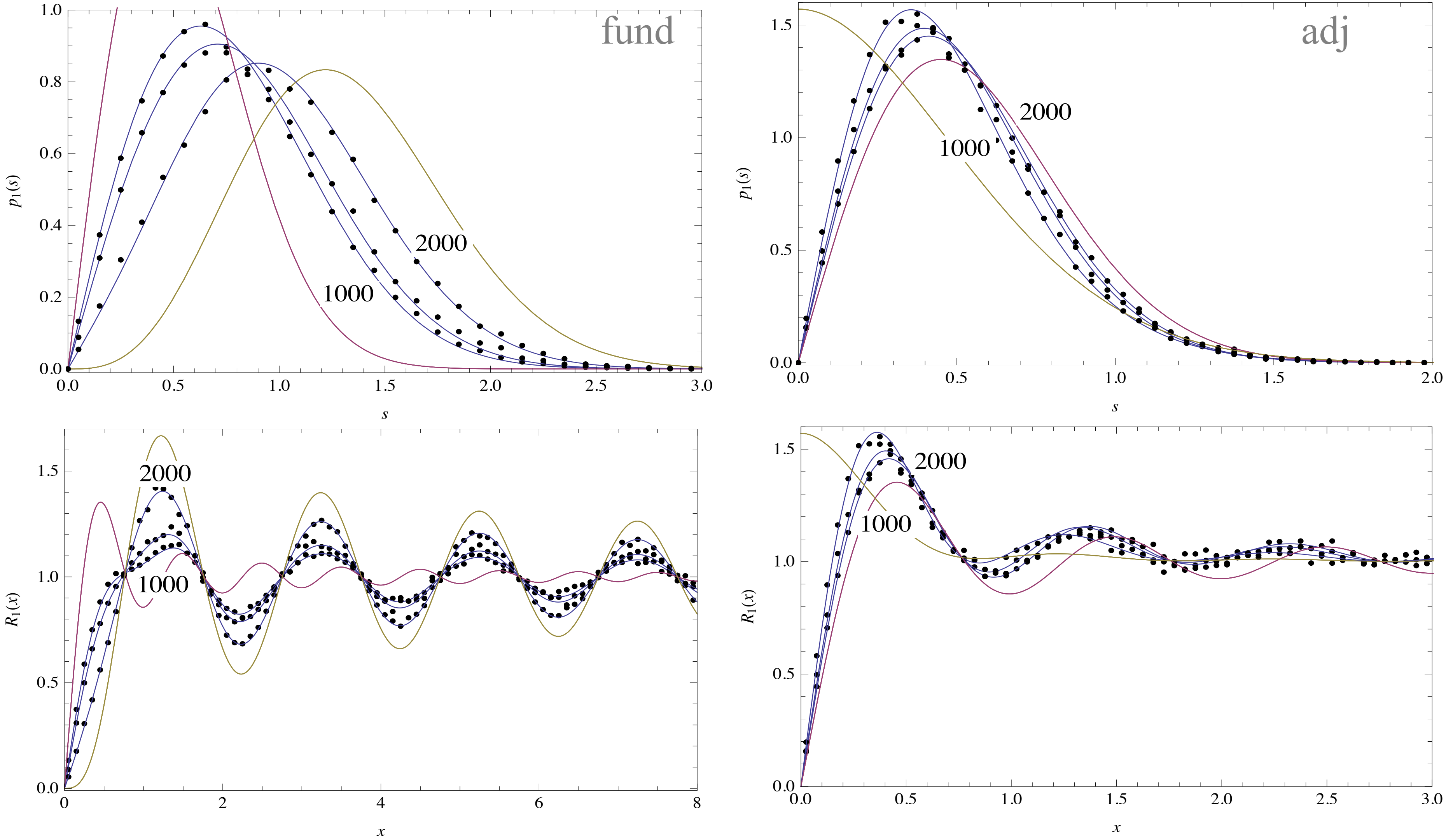




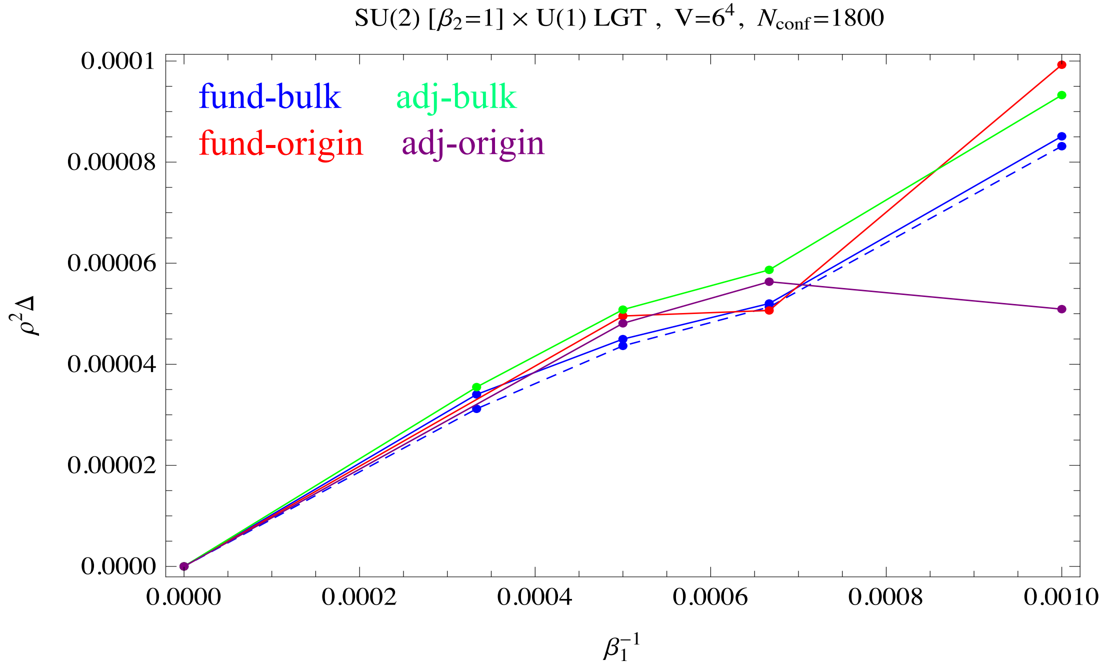


SU(2)×U(1) LGT - origin

$V = 4^4,\ N_{\text{conf}} = 40000,\ \beta_2 = 1,\ \beta_1 = 1000, 1500, 2000$



SU(2)×U(1) LGT



- fittable, fair determin. ρ
- Xover parameter ρ scales as expected :

$$\rho = \underset{\substack{\uparrow \\ \text{LEC}}}{\text{const}}\,\frac{\beta_1^{-1/2}}{\Delta\sqrt{V}}$$

4. Summary

☐ computed/plotted

- smallest EV distr for chG(O,S)E→chGUE crossover
- level spacing distr for G(O,S)E→ GUE crossover

☐ measured the SU(2)×U(1) KS Dirac spectra for

- random SU(2) flux × U(1) noise toy model
- pure LGT at strong β_2 × very weak β_1

☐ fitting and scaling

- parametric (ch)RMT fits SU(2)×U(1) well in all cases
- accurate measurement of crossover parameter ρ
  expected scaling ⇒ nontrivial continuum limit
  ⇒ determine LEC for the symm break. term in $L_{\rm ch}$

thanks to Taro Nagao @Nagoya