Brane-anti-brane Annihilations in Field Theory and BEC

基研研究会「場の理論と弦理論」@京大基礎物理学研究所

2012年7月24日

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(慶應義塾大学/Keio U.)

Topological Quantum Phenomena in
Condensed Matter with Broken Symmetries

Keio University
1858
CALAMVS
GLADIO
FORTIOR

Field Theory

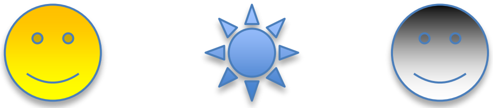
① **MN, Phys.Rev.D85 (2012) 101702** [arXiv:1205.2442 [hep-th]]
② **MN, Phys.Rev.D85 (2012) 121701** [arXiv:1205.2443 [hep-th]]
③ **MN, arXiv:1206.5551 [hep-th]**

Bose-Einstein Condensates(BEC)

Hiromitsu Takeuchi (Hiroshima U.)
Kenichi Kasamatsu(Kinki U.), Makoto Tsubota (Osaka City U.)

① **Phys.Rev.A85(2012)053639**[arXiv:1203.4896 [cond-mat.quant-gas
② **arXiv:1205.2330 [cond-mat.quant-gas]**
③ **J.Low.Temp.Phys.162(2011)243** [arXiv:1205.2328 [cond-mat.quant-gas]]
④ **JHEP 1011 (2010) 068** [arXiv:1002.4265 [cond-mat.quant-gas]]

Pair annihilations of particle and anti-particle (hole) turn to energy.



How about extended objects?

In string theory, pair annihilations of D-brane and anti-D-brane were studied extensively. A.Sen etc

D*p*-brane + anti-D*p*-brane → D(*p*-2)-branes

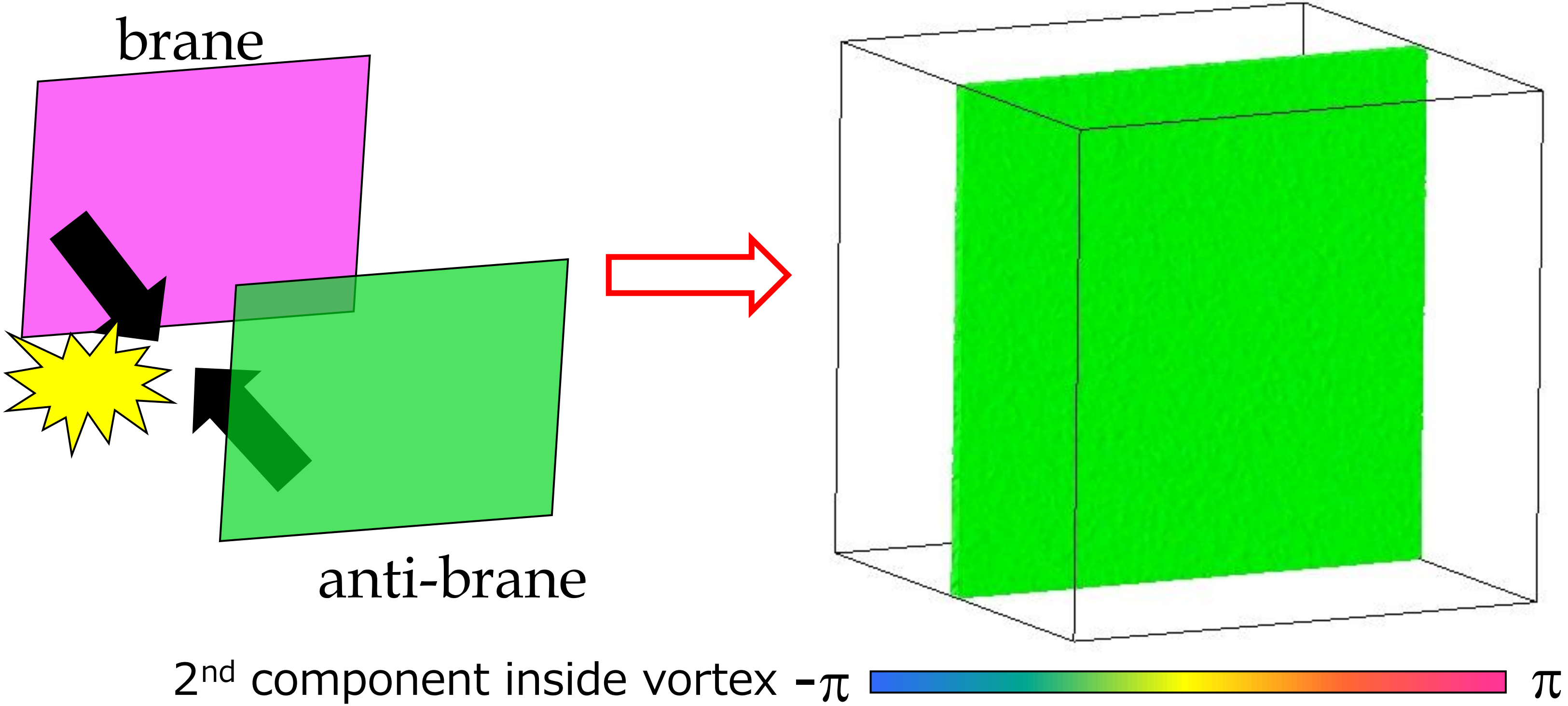
How about extended objects in field theory (& cond-mat)?

Brane-anti-brane annihilation in BEC

Takeuchi-Ksamatsu-MN-Tsubota, J.Low.Temp.Phys.162(2011)243
[arXiv:1205.2328 [cond-mat.quant-gas]]

Simulation by Takeuchi

closed string production by brane pair annihilation



Plan of my talk

§ 1 Introduction (2p)

§ 2 Domain wall annihilation (12+3p)

§ 3 Monopole-string annihilation (6+1p)

§ 4 Knot/Vorton/Knotted instanton (4+3p)

§ 5 Conclusion (1p)

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O(3) sigma model

1. (Truncated model of) 2component BECs
2. Ferromagnet

$$\mathcal{L} = \frac{1}{2}\partial_\mu \mathbf{n}\cdot\partial^\mu \mathbf{n} - \boxed{m^2(1-n_3^2)}$$

$$\mathbf{n}(\mathbf{x})=(\mathbf{n}_1,\mathbf{n}_2,\mathbf{n}_3) \quad \mathbf{n}^2 = 1$$

Target space = S^2

equivalent to

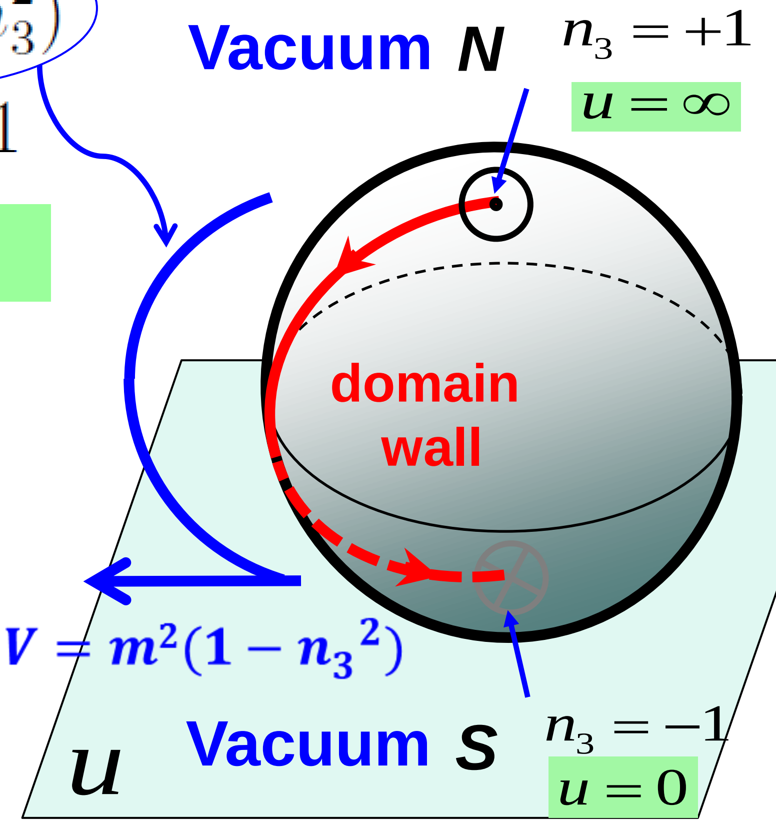
CP¹ model

$$\mathcal{L} = \frac{\partial_\mu u^* \partial^\mu u - m^2|u|^2}{(1+|u|^2)^2}$$

Stereographic coordinate u

$$u = \frac{n_x - in_y}{1-n_z}$$

$$\Phi^T = (1,u)/\sqrt{1+|u|^2} \qquad \mathbf{n} = \Phi^\dagger \sigma \Phi$$



Vacuum N $n_3 = +1$ $u = \infty$

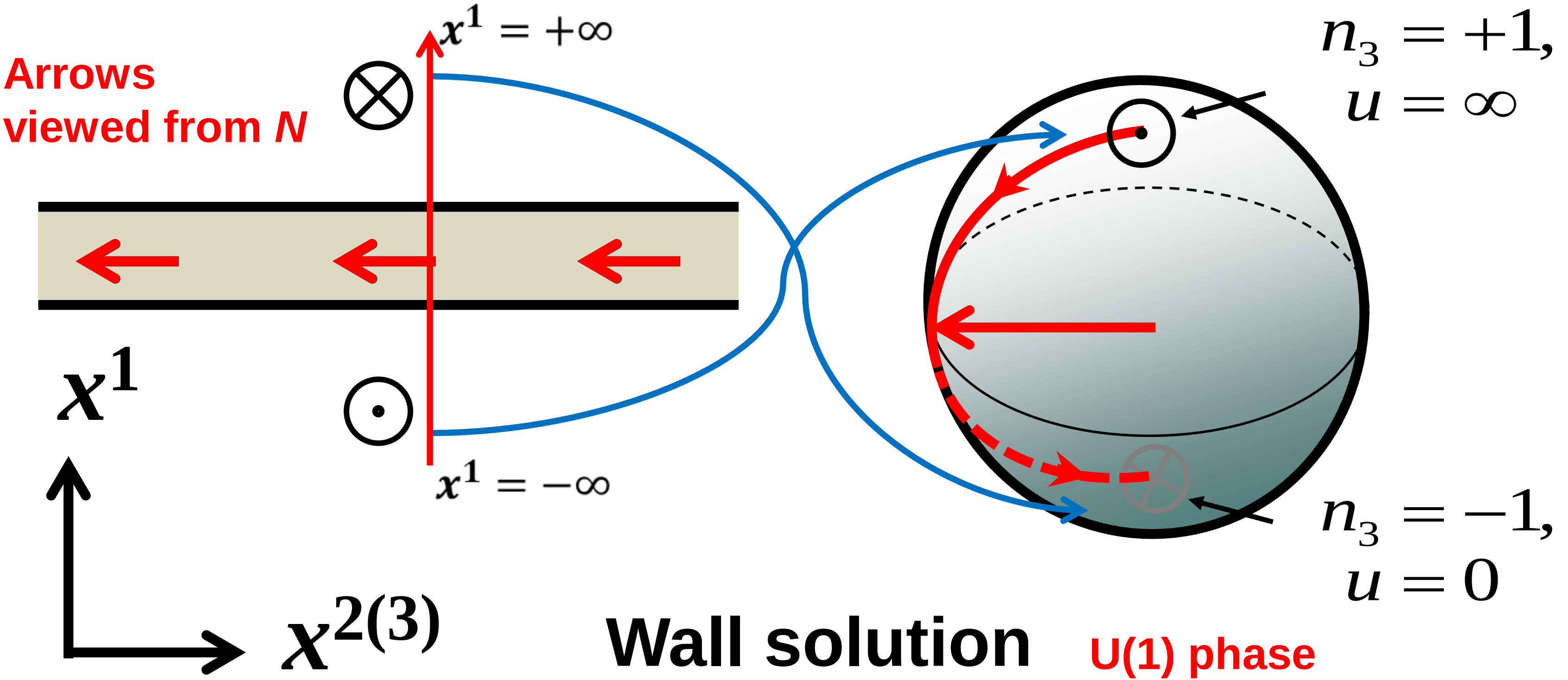
domain wall

$V = m^2(1-n_3{}^2)$

Vacuum S $n_3 = -1$ $u = 0$

u

Single domain wall



Wall solution

U(1) phase

$$u_{\rm W} = e^{\mp m(x^1 - x^1_0) + i\varphi}$$

Bogomol'nyi completion for domain wall

$$E = \int dx^1 \frac{\sum_\alpha |\partial_\alpha u|^2 + m^2 |u|^2}{\left(1+|u|^2\right)^2}$$

$$= \int dx^1 \left[\frac{|\partial_1 u \mp 2mu|^2}{\left(1+|u|^2\right)^2} \pm \frac{2m\left(u^* \partial_1 u + u \partial_1 u^*\right)}{\left(1+|u|^2\right)^2} \right]$$

$$\geq |T_W|$$

Topological charge

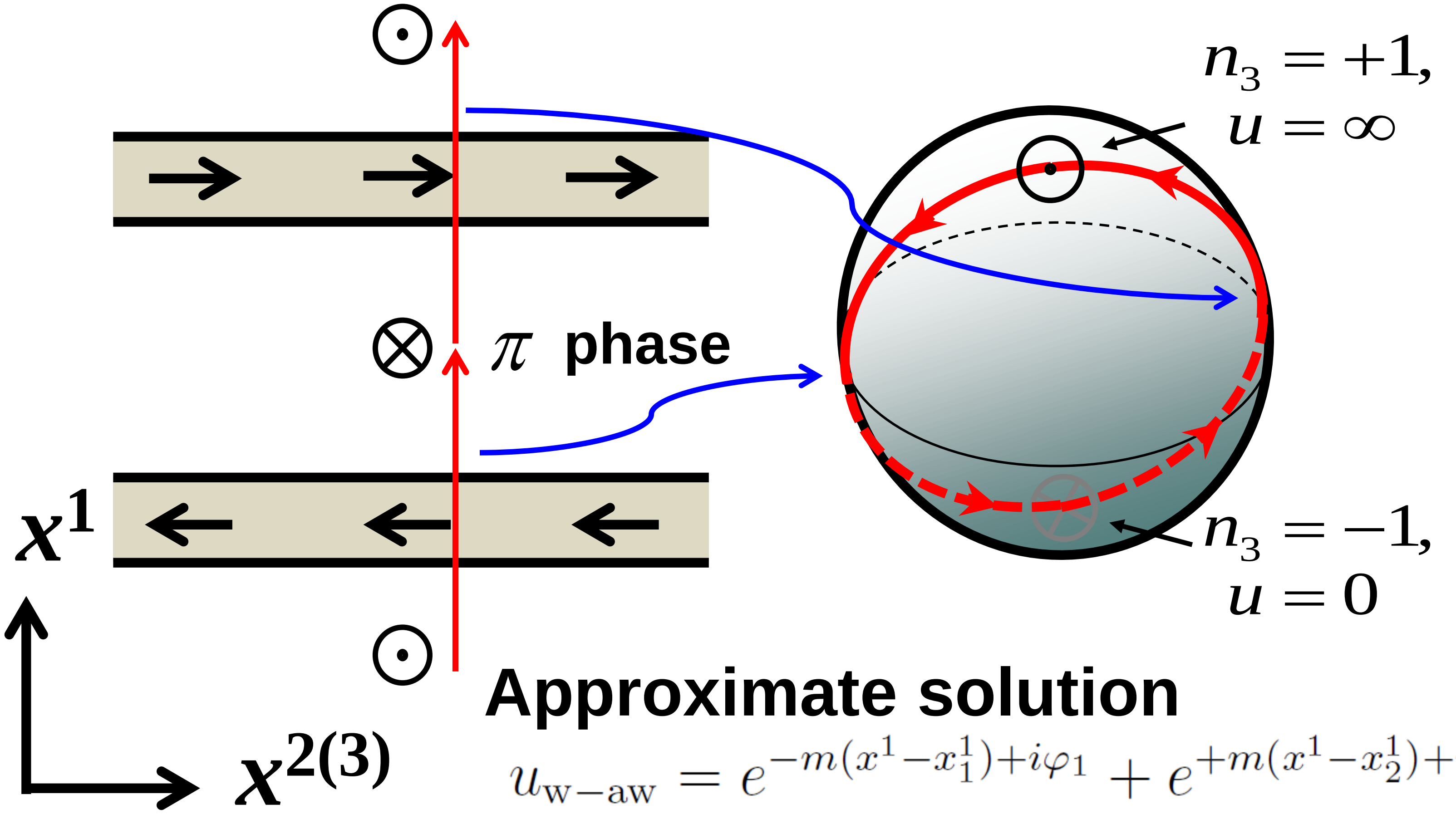
$$T_W = \pm \int dx^1 \frac{2m\left(u^* \partial_z u + u \partial_z u^*\right)}{\left(1+|u|^2\right)^2}$$

$$= \pm m \int dx^1 \partial_1 \left(\frac{1-|u|^2}{1+|u|^2} \right) = \pm m \left[\frac{1-|u|^2}{1+|u|^2} \right]_{x^1=-\infty}^{x^1=+\infty}$$

BPS equation

$$\partial_1 u \mp mu = 0$$

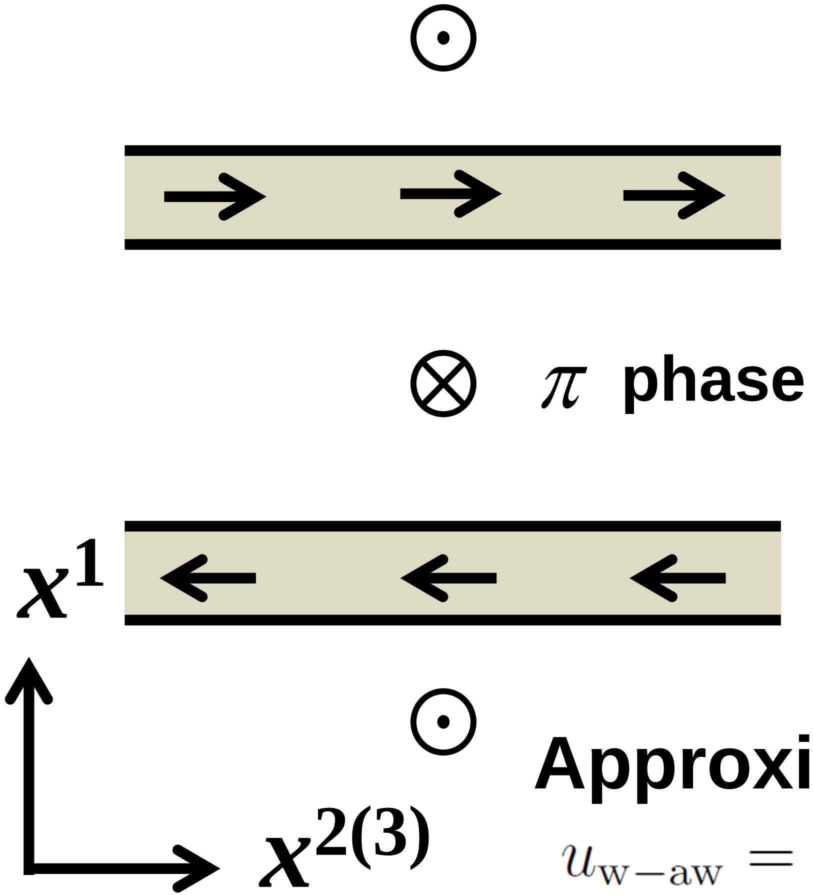
A pair of a domain wall and an anti-domain wall



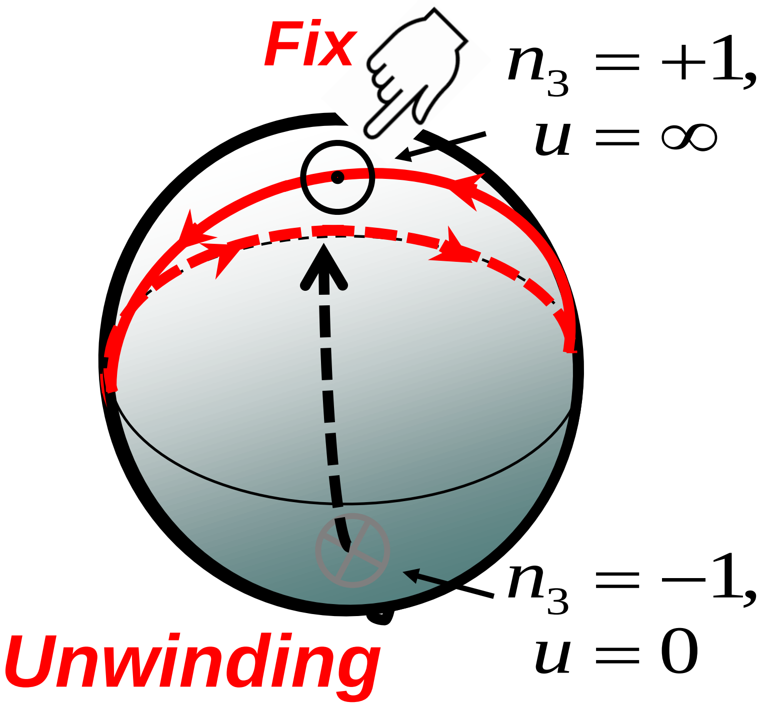
Approximate solution

$$u_{\mathrm{w-aw}} = e^{-m(x^1-x^1_1)+i\varphi_1} + e^{+m(x^1-x^1_2)+i\varphi_2}$$

A pair of a domain wall and an anti-domain wall



π phase



Fix

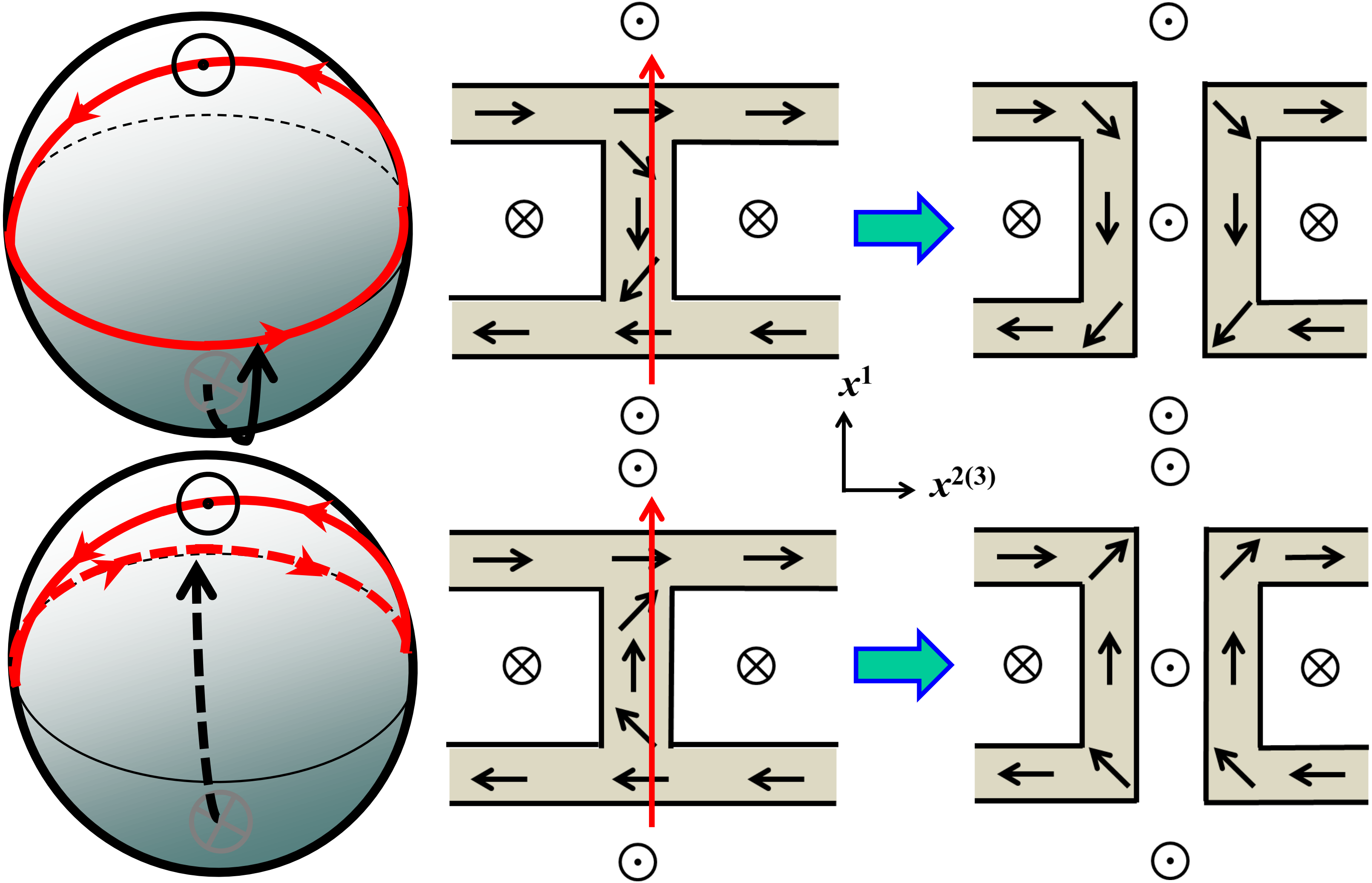
$n_3 = +1,$
$u = \infty$

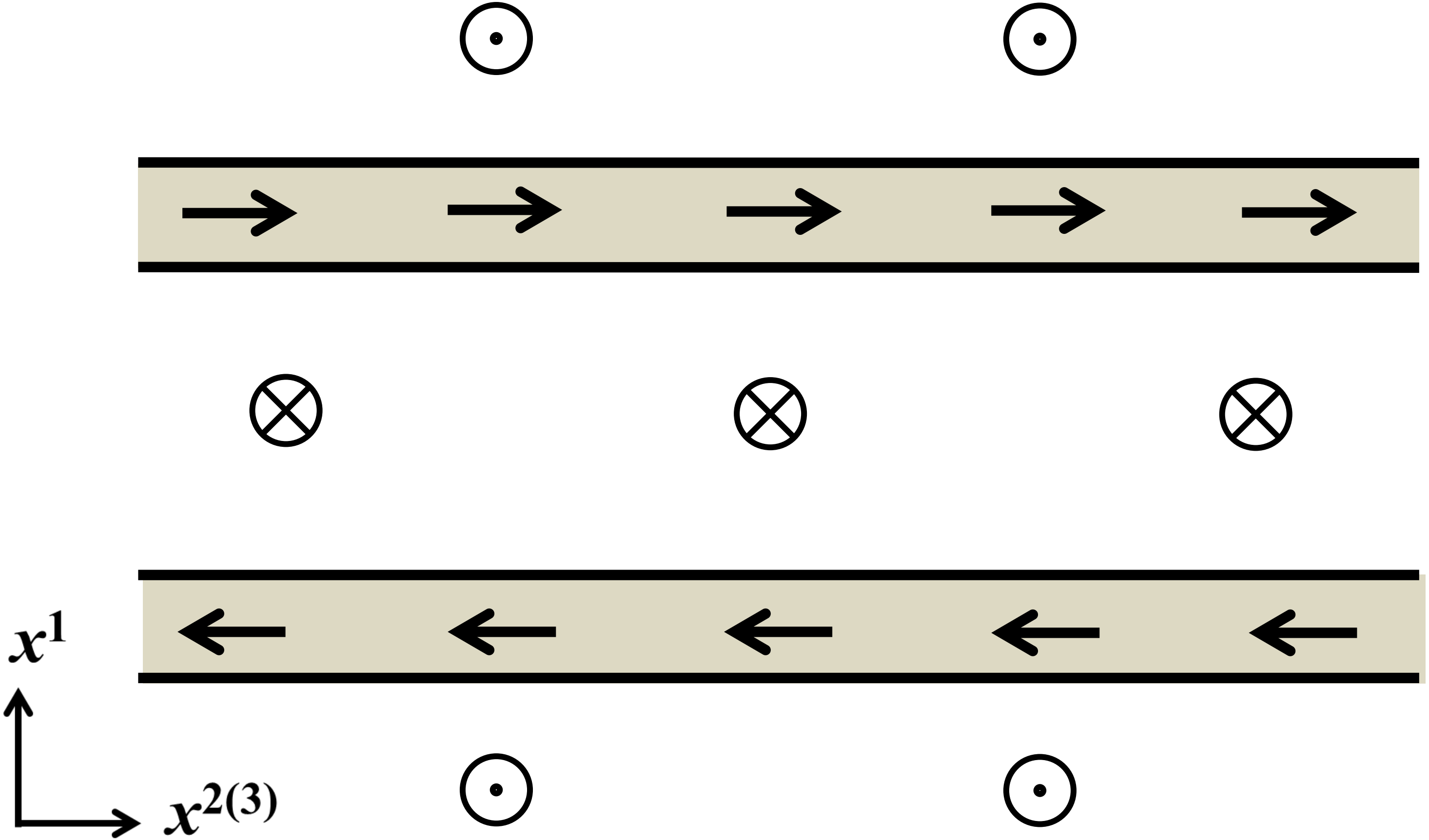
$n_3 = -1,$
$u = 0$

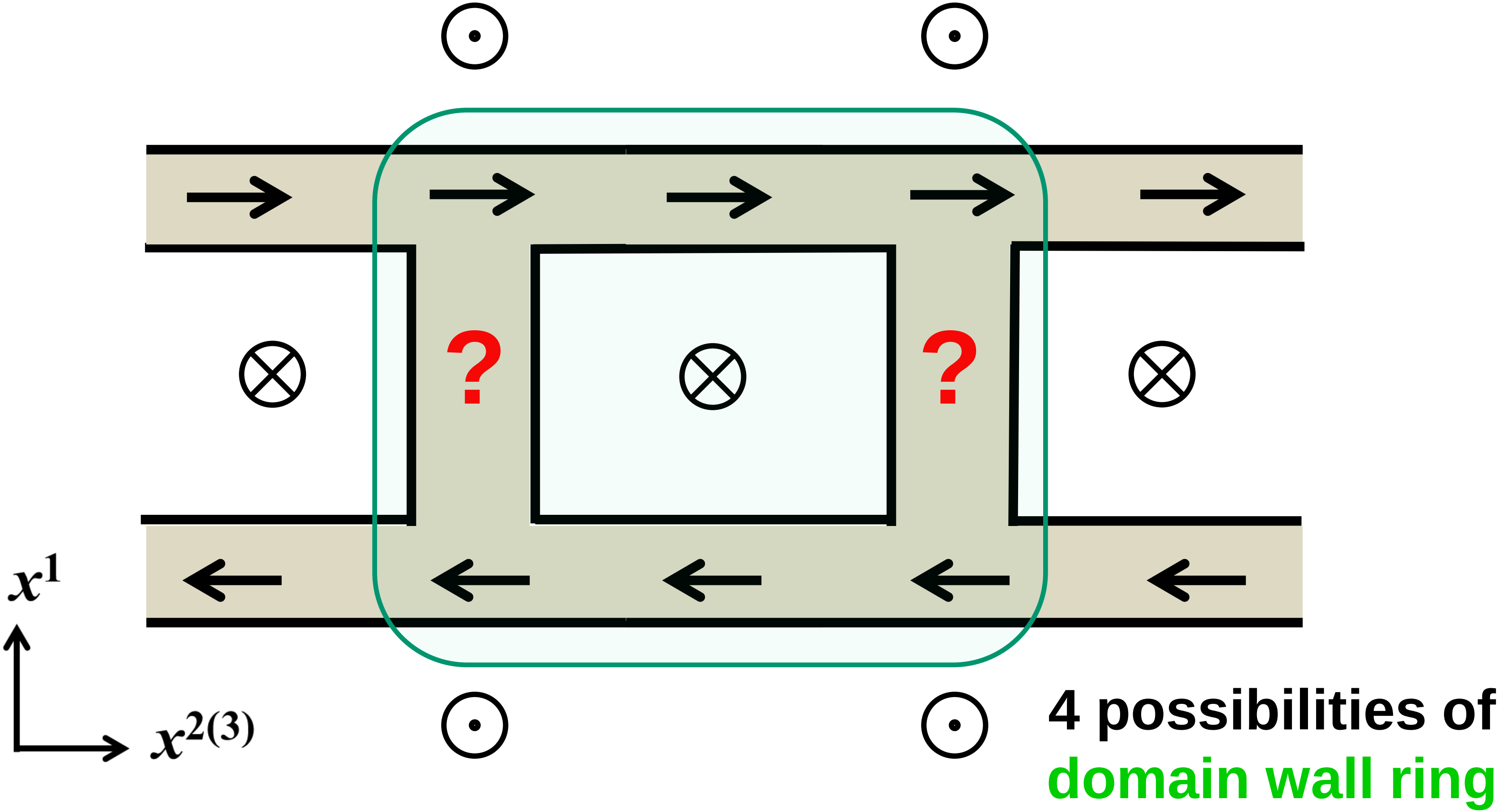
Unwinding

Approximate solution

$$u_{\rm w-aw} = e^{-m(x^1-x^1_1)+i\varphi_1} + e^{+m(x^1-x^1_2)+i\varphi_2}$$

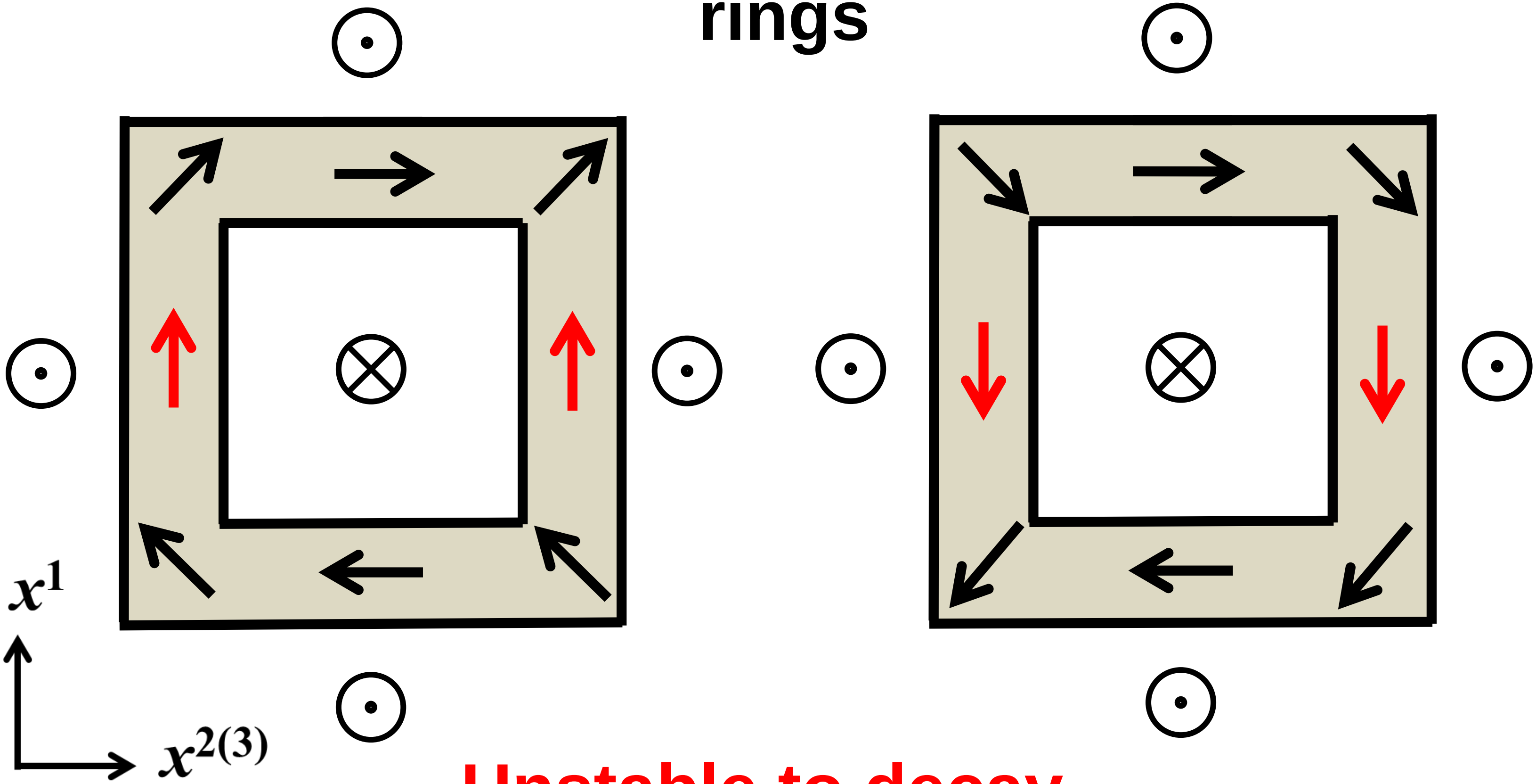

x^1
$x^{2(3)}$


x^1
$x^{2(3)}$



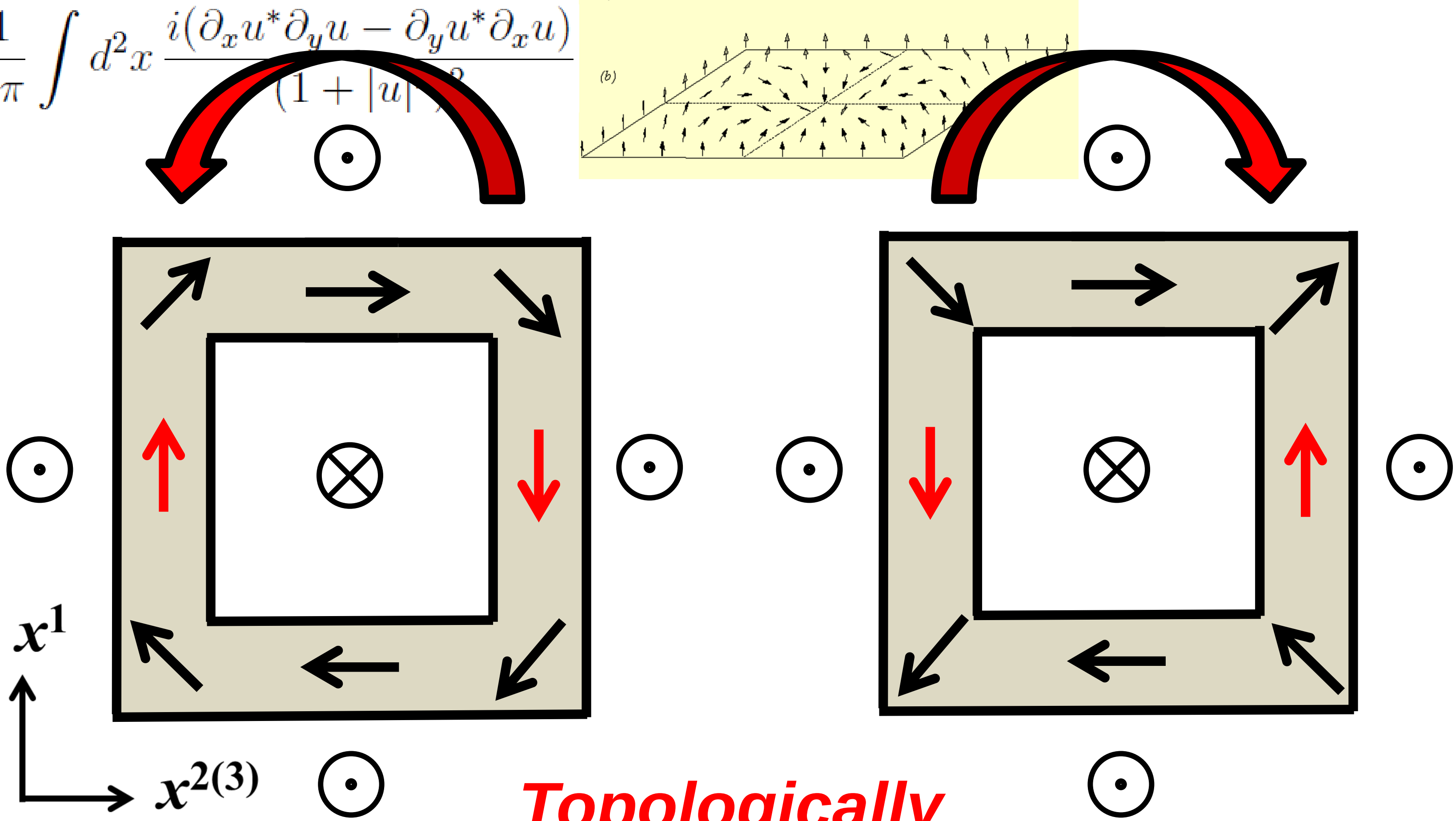
4 possibilities of domain wall ring

Domain wall rings



Unstable to decay

$$\frac{1}{2\pi}\int d^2x\,\frac{i(\partial_x u^*\partial_y u-\partial_y u^*\partial_x u)}{(1+|u|^2)^2}$$



$+1\in\pi_2(S^2)=\mathbf{Z}$

Topologically stable lump

$-1\in\pi_2(S^2)=\mathbf{Z}$

Bogomol’nyi completion for lumps

$$E=\int d\mathbf{r}\,\frac{\sum_\alpha\left|\partial_\alpha u\right|^2+M^2\left|u\right|^2}{\left(1+\left|u\right|^2\right)^2}$$

$$=\int d\mathbf{r}\left[\frac{\boxed{\left|\partial_x u\mp i\partial_y u\right|}^2}{\left(1+\left|u\right|^2\right)^2}\pm\frac{i\left(\partial_x u^*\partial_y u-\partial_y u^*\partial_x u\right)}{\left(1+\left|u\right|^2\right)^2}\right]$$

$$\geq\left|T_L\right|$$

BPS equation

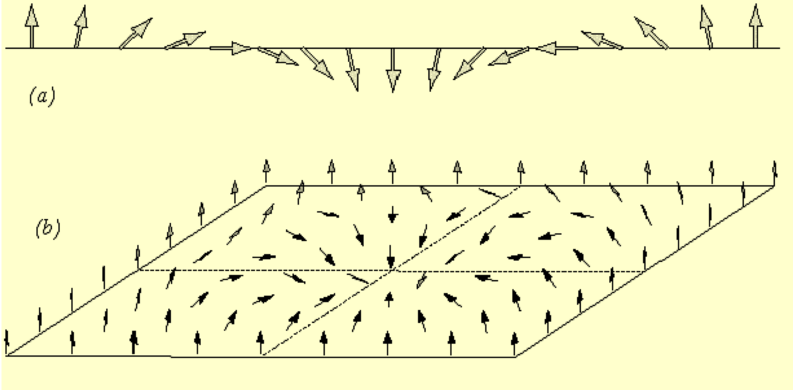
$$\partial_x u\mp i\partial_y u=0$$

Lump topological charge

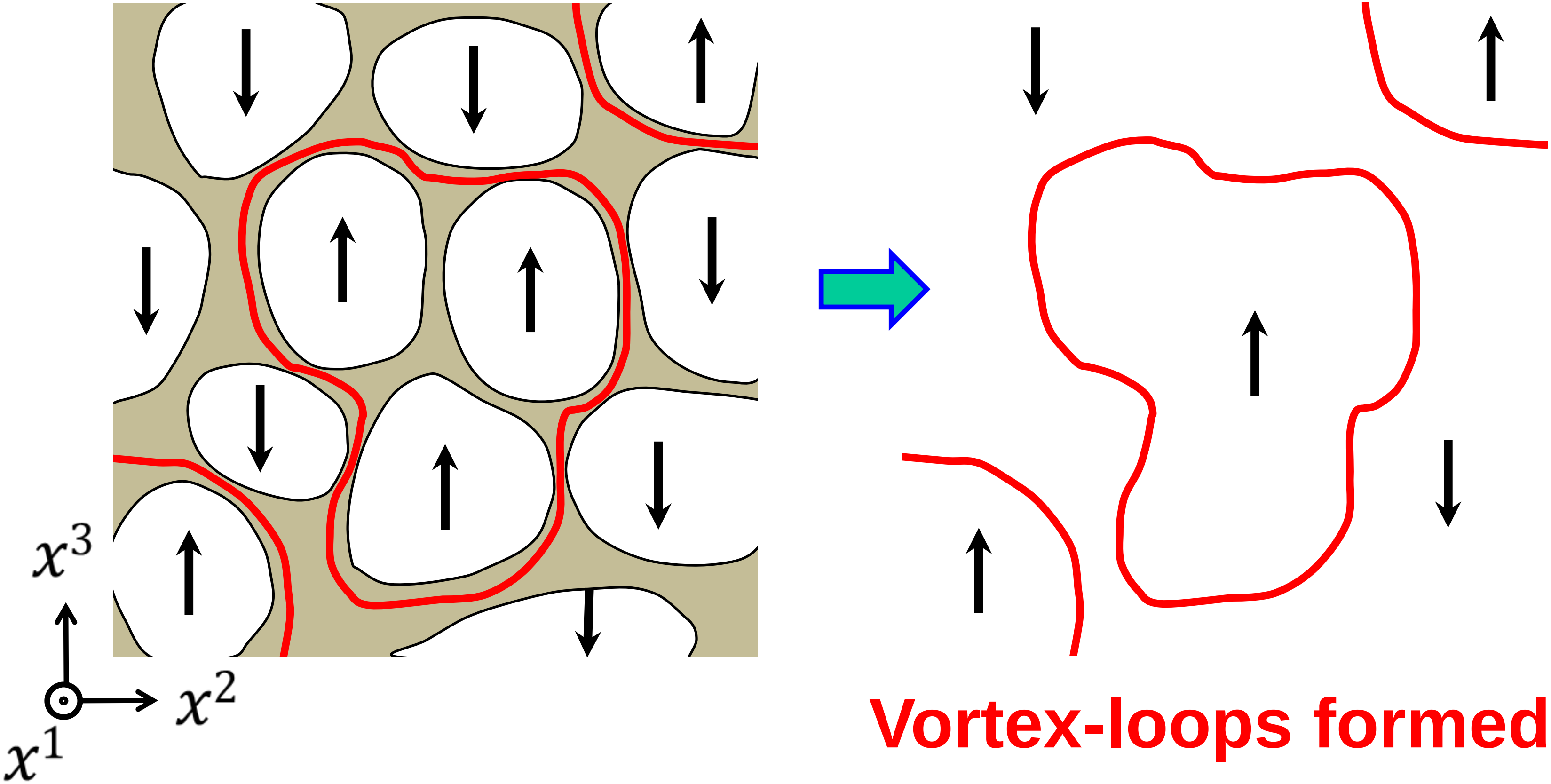
$$T_L=\pm\int d^2x\,\frac{i\left(\partial_x u^*\partial_y u-\partial_y u^*\partial_x u\right)}{\left(1+\left|u\right|^2\right)^2}$$

$$=2\pi k$$

$$k\in\pi_2(S^2)=\mathbf{Z}$$



Wall annihilations in 3+1 dimensions



Vortex-loops formed

U(1) Gauge theory with N_f =2 (extended Abelian-Higgs model)

This can be extended to **U(N_c), N_f**

$$\mathcal{L} = -\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{e^2}(\partial_\mu\Sigma)^2 + |D_\mu\Phi|^2 - V,$$

$$V = \frac{e^2}{2}(\Phi^\dagger\Phi - v^2)^2 + \Phi^\dagger(\Sigma\mathbf{1}_2 - M)^2\Phi$$

complex scalar fields $\Phi = (\phi^1, \phi^2)^T$

a real scalar field Σ

Mass matrix $M = \text{diag.}(m_1, m_2)$

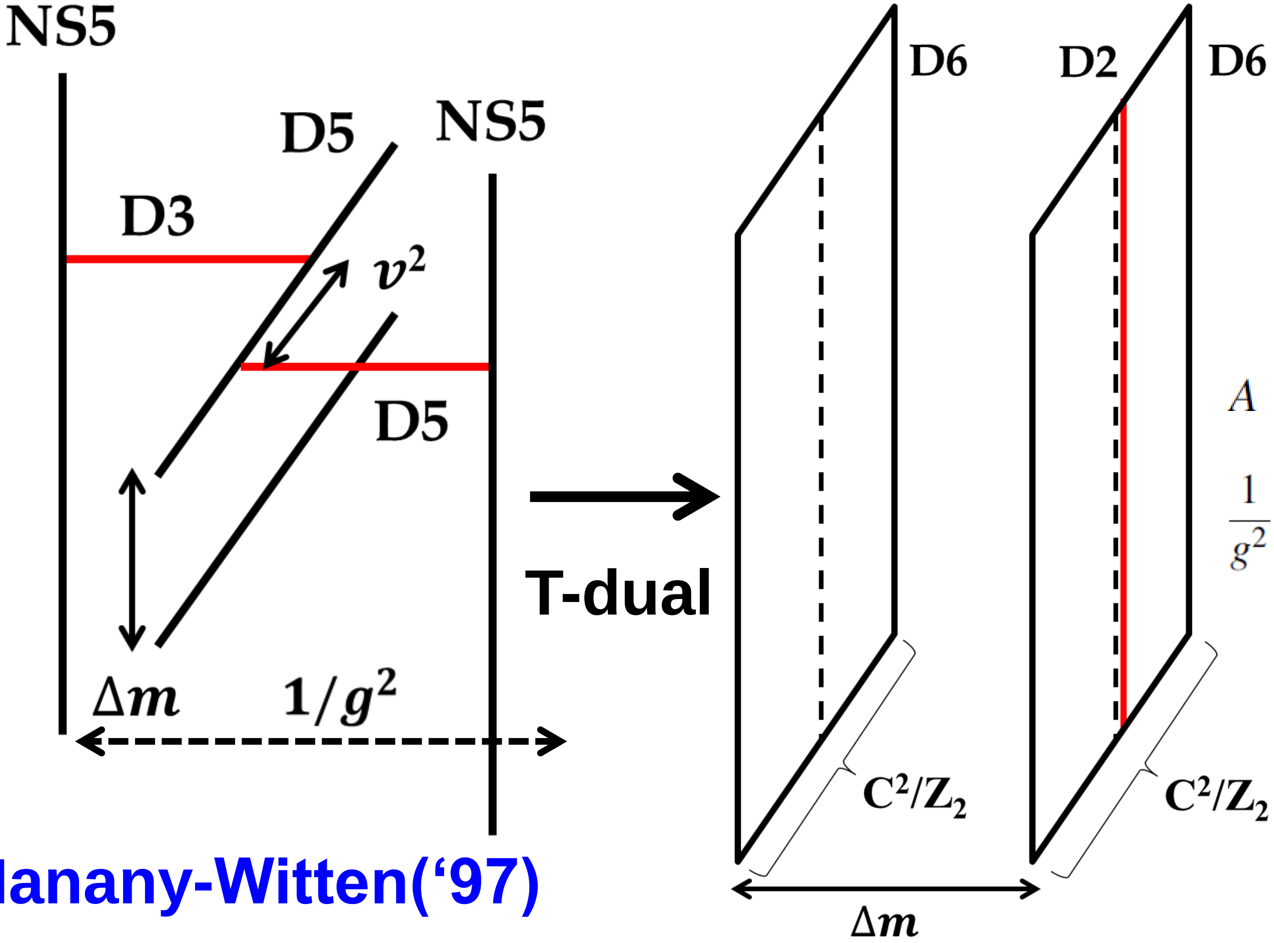
Fayet-Illiopoulos parameter v^2

$$m_1 - m_2 = m$$

Lumps = Vortices

Embedding into String theory

Eto,MN,Ohashi,Ohta&Sakai,
Phys.Rev. D71 (2005) 125006
[hep-th/0412024]



Hanany-Witten('97)

$(p = 2)$

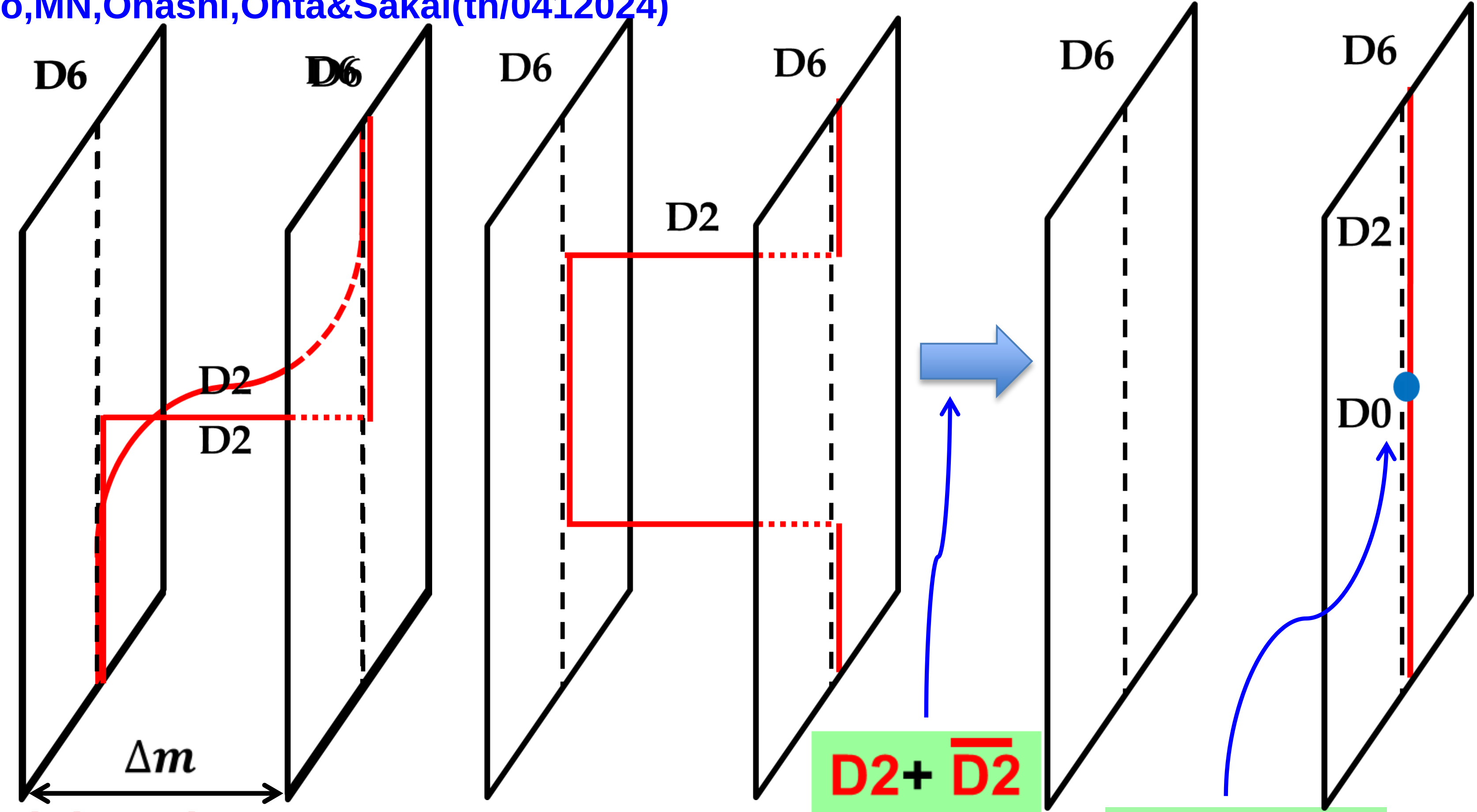
$$A = \text{Area}(S^2) \sim c\, l_s^{p+1}$$

$$\frac{1}{g^2} = b\tau_{p+2} l_s^2 = \frac{b}{g_s l_s^{p-3}}$$

$$b \sim A B_{ij}$$

$$\tau_{p+2} = 1/g_s l_s^{p+3}$$

Eto,MN,Ohashi,Ohta&Sakai(th/0412024)



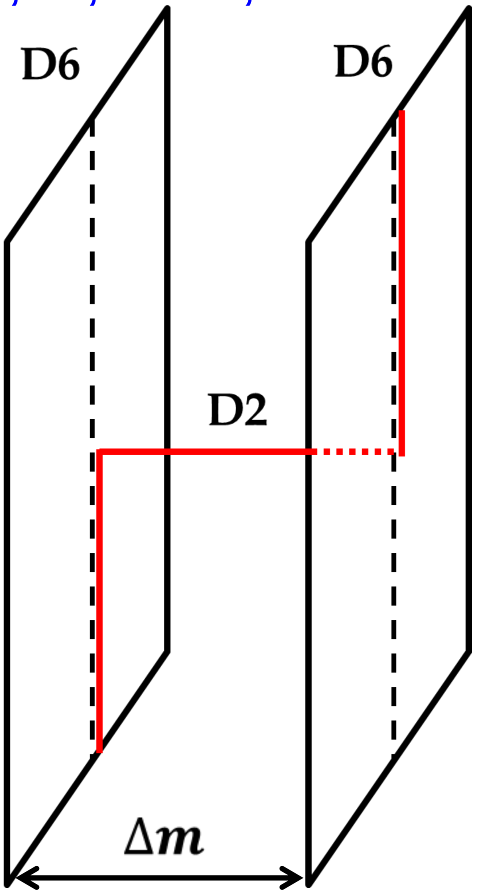
Kinky D-brane
for a domain wall

Kinky D-brane
for wall-anti-wall

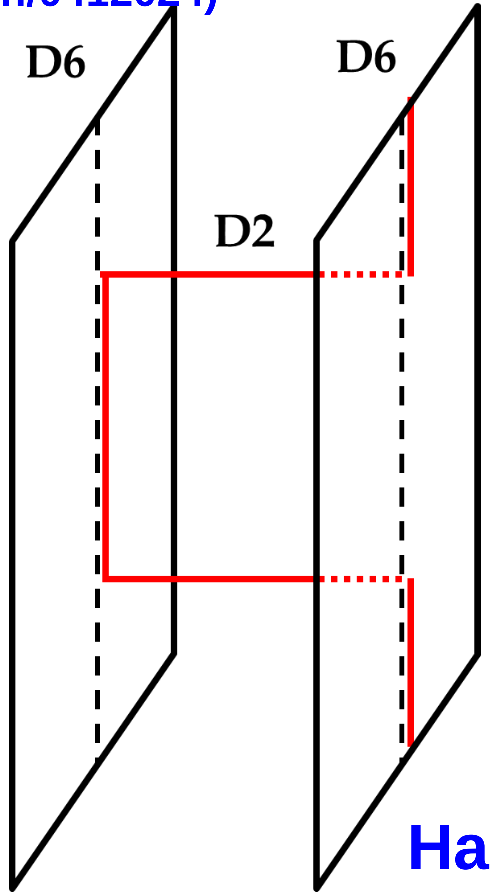
D2+ $\overline{\text{D2}}$
→D0

Vortex!!

Eto,MN,Ohashi,Ohta&Sakai(th/0412024)

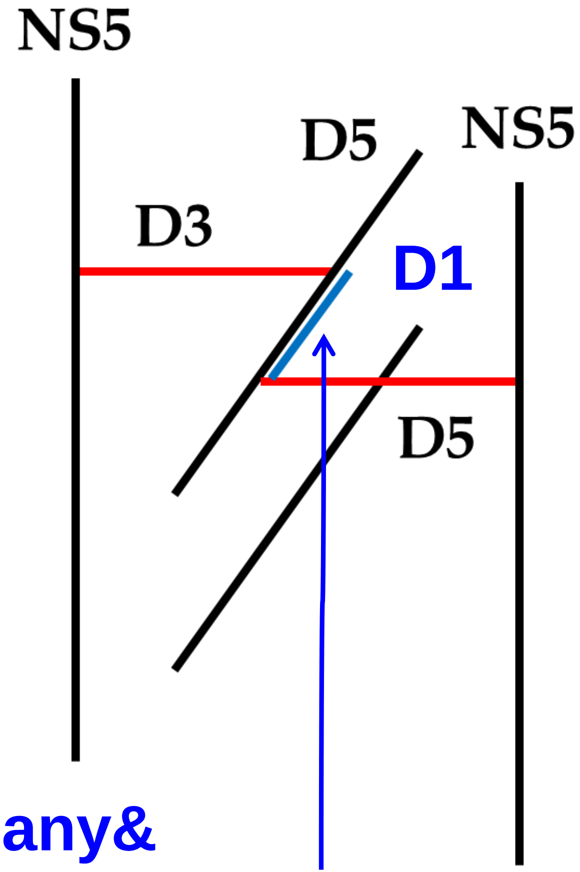


Kinky D-brane
for a **domain wall**



Kinky D-brane
for **wall-anti-wall**

Hanany&
Tong('03)



Vortex!!

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Non-Abelian Gauge theory

(Non-Abelian-Higgs model)

U(2) gauge theory $N_F=2$
in d=3+1, 4+1 or 5+1

$$\mathcal{L} = -\frac{1}{4g^2}\mathrm{tr}\, F_{AB}F^{AB} + \frac{1}{2g^2}\mathrm{tr}\,(D_A\Sigma)^2 + \mathrm{tr}\, D_A H^\dagger D^A H - V,$$

$$V = g^2\mathrm{tr}\,(HH^\dagger - v^2\mathbf{1}_2)^2 + \mathrm{tr}\,\left[H(\Sigma - M)^2 H^\dagger\right],$$

$N_F=2$ fundamental Higgs H

real adjoint Higgs Σ

$m_1 - m_2 = m$

Mass matrix $M = \mathrm{diag.}(m_1, m_2)$

Fayet-Illiopoulos parameter v^2

vacuum

$$H = v^2\mathbf{1}_2$$
$$\Sigma = M$$

Color-flavor locked vacuum

$U(2)_C \times SU(2)_F$
$\to SU(2)_{C+F}$

$m=0$ Non-Abelian vortex

Hanany-Tong, Konishi et.al ('03)

We can embed the ANO solution $H^{\rm ANO}(z), F_{12}^{\rm ANO}(z)$

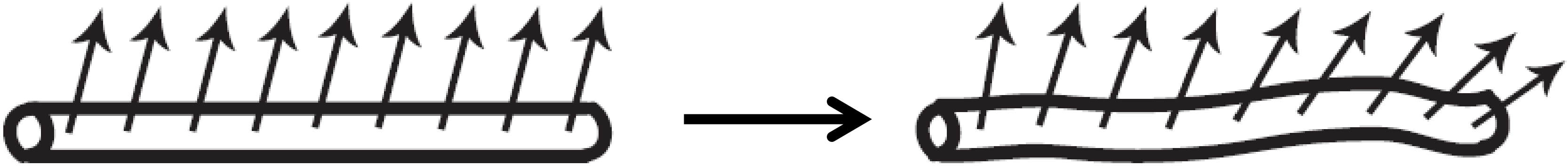
$$H=\left(\begin{array}{c|c} H^{\rm ANO}(z-\underline{z_0}) & \\ \hline & v \end{array}\right),\quad F_{12}=\left(\begin{array}{c|c} F_{12}^{\rm ANO}(z-\underline{z_0}) & \\ \hline & 0 \end{array}\right)$$

This solution breaks $SU(2)_{\rm C+F}\rightarrow U(1)$

The moduli space of Nambu-Goldstone modes:

$$\underset{\substack{\uparrow \\ z_0\ \text{Translation}}}{\mathbf{C}}\times\underbrace{\frac{SU(2)_{\rm C+F}}{U(1)}}_{\text{Internal symmety}}\cong\mathbf{C}\times\mathbf{C}P^1\cong\mathbf{C}\times S^2$$

The effective theory is the $\mathbf{C}P^1$ model



“vacuum state”

fluctuation of zero modes

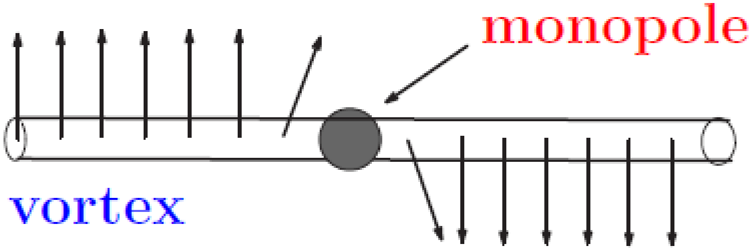
$$\mathcal{L}_{\text{vort.eff.}} = 2\pi v^2 \left|\partial_\mu z_0\right|^2 + \frac{4\pi}{g^2}\left[\frac{\partial_\mu u^* \partial^\mu u}{(1+|u|^2)^2}\right]$$

$$m \neq 0 << v$$

Confined monopole

Tong('03), Hanany-Tong, Shifman-Yung ('04)
Eto,Isozumi,MN&Sakai('04)

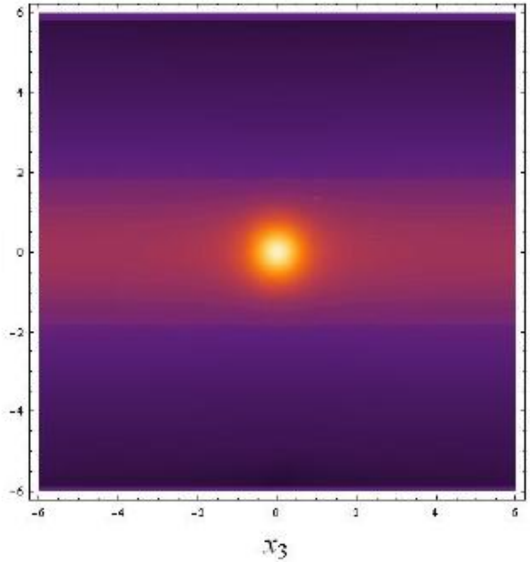
Kink on a NA vortex = monopole



$$E_{\rm dw} = \frac{4\pi}{g^2} \times m = E_{\rm m}$$

Domain wall tension

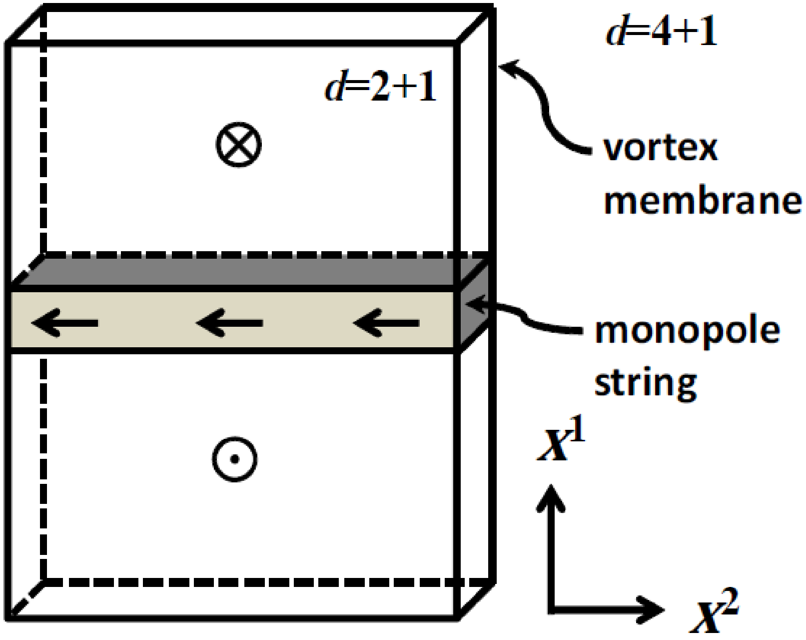
Monopole mass



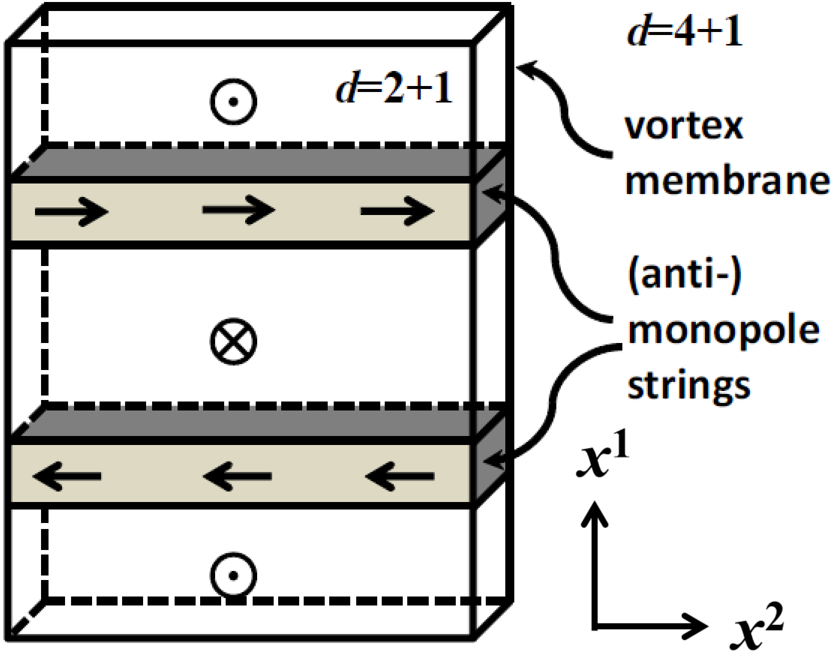
Numerical solution by Fujimori

d=4+1

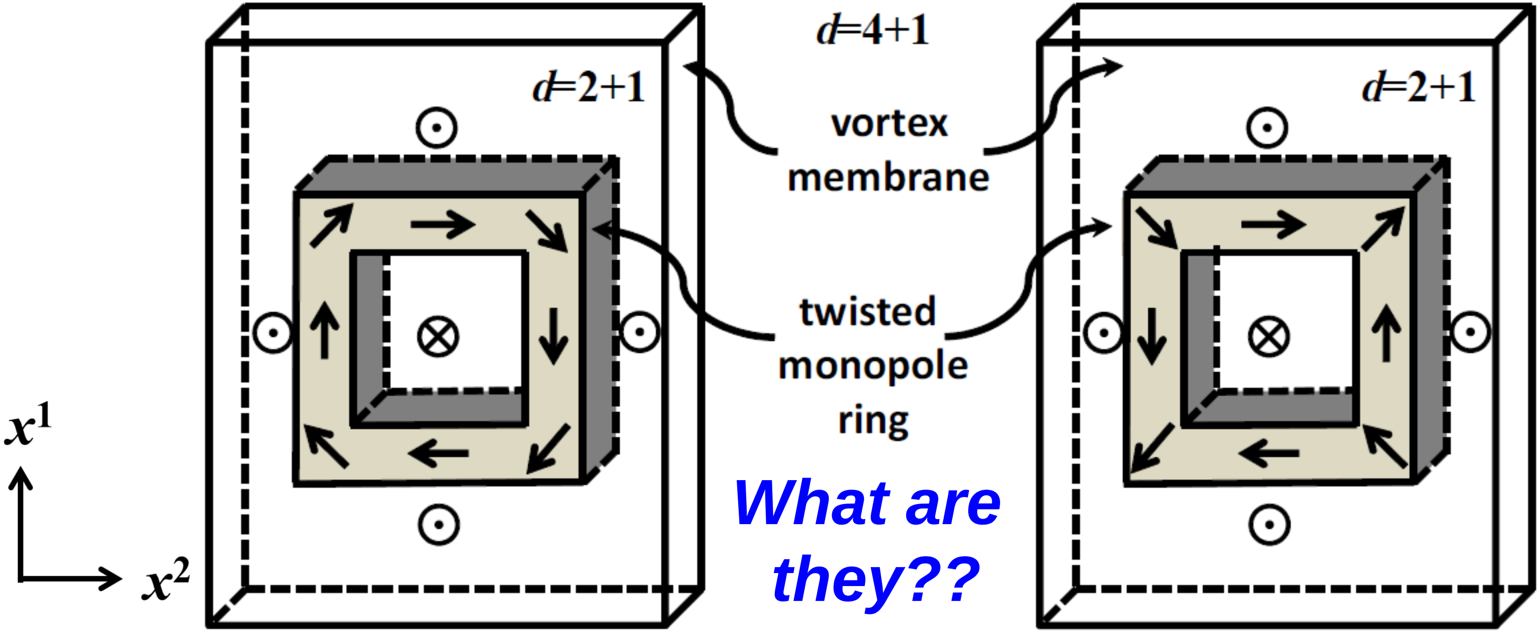
Monopole string in vortex membrane



Monopole-anti-monopole in vortex membrane



Twisted monopole-rings are created in *d=4+1*



What are they??

Answer: Yang-Mills instantons (particles)

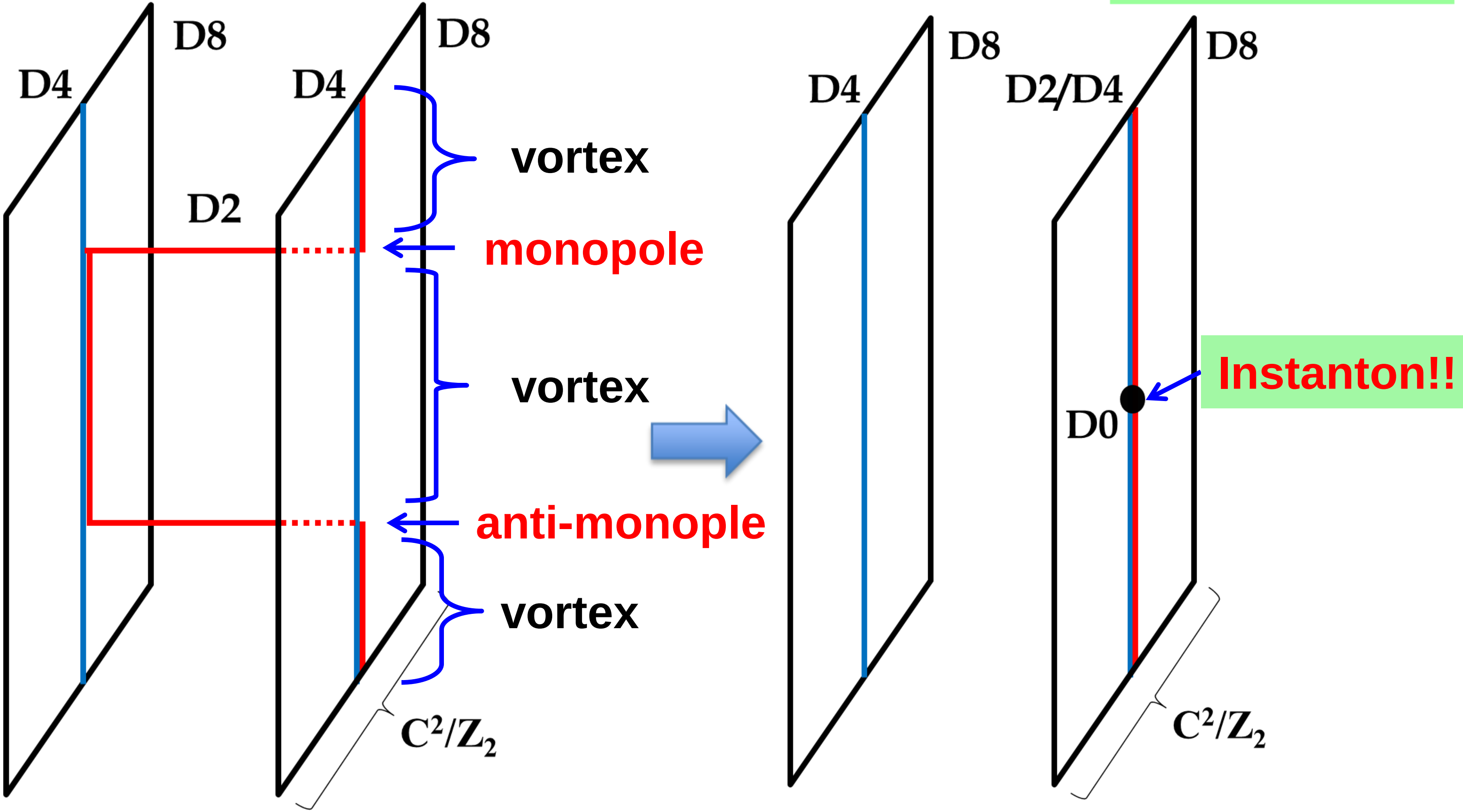
Lumps in NA vortex
= YM instantons in bulk

$$E_l = \frac{4\pi}{g^2} \times 2\pi k = \frac{8\pi^2}{g^2} = E_i$$

Eto-Isozumi-MN
-Ohashi-Sakai
PRD72 (2005)
[hep-th/0412048]

Embedding into String theory

$\mathrm{D2}+\overline{\mathrm{D2}}\rightarrow\mathrm{D0}$



vortex

monopole

vortex

anti-monople

vortex

Instanton!!

Summary up to here

Codimension→ ↓World-volume	Domain wall and Anti-domain wall	Monopole and Anti-monopole
Strings	(anti-)vortices	(anti-)Yang-Mills instantons
Sheets (membranes)	Closed vortex strings	Closed instanton strings

String theory $D2+\overline{D2}\rightarrow D0$

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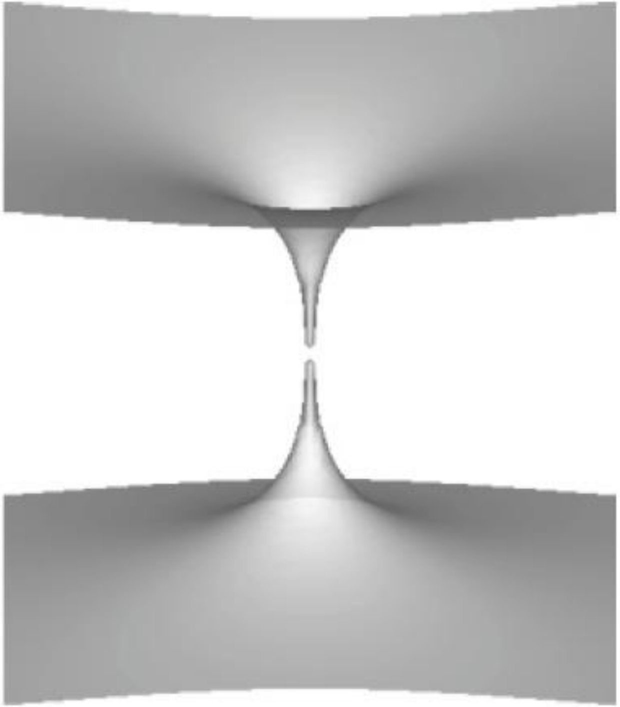
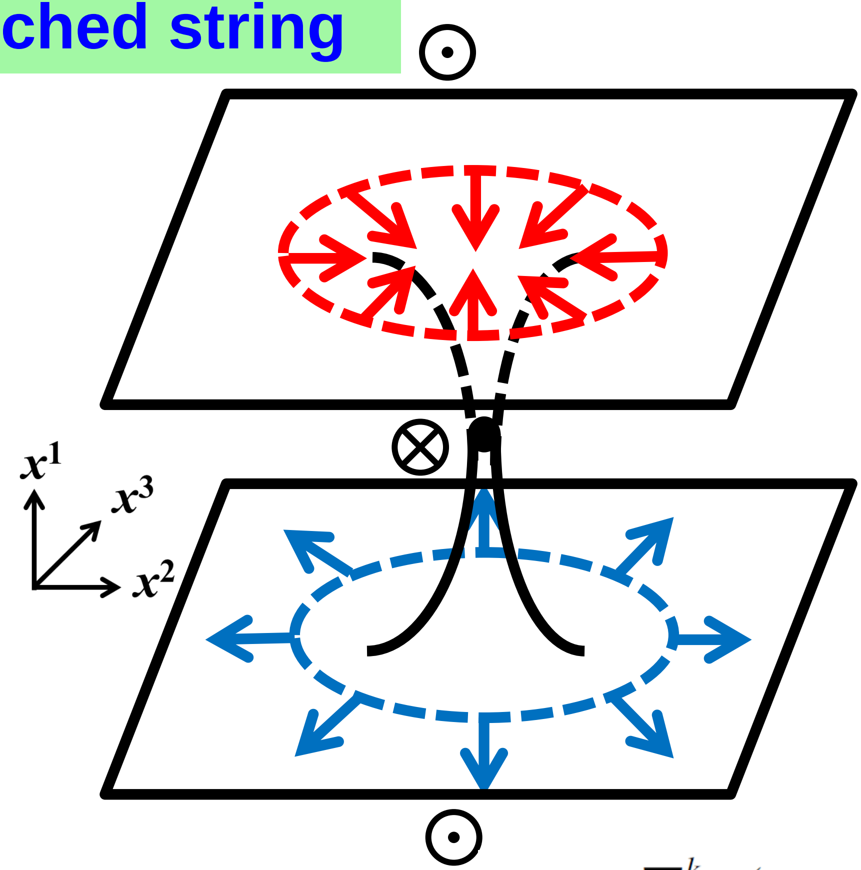
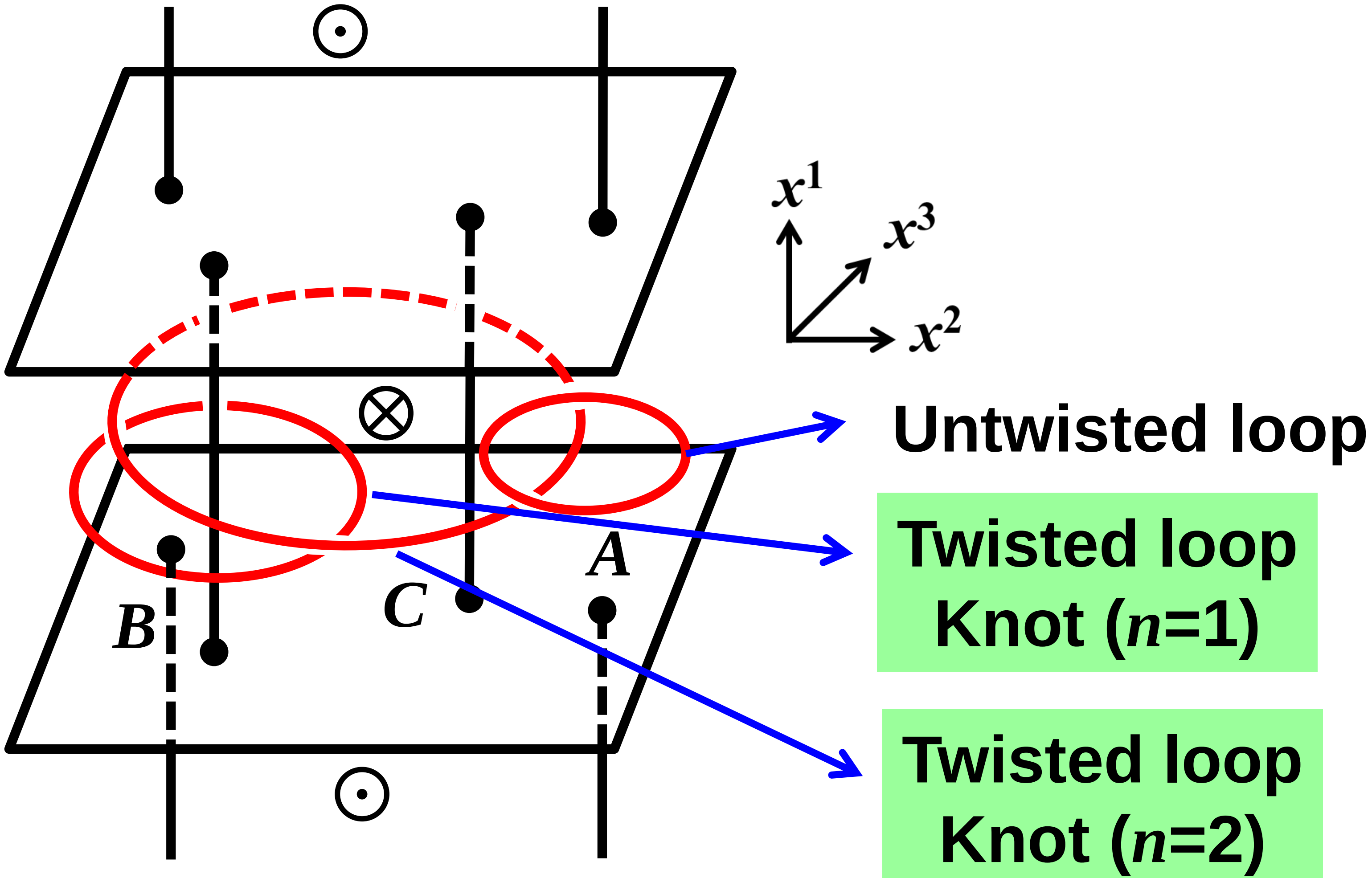
Brane-anti-brane with stretched string

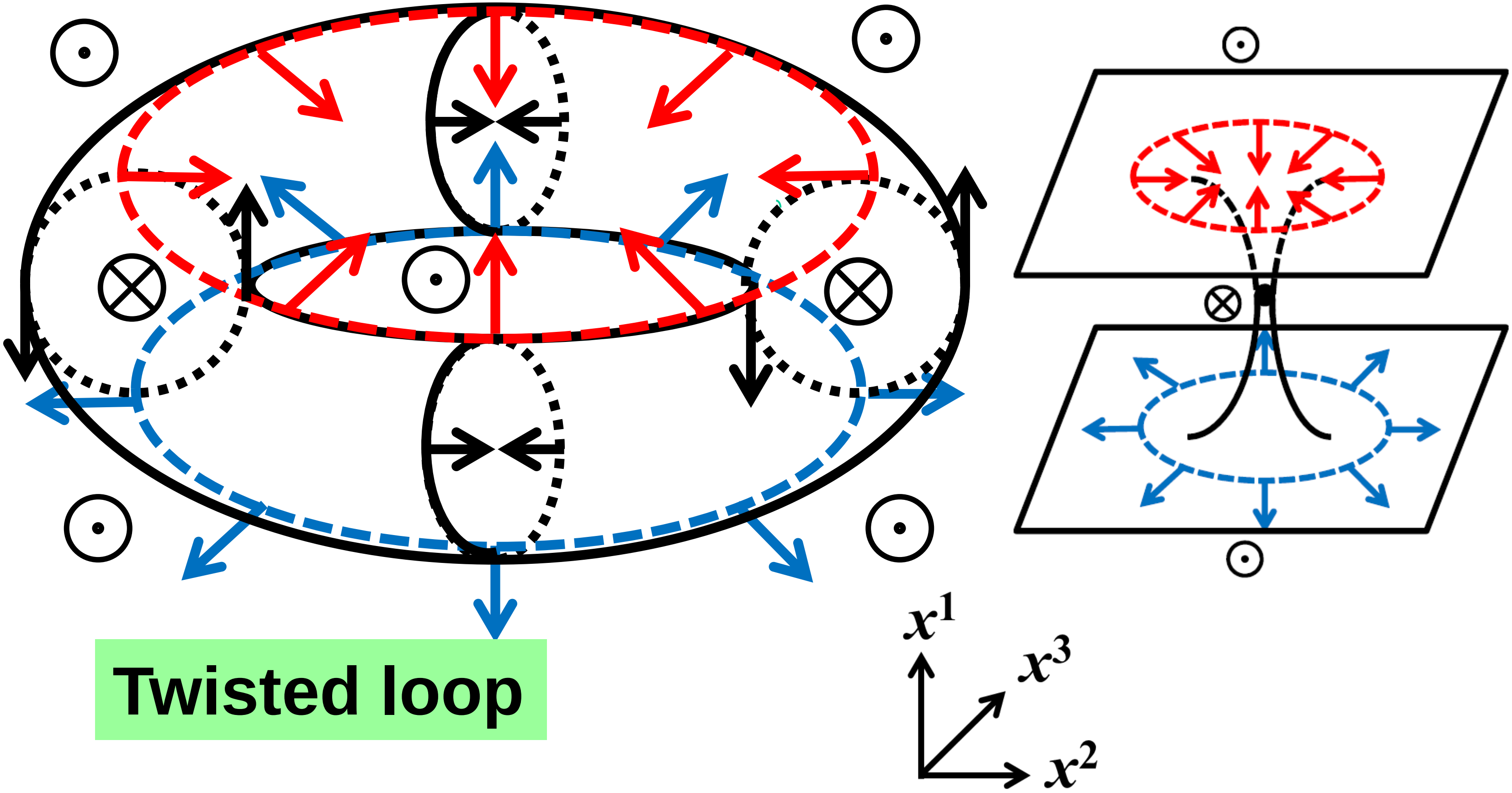


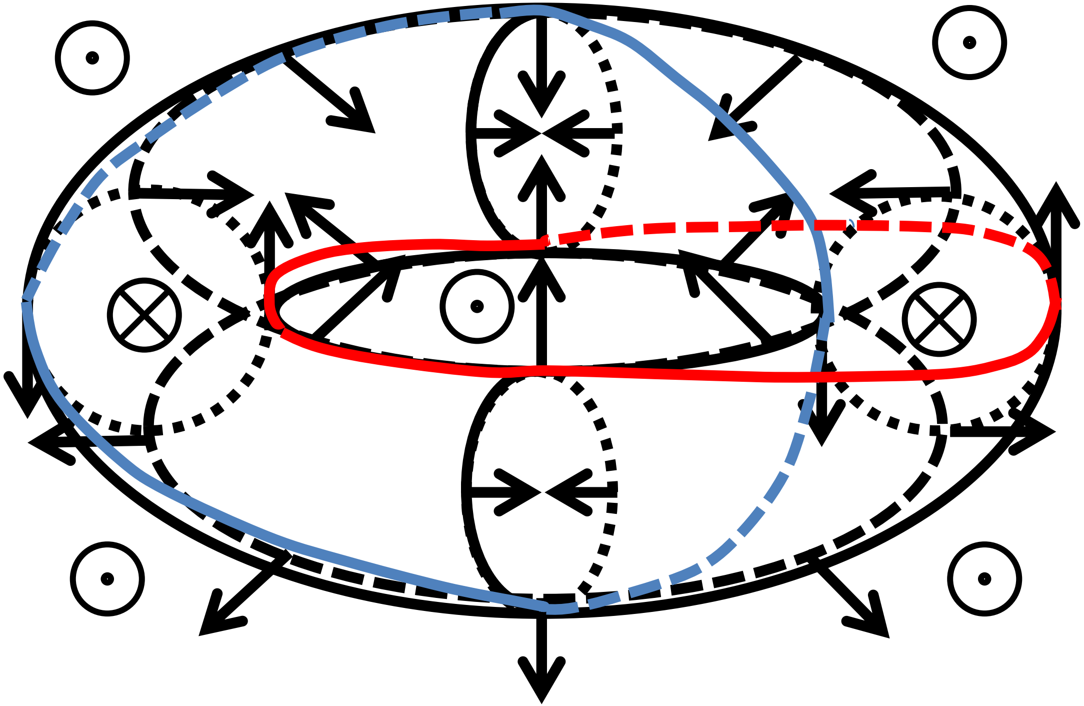


Approximate analytic solution

$$u_{\text{w-v-aw}} = \left(e^{-m(x^1-x^1_1)+i\varphi_1} + e^{+m(x^1-x^1_2)+i\varphi_2}\right)Z(z) \qquad Z(z) = \frac{\prod_{j=1}^{k_+}(z-z_j^+)}{\prod_{i=1}^{k_-}(z-z_i^-)}$$


x^1
x^3
x^2
A
B
C
Untwisted loop
Twisted loop
Knot (n=1)
Twisted loop
Knot (n=2)

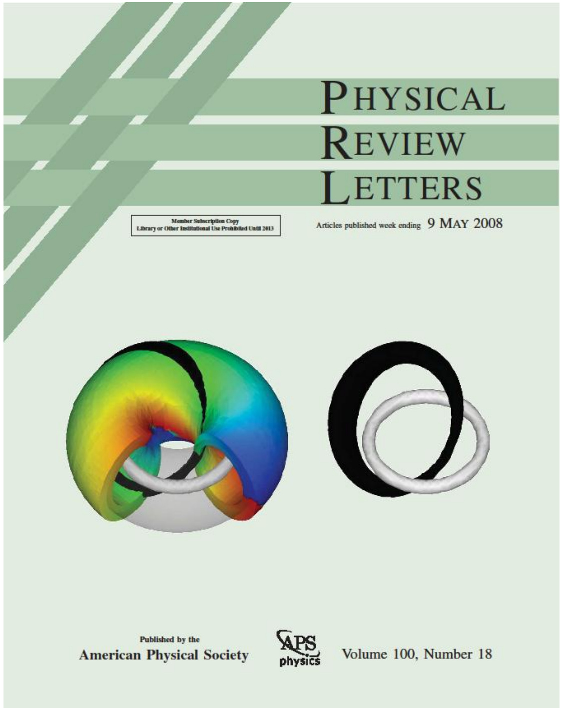

Twisted loop
x^1
x^3
x^2



Faddeev-Niemi, Nature 387 (1997) 58

Knot soliton (Hopfion)

Linking number = 1



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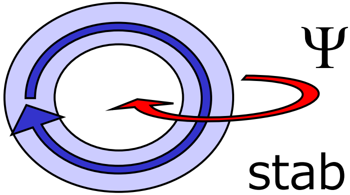
APS physics

Volume 100, Number 18

Knots in Spin 1 BEC

Kawaguchi-MN-Ueda
PRL('08)
[arXiv:0802.1968
[cond-mat.other]]

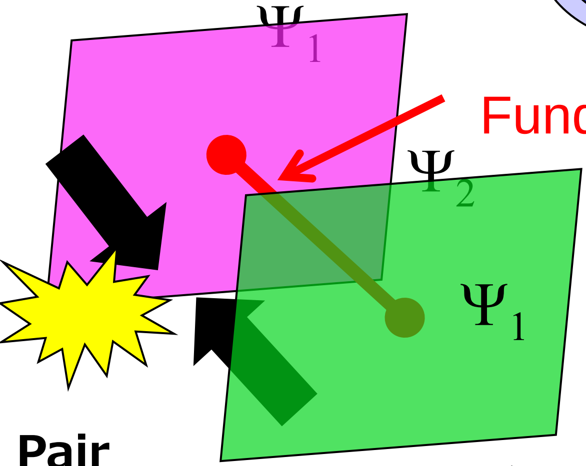
Vorton creation in BEC = 3D Skyrmion



Ψ_2 has superfluid flow inside a vortex ring of Ψ_1

stable vortex ring(**vorton**)

brane

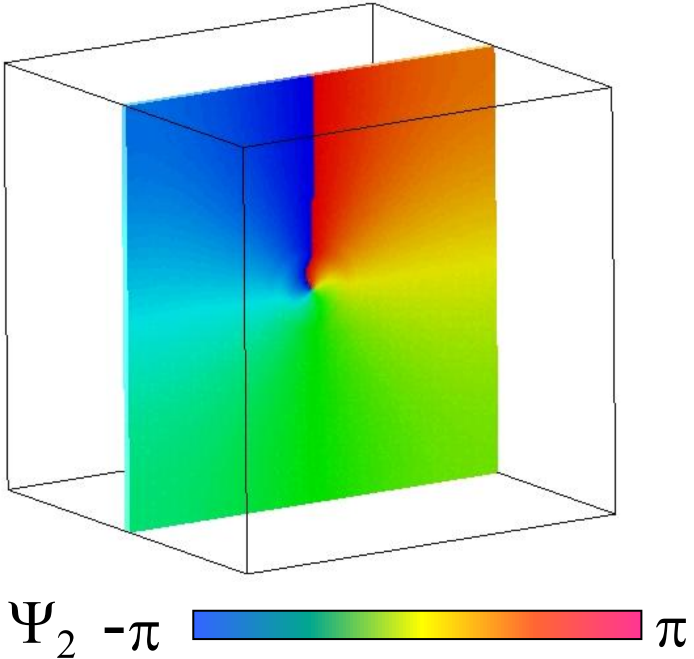


Fundamental string

Pair anihilation

anti-brane

Simulation by Takeuchi



Phase of Ψ_2

MN-Takeuchi-Kasamatsu-Tsubota

Phys.Rev.A85(2012)053639

[arXiv:1203.4896 [cond-mat.quant-gas]]

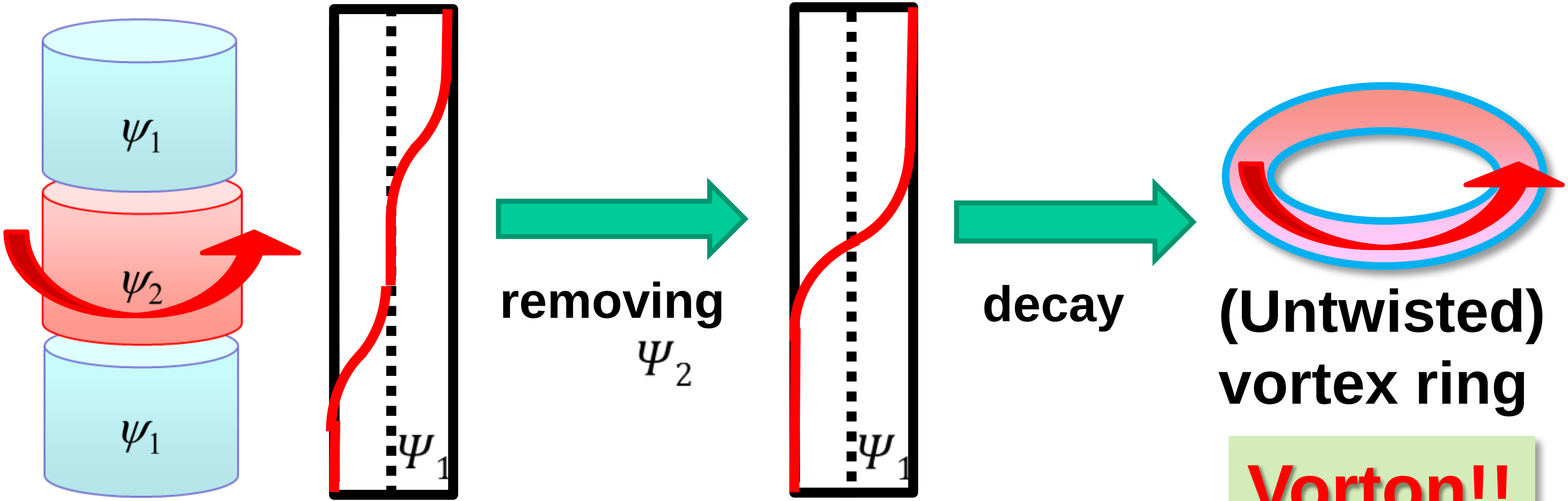
Vorton creation in BEC

Experiments in BEC

Watching Dark Solitons Decay into Vortex Rings in a Bose-Einstein Condensate

B. P. Anderson *et.al.*, Phys. Rev. Lett. 86, 2926–2929 (2001)

(**JILA,** National Institute of Standards and Technology and Department of Physics, University of Colorado, Boulder, Colorado)



removing Ψ_2

decay

(Untwisted) vortex ring

Dark soliton

Vorton!!

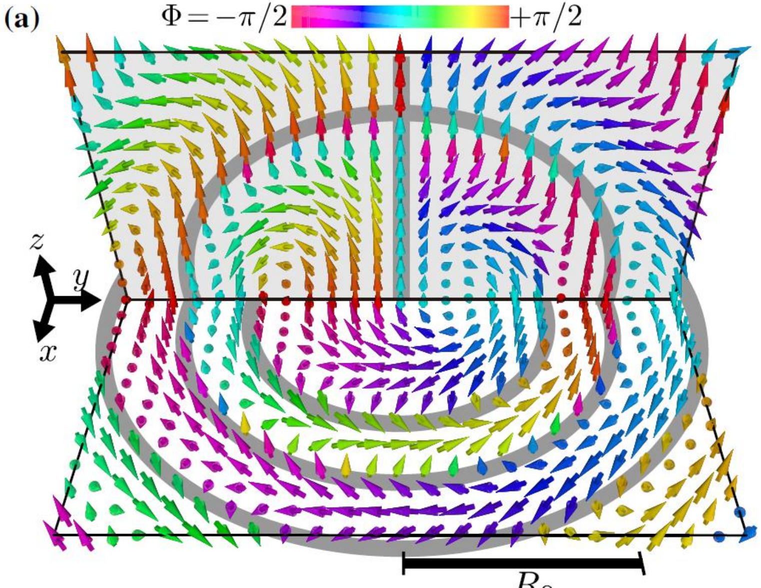
3D Skyrmion

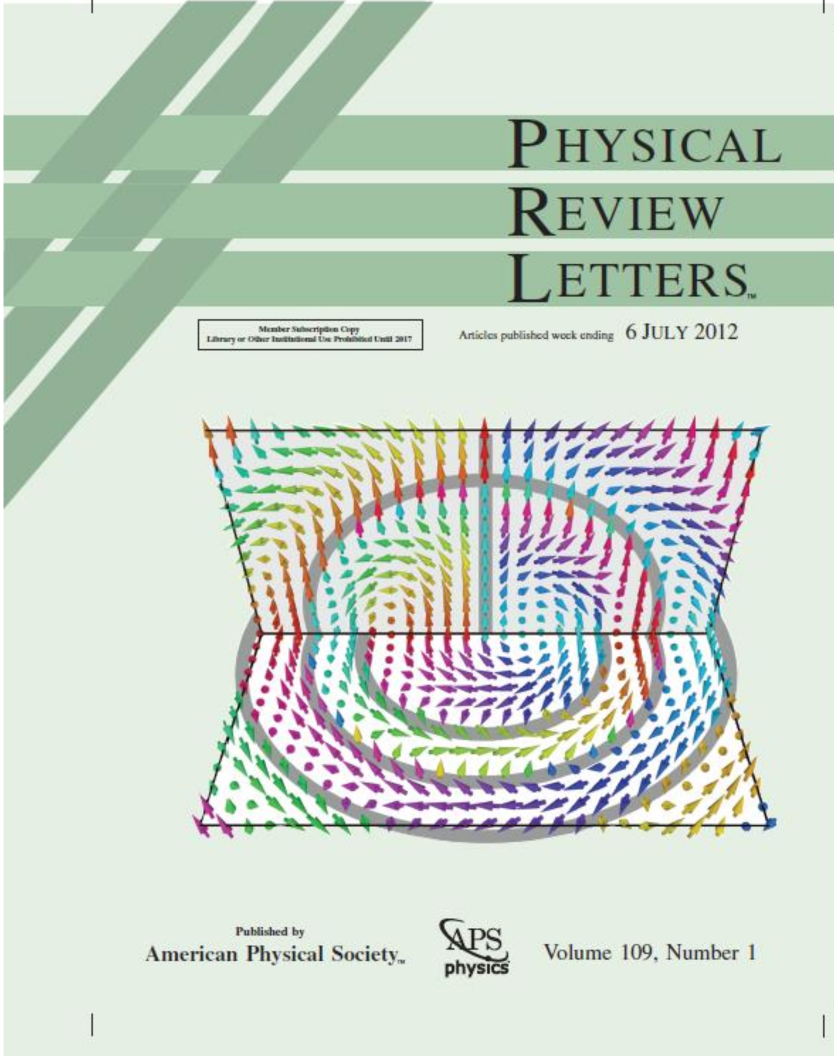
advertisement

unstable ***or at most*** *metastable*

Artificial "SU(2) gauge field" stabilizes 3D Skyrmion

Kawakami,Mizushima,MN&Machida
Phys. Rev. Lett. 109, 015301 (2012)





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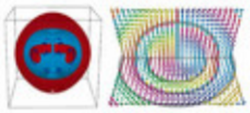
Volume 109, Number 1

岡山大など、概念上の素粒子「スカーミオン」を安定に作り出すことを提唱

マイナビニュース 7月20日(金)16時10分配信



拡大写真

（写真：マイナビニュース）

岡山大学と慶應義塾大学(慶応大)は7月19日、陽子や中性子のような「核子」と呼ばれる粒子を理解するために導入された数学的概念であり、未だにその性質に謎が多く、素粒子理論に不可欠な「トポロジカル構造」である素粒子「スカーミオン」の理解に不可欠な構造を、現実に数ナノケルビン程度まで冷却された原子気体において安定に作り出すことを世界で初めて提唱したことを発表した。

コンピュータトピックス
・キヤノン ミラーレス
・au「CDMA 1X」22日
・最新GALAXYデバ

Plan of my talk

§ 1 Introduction (2p)

§ 2 Domain wall annihilation (12+3p)

§ 3 Monopole-string annihilation (6p)

§ 4 Knot/Vorton/Knotted instanton (4+3p)

§ 5 Conclusion (1p)

① Brane-anti-brane annihilations in field theory

	Domain wall and Anti-domain wall	Monopole and Anti-monopole
Strings	(anti-)vortices	(anti-)Yang-Mills instantons
Sheets (membranes)	Closed vortex strings	Closed instanton strings
Sheets with stretched strings	Knots (Hopfions)	Knotted instantons

② D2+ $\overline{\mathrm{D2}}$→D0 in string theory

③ Testable in laboratory using BEC

D-brane-like object in Field Theory

massive $O(3)$ sigma model

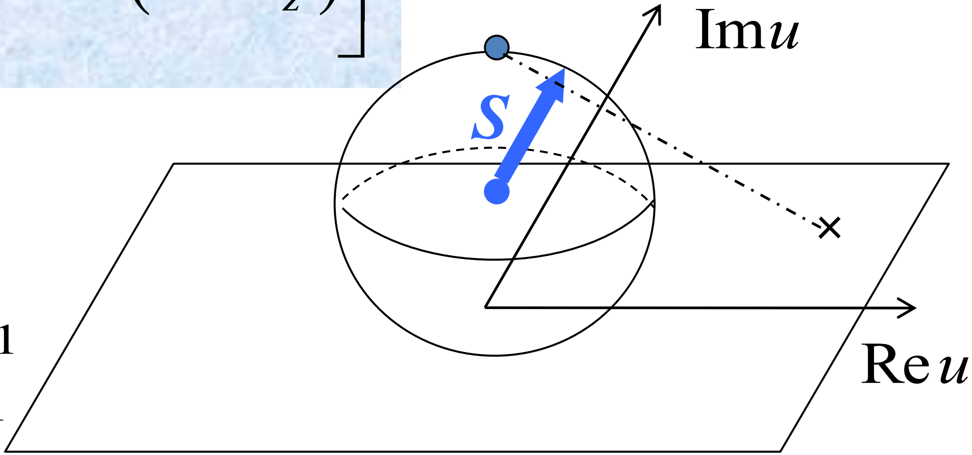
$$E = \frac{1}{4}\int d\mathbf{r}\left[\sum_{\alpha}(\nabla S_{\alpha})^2 + M^2\left(1 - S_z^2\right)\right]$$

stereographic coordinate

$$u = \frac{S_x - iS_y}{1 - S_z}$$

$$u = \infty \quad \text{for} \quad S_z = +1$$

$$u = 0 \quad \text{for} \quad S_z = -1$$



$$E = \int dr \frac{\sum_{\alpha}\left|\partial_{\alpha}u\right|^2 + M^2|u|^2}{\left(1+|u|^2\right)^2}$$

Bogomol’nyi-Prasad-Sommerfield bound

$$E=\int d\mathbf{r}\,\frac{\sum_{\alpha}\left|\partial_{\alpha}u\right|^{2}+M^{2}\left|u\right|^{2}}{\left(1+\left|u\right|^{2}\right)^{2}}$$

$$=\int d\mathbf{r}\left[\frac{\left|\partial_{x}u\mp i\partial_{y}u\right|^{2}}{\left(1+\left|u\right|^{2}\right)^{2}}\pm\boxed{\frac{i\left(\partial_{x}u^{*}\partial_{y}u-\partial_{y}u^{*}\partial_{x}u\right)}{\left(1+\left|u\right|^{2}\right)^{2}}}\right.$$

$$\left.+\frac{\left|\partial_{z}u\mp 2Mu\right|^{2}}{\left(1+\left|u\right|^{2}\right)^{2}}\pm\boxed{\frac{2M\left(u^{*}\partial_{z}u+u\partial_{z}u^{*}\right)}{\left(1+\left|u\right|^{2}\right)^{2}}}\right]$$

$$\geq\left|T_{W}\right|+\left|T_{V}\right|$$

→ $T_V = 2\,\pi\,N_V$

vortex charge

→ $T_W = \pm\,M,\ 0$

domain wall charge

BPS equations

½ BPS solutions

$$\partial_x u \mp i\partial_y u = 0 \quad \xrightarrow{\eta = x + iy} \quad u_V = \frac{\prod_{j=1}^{N_{k1}}\left(\eta - \eta_j^{(1)}\right)}{\prod_{j=1}^{N_{k2}}\left(\eta - \eta_j^{(2)}\right)}$$

vortices

$$\partial_z u \mp 2Mu = 0 \quad \rightarrow \quad u_W = e^{\mp M(z-z_0)-i\phi_0}$$

domain wall

¼ BPS solution

$$u(\eta, z) = e^{mM(z-z_0)-i\phi_0} \frac{\boxed{\prod_{j=1}^{N_{k1}}\left(\eta - \eta_j^{(1)}\right)}}{\boxed{\prod_{j=1}^{N_{k2}}\left(\eta - \eta_j^{(2)}\right)}}$$

→ vortices in ψ_1

→ vortices in ψ_2

localized U(1) Nambu-Goldstone mode

moduli $(z_0, \phi_0) \in$ **R x S^1**

Y.Isozumi, M.Nitta, K.Ohashi, N.Sakai
Phys.Rev. D71 (2005) 065018

duality

$$I = -2\int d^3\xi \sqrt{-\det\left(g_{ij} + F_{ij}\right)}$$

$$\partial_i \phi_0 = \varepsilon_{ijk}\partial_j A_k$$

DBI action → endpoints of vortices are electric charges

J.P.Gauntlett, R.Portugues,
D. Tong, P.K. Townsend
Phys.Rev. D63 (2001) 085002

BIon

O(3) sigma model

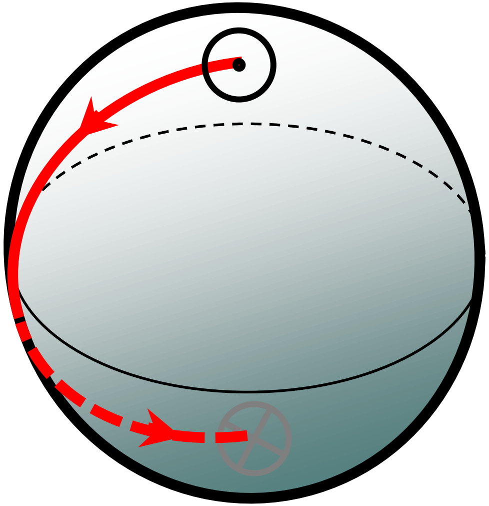
$$\mathcal{L} = \frac{1}{2}\partial_\mu \mathbf{n} \cdot \partial^\mu \mathbf{n} - m^2(1 - n_3^2)$$

$$\mathbf{n}^2 = 1$$

CP¹ model

$$\mathcal{L} = \frac{\partial_\mu u^* \partial^\mu u - m^2|u|^2}{(1+|u|^2)^2}$$

Vacuum *N* $n_3 = +1,$ $u = \infty$



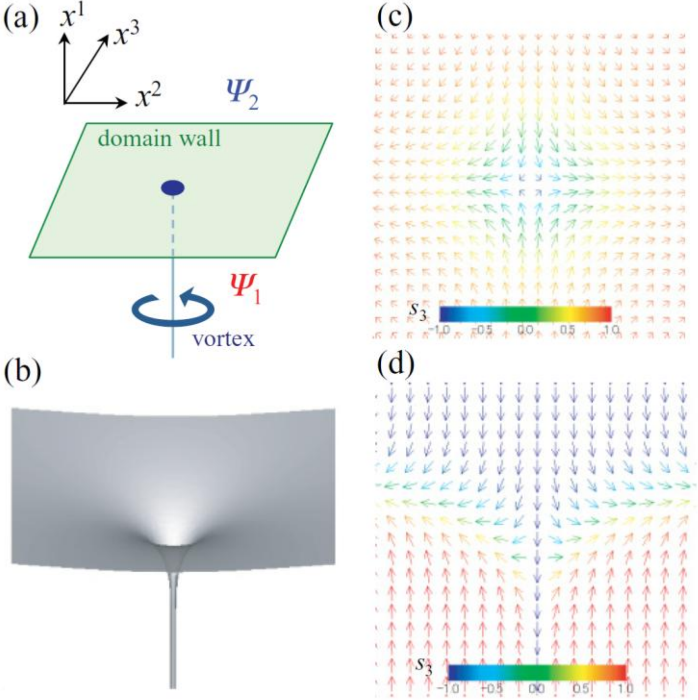
Vacuum *S* $n_3 = -1,$ $u = 0$

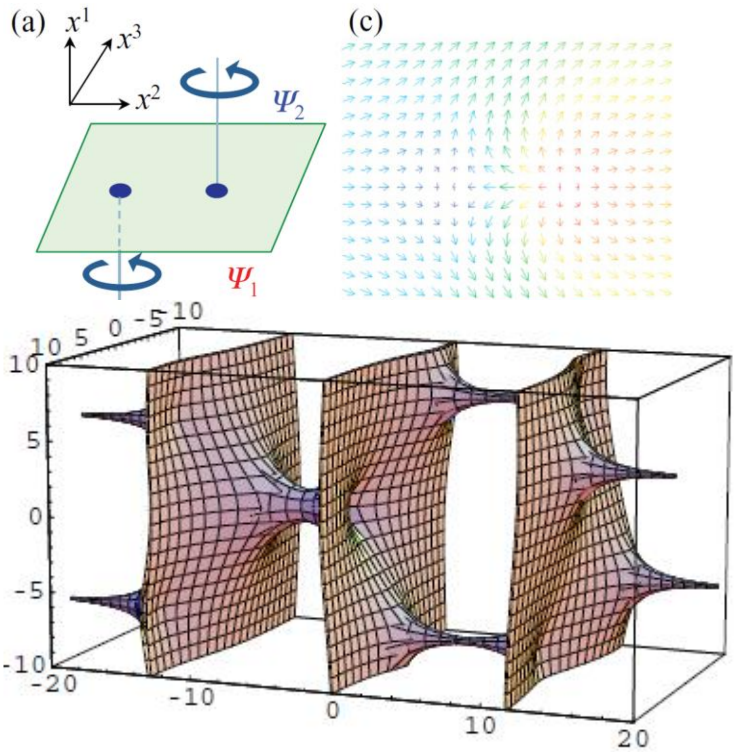
Adding 4 derivative term

$$\mathcal{L}_4(\mathbf{n}) = \kappa[\mathbf{n} \cdot (\partial_\mu \mathbf{n} \times \partial_\nu \mathbf{n})]^2 = \kappa(\partial_\mu \mathbf{n} \times \partial_\nu \mathbf{n})^2$$

$$= -2\kappa \frac{(\partial_\mu u^* \partial^\mu u)^2 - |\partial_\mu u \partial^\mu u|^2}{(1+|u|^2)^4}$$

Faddeev-Skyrme model





Exact analytic solutions

$$u(x^1,z) = u_{\mathrm{W}}(x^1)u_{\mathrm{V}}(z),$$

$$u_{\mathrm{W}}(x^1) = e^{\mp M(x^1-x_0^1)-i\phi_0},\quad u_{\mathrm{V}}(z) = \frac{\prod_{j=1}^{N_{v_1}}\left(z-z_j^{(1)}\right)}{\prod_{j=1}^{N_{v_2}}\left(z-z_j^{(2)}\right)}$$

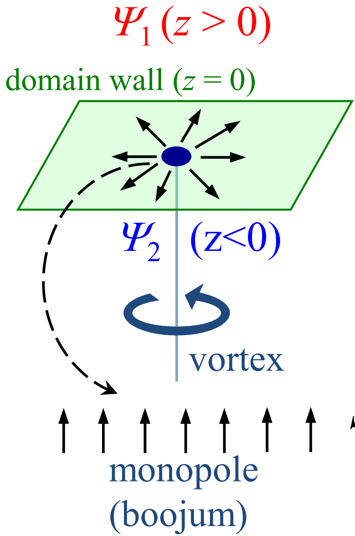
All exact(analytic) solutions of ¼ BPS wall-vortex states

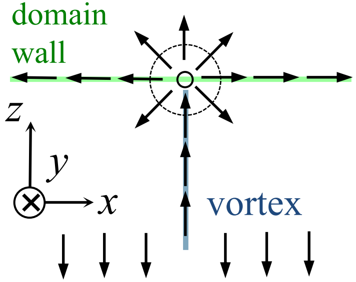
Y.Isozumi, MN, K.Ohashi, N.Sakai
Phys.Rev. D71 (2005) 065018
[hep-th/0405129]

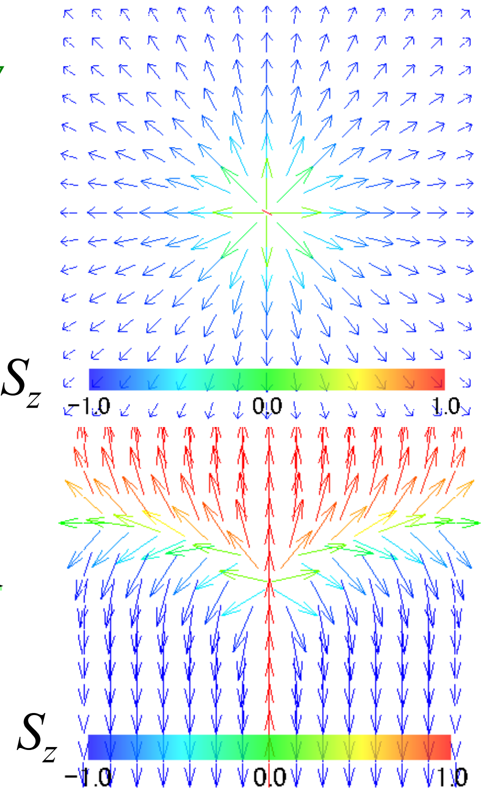
D-brane in a laboratory

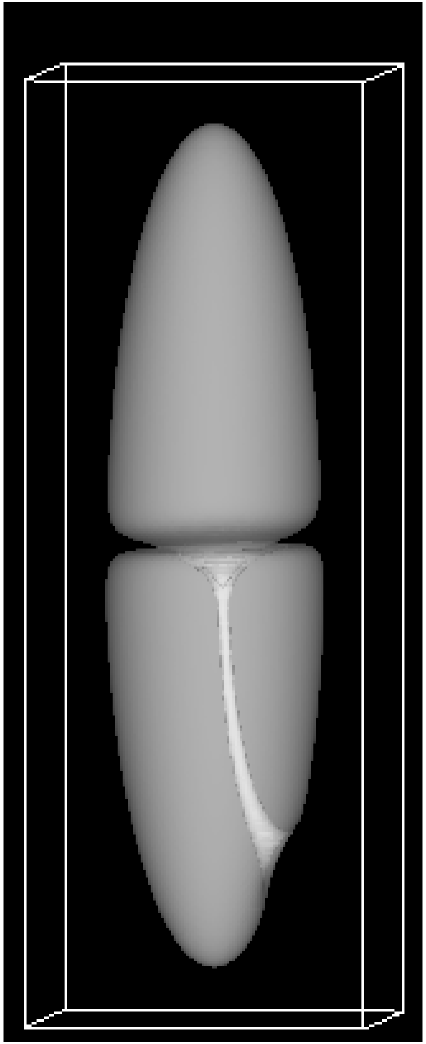
Kasamatsu-Takeuchi-MN-Tsubota

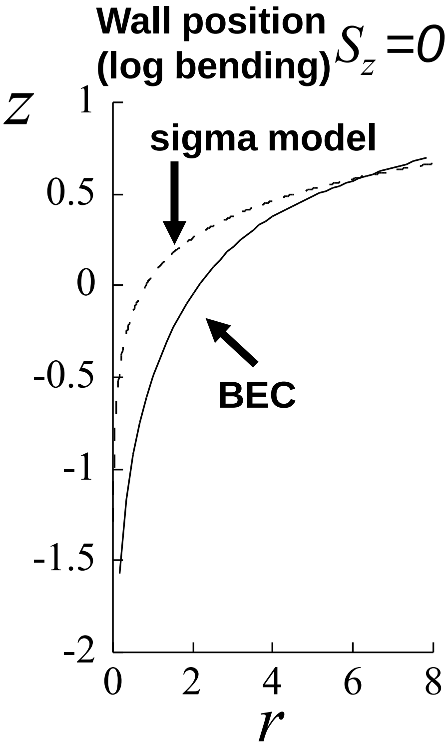
JHEP 1011:068,2010[arXiv:1002.4265]

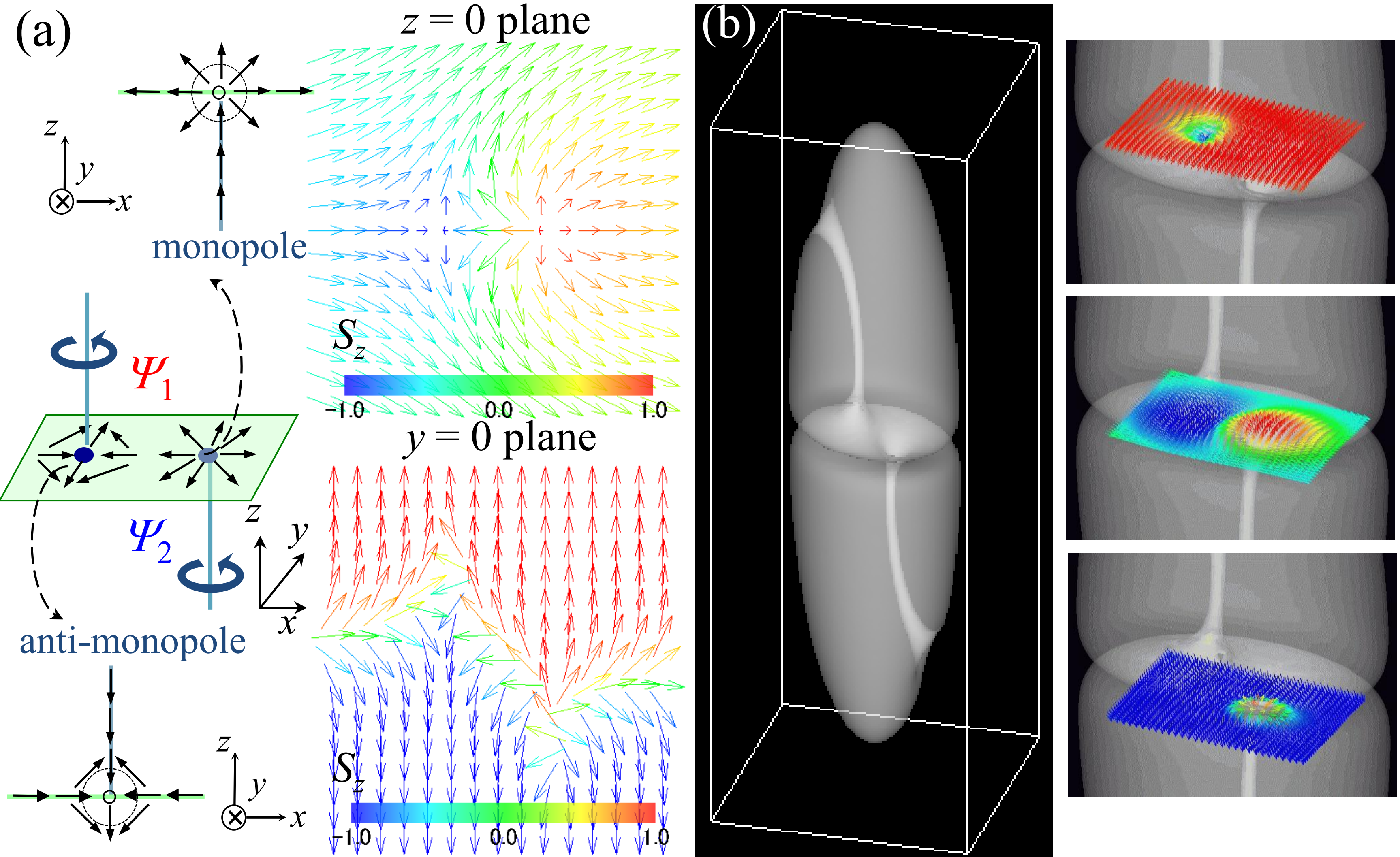




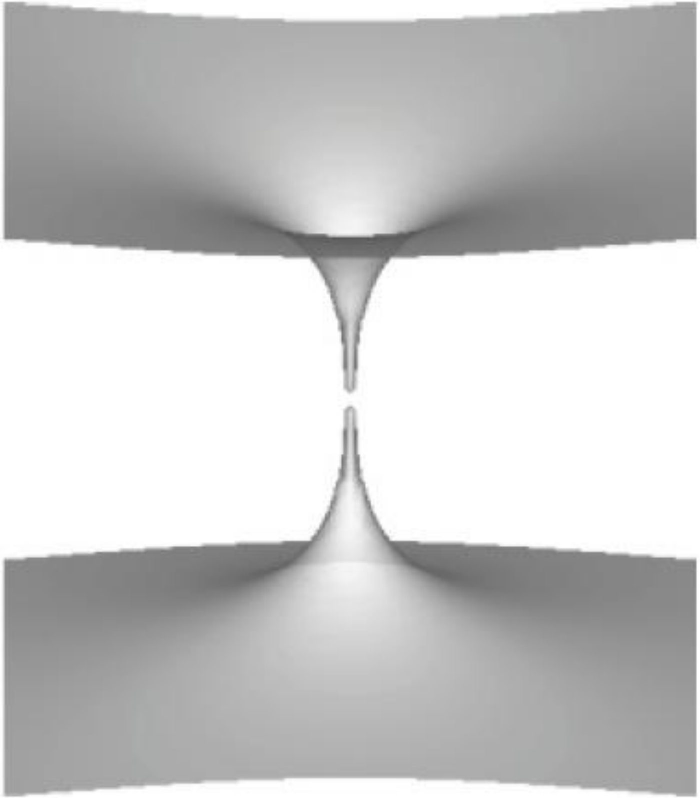


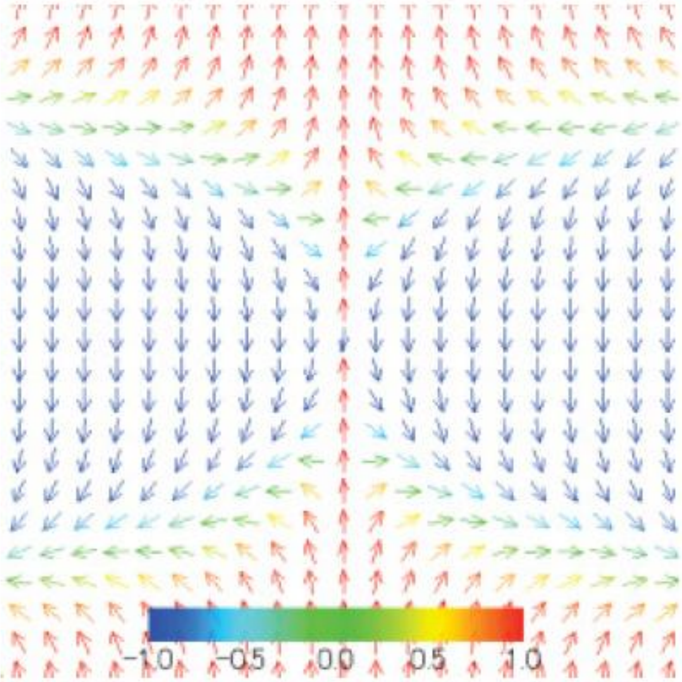





(a)
z = 0 plane
z
y
x
monopole
Ψ1
Ψ2
z
y
x
anti-monopole
z
y
x
S_z
-1.0
0.0
1.0
y = 0 plane
S_z
-1.0
0.0
1.0
(b)

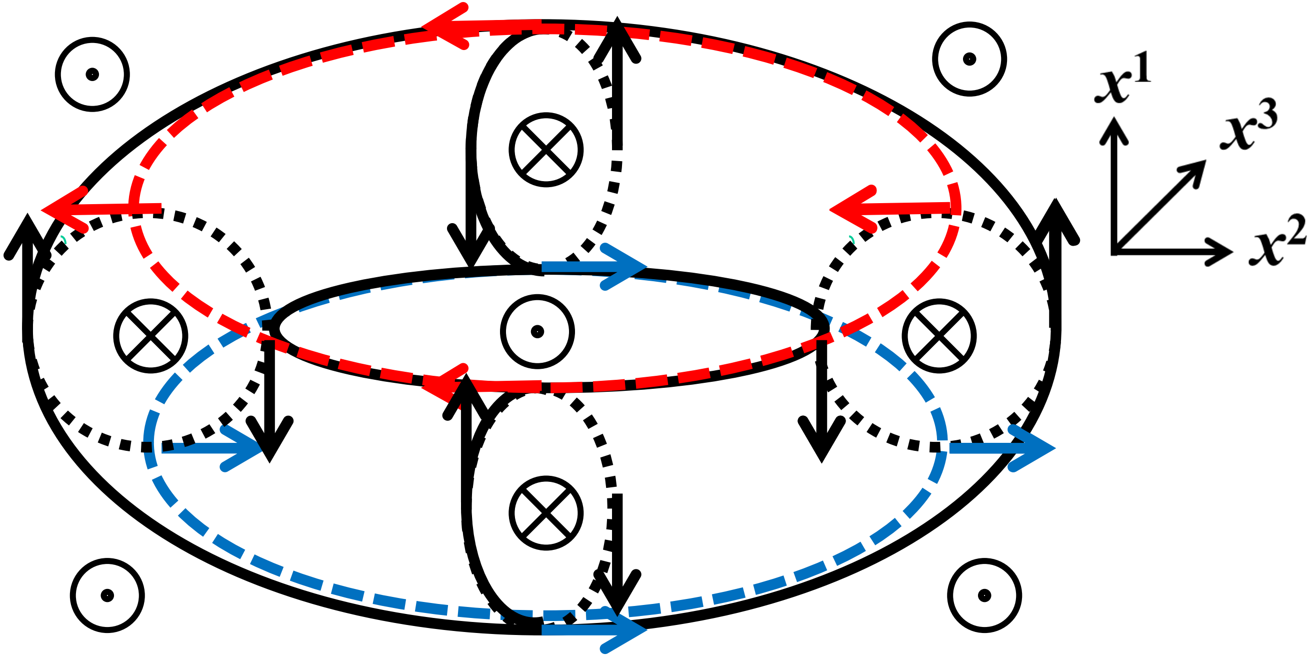
Analytic (approx) solution of brane-anti-brane with strings





$$Z(z) = \frac{\prod_{j=1}^{k_+}(z - z_j^+)}{\prod_{i=1}^{k_-}(z - z_i^-)}$$

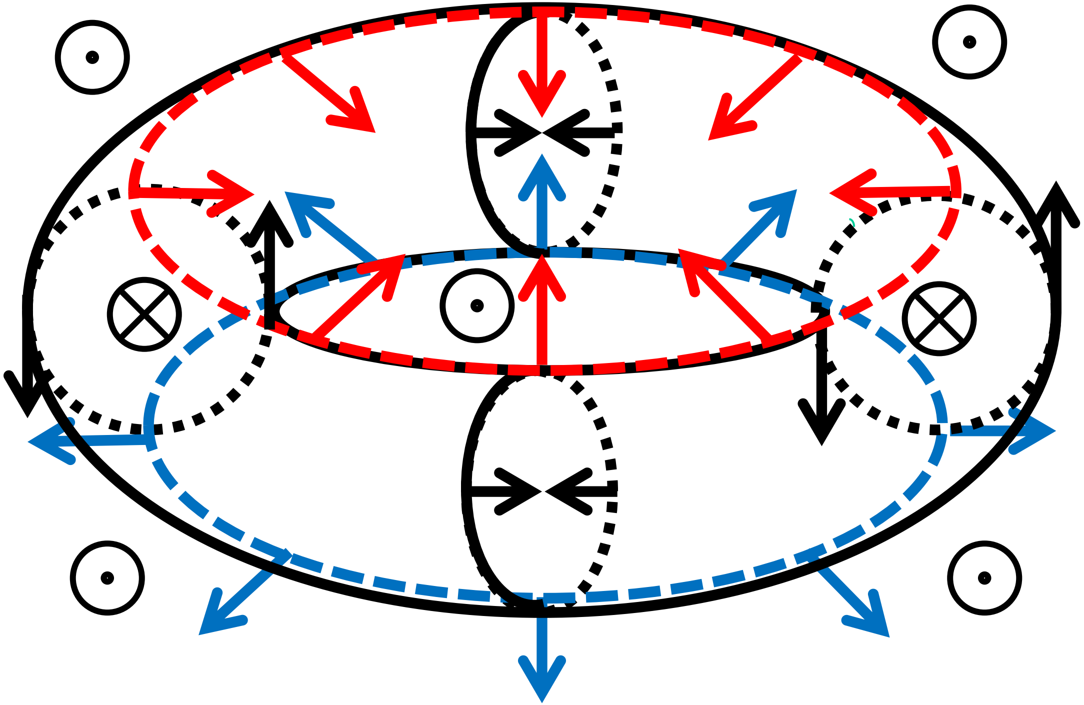
$$u_{\text{w-v-aw}} = \left(e^{-m(x^1 - x_1^1) + i\varphi_1} + e^{+m(x^1 - x_2^1) + i\varphi_2}\right)Z(z)$$

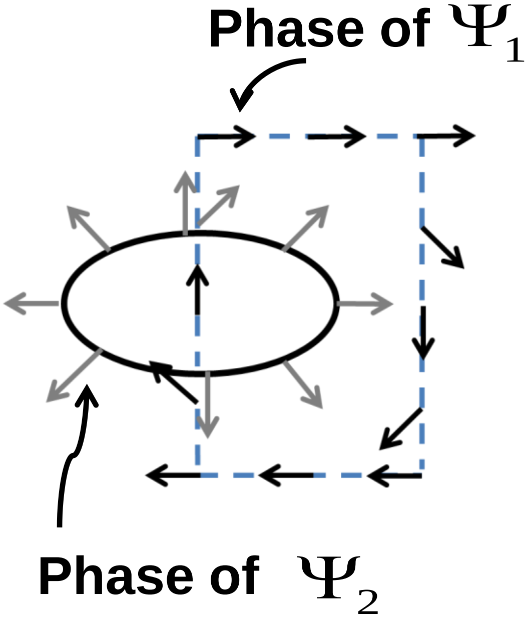


Untwisted loop

Unstable to decay

Vorton creation in BEC





Vorton

MN-Takeuchi-Kasamatsu-Tsubota

Phys.Rev.A85(2012)053639

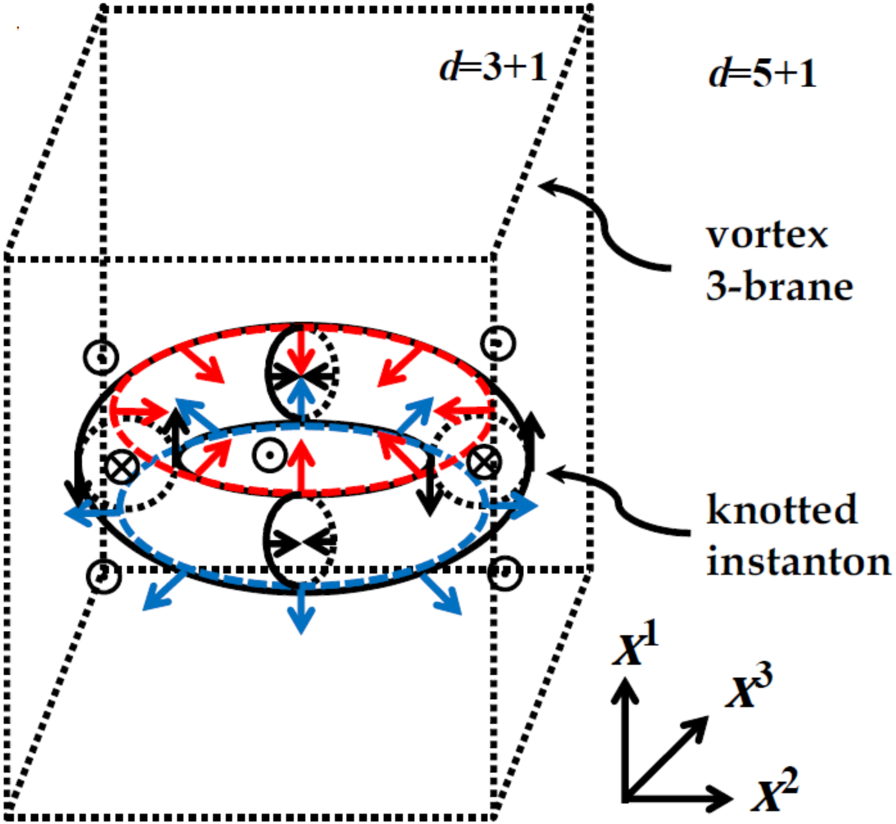
[arXiv:1203.4896 [cond-mat.quant-gas]]

Inside a NA vortex in *d=5+1*

$$\mathcal{L} = \frac{4\pi}{g^2}\frac{\partial_\mu u^* \partial^\mu u - m^2|u|^2}{(1+|u|^2)^2} + c\frac{|\partial_\mu u \partial^\mu u|^2}{(1+|u|^2)^4}$$

4 derivative correction

Eto-Fujimori-MN-Ohashi-Sakai
PTP(2012)
[arXiv:1204.0773[hep-th]]



Knotted instanton

MN,arXiv:1206.5551 [hep-th]