Higher Derivative Corrections to the effective theory of an Non-Abelian Vortex

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§1. Introduction

- **Non-Abelian Vortex** [Hannany, Tong, 2003]
  [Auzzi, et al, 2003]

- Abrikosov-Nielsen-Olesen (ANO) Vortex
  → color-flavor locked vac.
  of $U(N)$ gauge theory with $N$ flavor

- $\frac{1}{2}$ BPS config. in 4-dim. $N=2$ theory

- Orientational zero mode

$$\mathbb{C}P^{n-1} = \frac{SU(N)}{SU(N-1) \times U(1)}$$

Nambu-Goldstone zero mode
Exact correspondence of BPS spectra
(Kimura, Fujimori's talk)

- 4-d bulk theory
  - $N=2$, $U(N)$ gauge theory
  - $+N$ flavors

- 2-d vortex worldsheet theory
  - $N=(2,2)$ $\mathbb{C}P^{N-1}$ sigma model
  - + higher derivative corrections

\[ \uparrow \text{today's talk} \]

- vortex
- Yang-Mills Instantons
- vacua
- lumps
  - (O-model instanton)
  - holomorphic map
  - $\mathbb{C} \rightarrow 2$-cycle in $\mathbb{C}P^{N-1}$
Example: a particle on $\mathbb{R}^2$

$$L = \frac{m}{2} \left( \dot{r}^2 + r^2 \dot{\theta}^2 \right) - \frac{\Lambda}{2} \left( r^2 - r_0^2 \right)^2$$

vac. $r = r_0$, $\Theta$: moduli parameter

$\rightarrow$ $L_{\text{eff}} = \frac{m}{2} r_0^2 \dot{\Theta}^2$ with $\Theta = \Theta(t)$

derivative correction $r = r_0 + r^{(1)} + r^{(4)} + \ldots$

$\rightarrow L^{(4)} = m r_0 r^{(2)} \dot{\theta}^2 - \frac{\Lambda}{2} (r^{(2)})^2 r_0^2$

e.g. $m$ of $r^{(2)} \rightarrow r^{(2)} = \frac{m}{2 \Lambda r_0} \dot{\theta}^2$

$\Rightarrow L^{(4)} = \frac{m^2}{2 \Lambda} \dot{\Theta}^4$
higher derivative correction for Orientifolding zero mode for $U(2)$

$$S = \frac{4\pi\alpha}{g^2} \int dx \frac{|b|^2}{(1 + |b|^2)^2} + S^{(4)} + S^{(6)} + \ldots$$

$b \in \mathbb{C}P^1$ F.S metric, $\alpha = 1, 2$.

Single lump (instanton) solution

$$b = \frac{x^1 + i x^2}{a}$$

size moduli $\epsilon \in \mathbb{C}$

$$S = \frac{8\pi^2}{g^2} + S^{(4)}(a) + \ldots.$$  

$$= \frac{8\pi^2}{g^2} + \text{const.} \cdot \frac{1}{a^2} + \ldots$$

Our main result

**Instability?**

We checked the correction vanishing!
§ 2. Non-Abelian Vortices

4-dim \( N = 2 \) U(\( N \)) gauge theory + \( N \) hypermultiplets in fun. rep.

\[
L = \text{Tr} \left[ -\frac{1}{2g^2} F_{\mu \nu} F^{\mu \nu} + D_x H (D_x H)^\dagger - \frac{g^2}{4} (H H^\dagger - \nu^2 1_N)^2 \right]
\]

\( g \) gauge coupling const. \( \nu \) FI para.

\( U(N) \times SU(N) \rightarrow SU(N) \textit{c+f} \)

vacuum. \( H^2 = \nu 1_N \) (color-flavor locking vacuum)

\( U(N) \times SU(N) \rightarrow SU(N) \textit{c+f} \)

BPS equation \( (Z = x^3 + i x^4) \)

\[ D_{\bar{z}} H = 0, \quad i F_{z \bar{z}} = \frac{g^2}{4} (\nu^2 - H H^\dagger) \]

with a tension for static config.

\[
T = -\nu^2 \int dz d\bar{z} \text{Tr} F_{z \bar{z}} = 2\pi \nu^2 \nu \frac{1}{N} \tau_{\geq 0}
\]
BPS solutions

\[ H = S^{-1} H_0(z) \]

\[ A_{\bar{z}} = -i S^{-1} \partial_{\bar{z}} S \]

with \( S \in \text{GL}(N, \mathbb{C}) \),

\[ \text{master equation for } \Omega = SS^+ \]

\[ \partial_{\bar{z}}(\Omega \partial_{\bar{z}} \Omega^{-1}) = \frac{g^2}{4} (H_0 H_0^+ - I_N) \]

Single vortex sol

\[ H_0(z) = \begin{pmatrix} I_{N-1} & -b \\ 0 & z - \bar{z} \end{pmatrix} \]

\[ \text{in homogeneous coord. of } \mathbb{C}P^{N-1} \]

\[ \text{position of vortex} \]

\[ \Rightarrow F_{2\bar{z}} = F_{z\bar{z}} \times \frac{1}{1 + |b|^2} \begin{pmatrix} b \otimes b^+ & b \\ b^+ & 1 \end{pmatrix} \]
Derivative expansion and effective action
($\partial \alpha$, $\alpha = 1, 2$)

$H(x^m) = H^{(0)} + H^{(2)} + O(\partial \alpha^4) \quad \omega_m^{\ i} \ x^{1+i} x^2$

$A_{\bar{z}}(x^m) = A_{\bar{z}}^{(0)} + A_{\bar{z}}^{(2)} + O(\partial \alpha^4)$

$A_{\bar{z}}(x^m) = A_{\bar{z}}^{(1)} + O(\partial \alpha^3)$

- the zeroth order solution.

$\{ \phi^i \}$ : complex coordinates on the moduli space

$Z, \bar{Z}$

$\phi^i \rightarrow$ chiral (super) field $\phi^i(x^m) (Z(x), \bar{Z}(x))$

$H^{(o)} = H(x^{5,4}, \phi(x^{1,2}))$

$A_{\bar{z}}^{(0)} = A_{\bar{z}}^{so(8)} (x^{3,4}, \phi(x^{1,2}))$
• the first order Gauss law equation

\[ A^{(1)}_{\alpha=1,2} = i (\delta_{\alpha}^i \delta_{\alpha}^j \delta^i \delta^j - \delta^i \delta^j \delta_{\alpha}^i \delta_{\alpha}^j) \] [Eto et al. 2006]

\[ \delta_{\alpha} = \partial_{\alpha} \phi \frac{\partial}{\partial \phi}, \quad \delta^i = \partial_{\phi} \frac{\partial}{\partial \phi} \]

• the 2-nd order effective action

\[ S_{\text{eff}}^{(2)} = -T \int d^2 x \]

\[ \text{vortex tension} \]

\[ S_{\text{eff}}^{(2)} = \int d^2 x \left[ \frac{1}{2} \partial_\alpha \bar{z} \partial^\alpha \bar{z} + \frac{4\pi}{g^2} g_{ij} \partial_a \bar{z} \partial_b \bar{z} \right] \]

with Fubini–Study metric

\[ g_{ij} = \frac{2}{\partial_i \bar{z} \partial_j \bar{z}} \log (1 + |\bar{z}|^2) \]

[Hannay-Tong, 2003], [Gorsky-Shifman-Tung, 2004]
E8. Higher derivative corrections

- the 2-nd order solution

\[ \psi^{(2)} = \begin{pmatrix} H^{(2)} \\ A^{(2)} \end{pmatrix} \]

contribution from massive modes

should be orthogonal to

gauge, and physical zero modes

\[ \Delta \psi_i = 0 , \quad \Delta \equiv \begin{pmatrix} iD_{\bar{z}} & -H^{(0)} \\ \frac{1}{4} A^+ H^{(0)} & iD_{\bar{z}} \end{pmatrix} \]

solution

\[ \partial_\alpha \phi^i \psi_i = \begin{pmatrix} D_\alpha H^{(0)} \\ F_{\alpha \bar{z}} \end{pmatrix} \]

E.o.M for \( \psi^{(2)} \)

\[ 4 \Delta^+ \Delta \psi^{(2)} + \partial_\alpha (\partial_\alpha \phi^i \psi_i) = \Lambda^i \psi_i \]

Lagrange multiplier
We fortunately find

$$\Delta \psi^{(2)} = \frac{i}{2} \left( \frac{4}{g^2} \varepsilon \phi \partial \phi \right) S^+ \left[ \nabla^i \frac{\partial}{\partial \phi} (\overline{\Omega} \Omega^{-1}) \right] H_0$$

the 4-th order effective action for $\mathbb{C}P^{N-1}$

$$S^{(4)}_{\text{eff}} = \frac{4 \pi C}{g^4 v^2} \int d^2 x \left( g_{ij} \partial^\alpha \phi^i \partial^\beta \phi^j \right) \left( g_{k\ell} \partial^\delta \phi^k \partial^\rho \phi^\ell \right)$$

$\alpha, \beta = 1, 2$  \quad $C = 0.830707$
Euclidean action with $\mathcal{CP}^1$ ($U(2)$ case)

$$w = x' + i x^2$$

$$S^{(2+4)}_{\text{eff}} = \frac{4\pi}{g^2} \int d^3x \left[ \frac{1}{(1 + |b|^2)^2} \left( |\partial_x b|^2 + |\partial_x \bar{b}|^2 \right) + 4c \frac{1}{(1 + |b|^2)^2} \frac{\partial_x b \partial_x \bar{b}}{(1 + |b|^2)^2} \right]$$

$$= \frac{16\pi}{g^2} \int d^3x \left[ 1 + 4c \frac{|\partial_x b|^2}{(1 + |b|^2)} \right] \frac{1}{(1 + |b|^2)^2} + \frac{8\pi^2}{g^2} k$$

\text{instanton (lump) number} \quad \Rightarrow \quad k = 0

$$k = \frac{i}{2\pi} \int db \partial_x \bar{b} \frac{1}{(1 + |b|^2)^2} \in \mathbb{Z}$$

The lower bound saturated by \underline{holomorphic map} $b(\omega)$, $(\partial_x b = 0)$

$$\Rightarrow \quad S^{(4)}_{\text{eff}} \bigg|_{b = b(\omega)} = 0$$
§4. Conclusion

- General Formula for the 4-th order derivative corrections to the non-Abelian vortex effective action

- Concrete result for the single vortex effective action.

- The instanton (lump) solutions and the nonpole (kink) solutions do not accept any correction from higher derivative terms. (at least, in the 4-th order.)