Localization and Large-N reduction for N=2 quiver CS theory on S^3

岡田 崇
(京大理)

共同研究者 伊敷吾郎、島崎信二、浅野侑磨

arXiv:1203.0559[hep-th]　Phys. Rev. D 85, 106003 (2012) に基づく

Our Results

S^3上のplanar SUSY(N=2) quiver CS theory (以下ABJM theory)
に対して large-N reductionが成り立つことを非摂動的に証明した。

ABJM theory
on S^3

Reduced model
(a matrix model)

もとの理論のplanar極限を
再現するか？

非摂動的証明

pure CS theory: Ishiki-Shimasaki-Tsuchiya
同様の研究: Honda-Yoshida

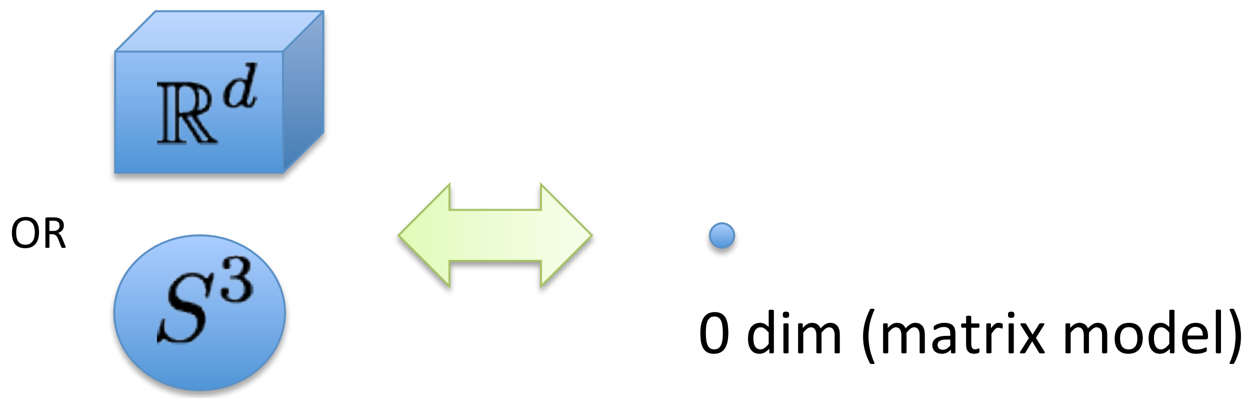
Plan

1. Large-N reduction (review)
2. Reduced model for ABJM theory
3. Localization method
4. Large-N equivalence
5. Conclusion

Large-N reduction [Eguchi-Kawai, Parisi, Gross-Kitazawa]

主張

large-N gauge theoryと、それをdimensional reductionして得られる行列模型(reduced model)は等価である。



意義

- large-N gauge theoryの、SUSYを保つ非摂動的定式化 (cf. 格子正則化)

- 超弦理論の行列模型による定式化への理解.

  - BFSS Matrix model [Banks-Fischler-Shenker-Susskind]
  - IIB 行列模型 [Ishibashi-Kawai-Kitazawa-Tsuchiya]

Reduction for scalar field on S^3

■ ϕ^4 theory (N×N hermitian)

$$S = \frac{1}{g^2}\int d\Omega_3 \mathrm{Tr}\left[-\frac{1}{2}(\mathcal{L}_a\phi(\Omega_3))^2 + \frac{m^2}{2}\phi(\Omega_3)^2 + \frac{1}{4}\phi(\Omega_3)^4\right]$$

Killing vectors on S^3
$a = 1, 2, 3$

Introduce a matrix [Ishii-Ishiki-Shimasaki-Tsuchiya]

$$L_a = \bigoplus_{s=-\frac{\Lambda}{2}}^{\Lambda/2} \mathbf{1}_N \otimes L_a^{[j_s]}$$

SU(2) generators in spin j_s rep.

$$2j_s + 1 = n + s$$

reduced model

$$S_{red} = \frac{1}{g_{red}^2}\mathrm{Tr}\left[-\frac{1}{2}[L_a,\phi]^2 + \frac{m^2}{2}\phi^2 + \frac{1}{4}\phi^3\right]$$

S^3上のLarge-N reductionの主張

Background

$$L_a = \bigoplus_{s=-\frac{\Lambda}{2}}^{\Lambda/2} \mathbf{1}_N \otimes L_a^{[j_s]}$$

$$2j_s + 1 = n + s$$

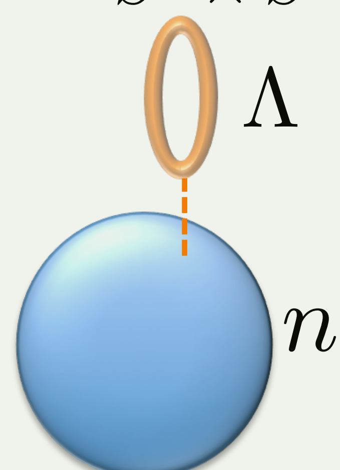
SU(2) generators in spin j_s rep.

Reduced modelと連続理論は以下の極限で等価

$$\begin{cases} n \to \infty & \text{(非可換 S}^2\text{ の可換極限)} \\ \Lambda \to \infty & \text{(S}^1\text{ fibration on S}^2\text{)} \\ N \to \infty & \text{(planar limit)} \\ g^2 N \equiv g_r^2 N/n := \lambda = \text{fixed} & \end{cases}$$

locally

$S^3 \sim S^2 \times S^1$



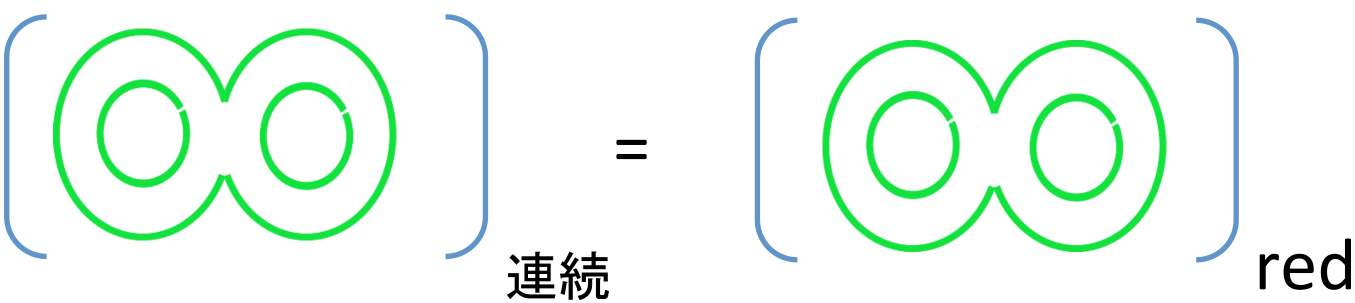
摂動論に基づく証明

■ ϕ^4 theory

連続理論 $$S = \frac{1}{g^2}\int d\Omega_3 \mathrm{Tr}\left[-\frac{1}{2}(\mathcal{L}_a\phi(\Omega_3))^2 + \frac{m^2}{2}\phi(\Omega_3)^2 + \frac{1}{4}\phi(\Omega_3)^4\right]$$

Reduced model $$S_{red} = \frac{1}{g_{red}^2}\mathrm{Tr}\left[-\frac{1}{2}[L_a,\phi]^2 + \frac{m^2}{2}\phi^2 + \frac{1}{4}\phi^3\right]$$

Reduced modelにおいて、対応するFeynman図を計算 → 連続理論の結果を再現



Reduction for YM theory on S^3

YM action on S^3

$$\int d\Omega_3 \mathrm{Tr} F_{ab}^2 = \int d\Omega_3 \mathrm{Tr}(\epsilon_{abc}A_c + i\mathcal{L}_a A_b - i\mathcal{L}_b A_a - i[A_a, A_b])^2$$

where $A = A_a e^a$ (expanded in left inv 1-from)

$$S_{red} = \mathrm{Tr}(\epsilon_{abc}A_c + i[L_a, A_b] - i[L_b, A_a] - i[A_a, A_b])^2$$

$$= \mathrm{Tr}(\epsilon_{abc}\mathcal{A}_c - i[\mathcal{A}_a, \mathcal{A}_b])^2 \quad (\mathcal{A}_a \equiv A_a \boxed{- L_a})$$

もとの作用をdimensional reductionしたもの

S^3を再現するbackground $\mathcal{A}_a^{bg} = \boxed{-L_a}$ のまわりで展開する

Comments

- 平坦な空間でのlarge-N reductionにはU(1)^D対称性の破れの問題がある。

  ([Bhanot–Heller–Neuberger], [Azeyanagi–Hanada–Unsal–Yacoby])

  → 一般にはLarge-N equivalenceが成り立たない

- S^３上におけるlarge-N reductionではこの問題はない
  - Moduliは離散的 （解はsu(2)の表現で指定)
  - Large-Nではtunnelingはない→特定の表現を取り出せる
  - susyによってprotect

Plan

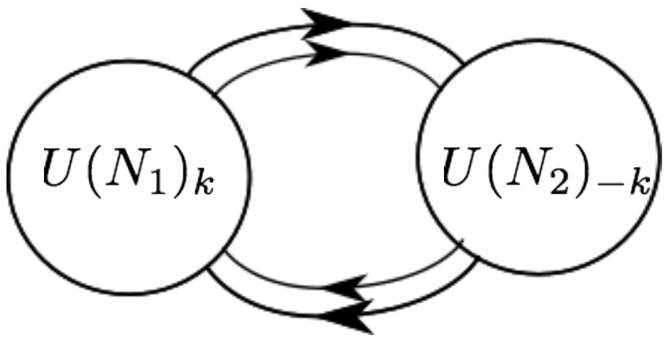
1. Large-N reduction (review) ✔
2. Large-N reduction for ABJM theory
3. Localization method
4. Large-N equivalence
5. Conclusion

ABJM theory on S^3

Fields

Vector mult.

$$(A_\mu, \sigma, \lambda, D)$$



Vector mult.

$$(B_\mu, \rho, \lambda', D')$$

σ, D : scalar場

λ : gaugino

chiral mult.

$$(\phi_i, \psi_i, F_i)_{i=1,2,3,4}$$

Reduced model for ABJM

$$S = S_{CS} + S_{matter}$$

$$S_{CS} = -\int d^3x \, \mathrm{tr}\Big[\varepsilon^{\mu\nu\lambda}(A_\mu \partial_\nu A_\lambda - \frac{2i}{3} A_\mu A_\nu A_\lambda) + \sqrt{g}(-\bar{\lambda}\lambda + 2D\sigma)\Big]$$

$$S_{matter} = \int d^3x \sqrt{g} \, \mathrm{tr}[D^\mu \bar{\phi} D_\mu \phi - i\bar{\psi}\gamma^\mu D_\mu \psi + \cdots]$$

Dimensional reduction

$$S^{red} = S^{red}_{CS} + S^{red}_{matter}$$

where

$$S^{red}_{CS} \equiv -\frac{1}{g^2}\mathrm{Tr}\left[A_a A^a - \frac{i}{3}\epsilon^{abc} A_a A_b A_c - \frac{1}{2}\bar{\lambda}\lambda + D\sigma\right] + \left[\begin{array}{c} (A_\mu, \sigma, \lambda, D) \\ \rightarrow (B_\mu, \rho, \lambda', D') \end{array}\right]$$

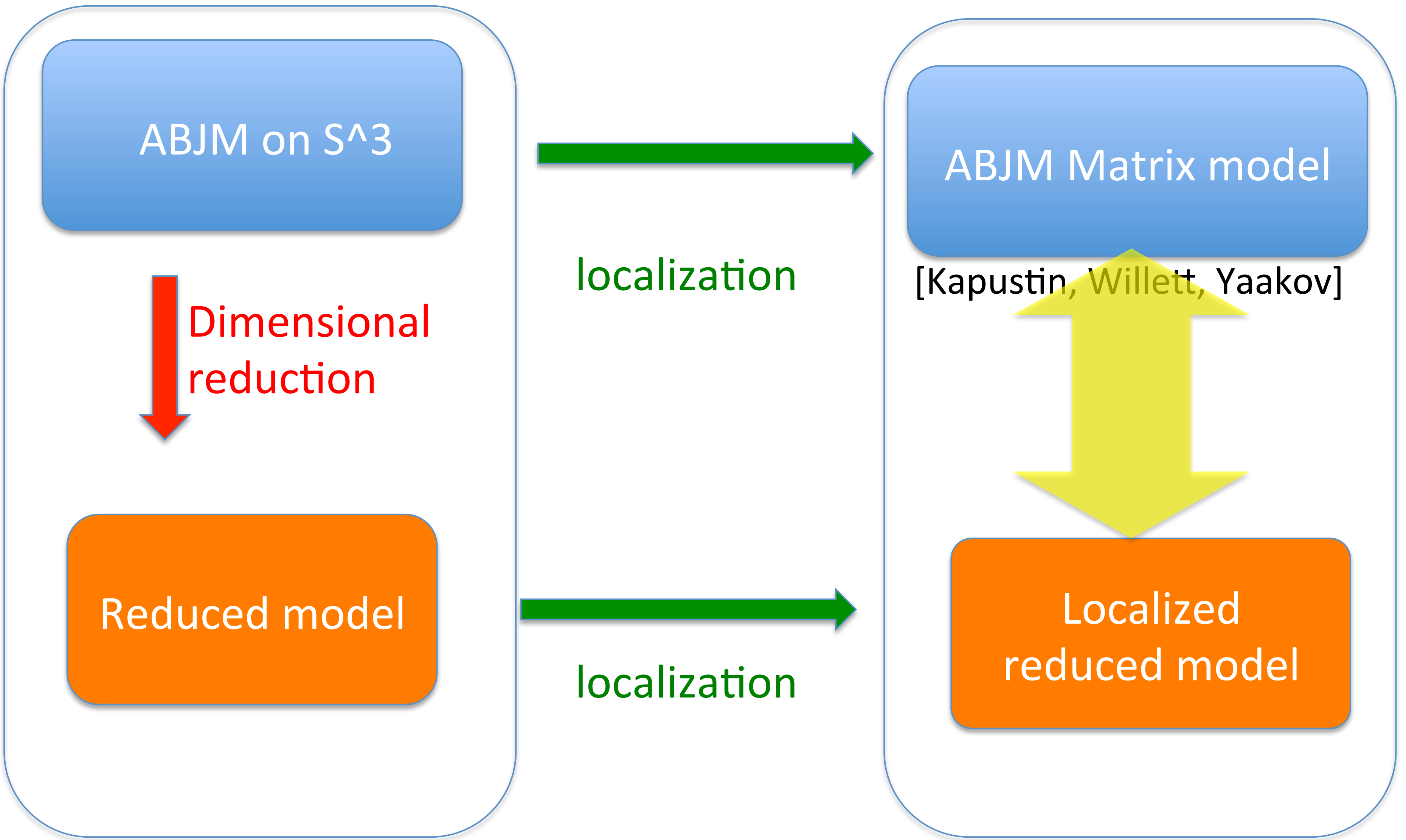
$$S^{red}_{matter} \equiv \mathrm{Tr} \; \bar{\phi} \, \nabla_{A,B} \, \phi - \bar{\psi}\gamma^a \nabla_{A,B} \, \psi + \cdots$$

where $\nabla_{A,B}\phi \equiv A\phi - \phi B$

Plan

1. Large-N reduction (review) ✔
2. Large-N reduction for ABJM theory ✔
3. Localization method
4. Large-N equivalence
5. Conclusion

Large-N equivalenceを非摂動的に証明したい



Localizationの一般論

$$\int \mathcal{D}\phi \, \exp(-S)$$

deform

$$Z(t) \equiv \int \mathcal{D}\phi \, \exp\left(-S - t\,\delta V\right)$$

with $\delta S = 0$ $\delta^2 = \mathcal{L}_B$ $\delta(\mathcal{D}\phi) = 0$ $\delta V|_{boson} \geq 0$

$$Z(t=0) = Z(t \to \infty)$$

$\delta V\Big|_{boson}$を最小にする場の配位に``localize''。1-loop exact。

Localization in reduced model

S^3上の quiver CS理論から得たreduced modelでは・・・

$$S^{red} = S^{red}_{CS} + \quad S^{red}_{matter}$$

$$S^{red} = S^{red}_{CS} + S^{red}_{matter} + tQV$$

SUSY-exactなdeformation

$$QV|_{boson} = \frac{1}{4}F_{ab}^2 + (\sigma + D)^2 - [A_a, \sigma]^2 + \cdots$$ を最小にする配位にlocalize

where $$F_{ab} \equiv 2\epsilon_{abc}A^c - i[A_a, A_b]$$

Localizing point

$$F_{ab} = 0,\ \sigma + D = 0,\ [A_a, \sigma] = 0$$ (matter場=0)

解 $A_a = -2\times$ (su(2)の任意の表現行列)_a

S^3に対応する背景を選ぶ: $A_a = -2 \bigoplus_{s=-\Lambda/2}^{\Lambda/2} L_a^{[j_s]} \otimes \mathbf{1}_N$

$$\sigma = \bigoplus_{s=-\Lambda/2}^{\Lambda/2} \mathbf{1}_{2j_s+1} \otimes \sigma_s$$

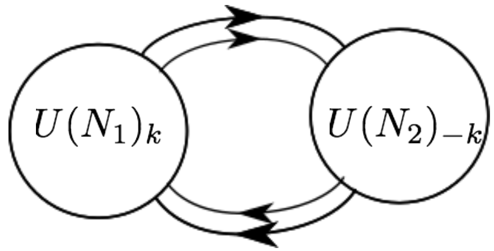
Λ＋１個のN×N行列

このlocalizing pointのまわりで1-loop積分を実行。

Localized reduced model for ABJM theory

scalar場

$$\sigma = \bigoplus_{s=-\Lambda/2}^{\Lambda/2} \mathbf{1}_{2j_s+1} \otimes \sigma_s$$



scalar場

$$\rho = \bigoplus_{s=-\Lambda/2}^{\Lambda/2} \mathbf{1}_{2j_s+1} \otimes \rho_s$$

moduli　Λ+1個の $N_1 \times N_1$ 行列 & Λ+1個の $N_2 \times N_2$ 行列

$$\int \prod_{s=-\Lambda/2}^{\Lambda/2} \left(\prod_{i=1}^{N_1} d\sigma_{si} \prod_{\alpha=1}^{N_2} d\rho_{s\alpha} \right) \mathcal{M}_{gauge} \mathcal{M}_{matter} \exp\left(-\frac{N_1}{t_1} \sum_{s,i} \sigma_{si}^2 + \frac{N_2}{t_2} \sum_{s,\alpha} \rho_{s\alpha}^2 \right)$$

where

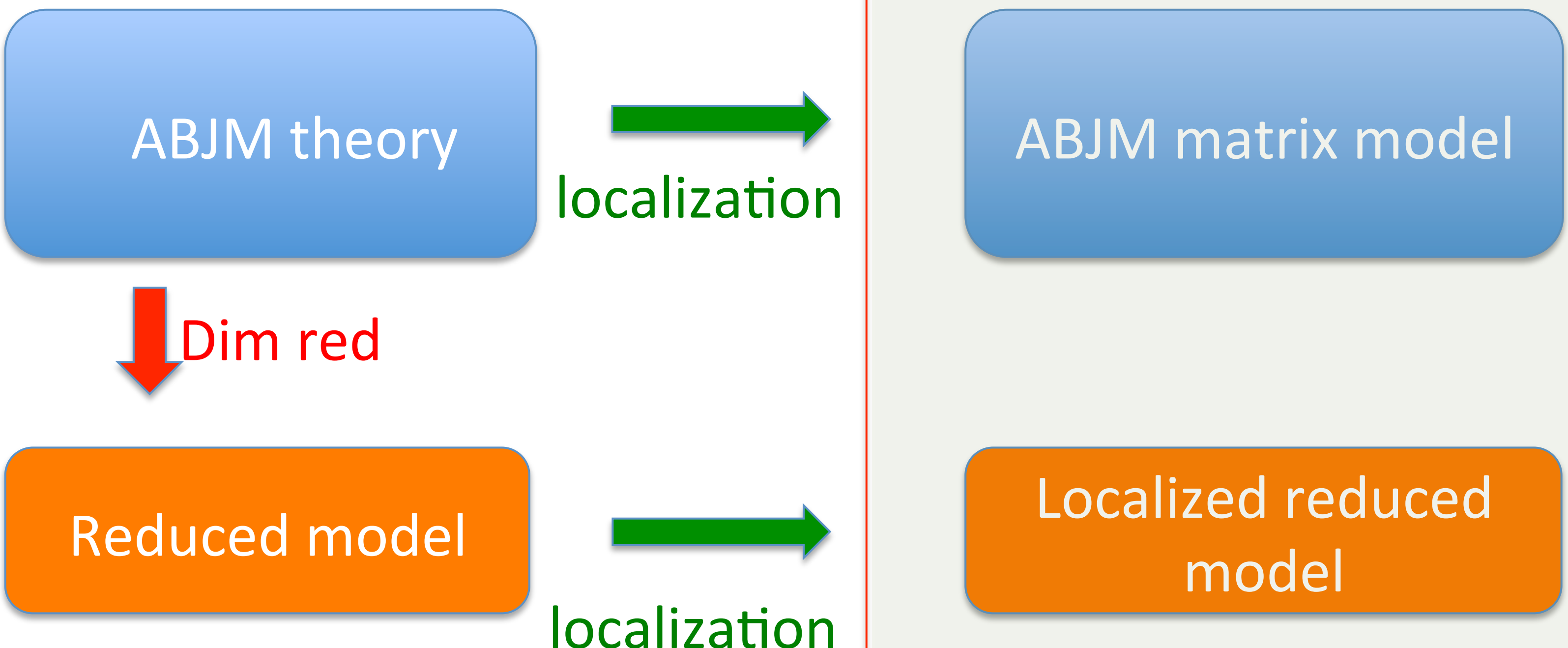
$$\mathcal{M}_{gauge} = \prod_{s} \prod_{i<j} (\sigma_{si} - \sigma_{sj})^2 \prod_{s<t} \prod_{i,j} \left[1 + \frac{(\sigma_{si} - \sigma_{tj})^2}{(s-t)^2} \right] \times \prod_{s} \prod_{\alpha<\beta} (\rho_{s\alpha} - \rho_{s\beta})^2 \prod_{s<t} \prod_{\alpha,\beta} \left[1 + \frac{(\rho_{s\alpha} - \rho_{t\beta})^2}{(s-t)^2} \right]$$

$$\mathcal{M}_{matter} = \prod_{s,t} \prod_{i,\alpha} \prod_{J=|s-t|/2}^{\infty} \left(\frac{(2J+\frac{3}{2})^2 + (\sigma_{si} - \rho_{t\alpha})^2}{(2J+\frac{1}{2})^2 + (\sigma_{si} - \rho_{t\alpha})^2} \right)^2$$

Plan

1. Large-N reduction (review) ✔
2. Large-N reduction for ABJM theory ✔
3. Localization method ✔
4. Large-N equivalence
5. Conclusion

Large-N reductionの非摂動的証明



Localized reduced modelは
ABJM MMを再現しているか？

- free energy
- Wilson loops

ABJM matrix model

$$\int \prod_{i=1}^{N_1} d\sigma_i \prod_{\alpha=1}^{N_2} d\rho_\alpha \frac{\boxed{\prod_{i<j}\sinh^2(\pi(\sigma_i-\sigma_j))}\,\boxed{\prod_{\alpha<\beta}\sinh^2(\pi(\rho_\alpha-\rho_\beta))}}{\boxed{\prod_{i,\alpha}\cosh^2(\pi(\sigma_i-\rho_\alpha))}} e^{-\frac{2\pi^2}{g_s}\sum_i \sigma_i^2+\frac{2\pi^2}{g_s}\sum_\alpha \rho_\alpha^2}$$

Localized reduced model

Λ+1個の行列 σ_s, ρ_s

$$\int \prod_{s=-\Lambda/2}^{\Lambda/2}\left(\prod_{i=1}^{N_1} d\sigma_{si}\prod_{\alpha=1}^{N_2} d\rho_{s\alpha}\right)\mathcal{M}_{gauge}\mathcal{M}_{matter}\exp\left(-\frac{N_1}{t_1}\sum_{s,i}\sigma_{si}^2+\frac{N_2}{t_2}\sum_{s,\alpha}\rho_{s\alpha}^2\right)$$

$$\mathcal{M}_{gauge}=\boxed{\prod_s\prod_{i<j}(\sigma_{si}-\sigma_{sj})^2\prod_{s<t}\prod_{i,j}\left[1+\frac{(\sigma_{si}-\sigma_{tj})^2}{(s-t)^2}\right]}\times\boxed{\prod_s\prod_{\alpha<\beta}(\rho_{s\alpha}-\rho_{s\beta})^2\prod_{s<t}\prod_{\alpha,\beta}\left[1+\frac{(\rho_{s\alpha}-\rho_{t\beta})^2}{(s-t)^2}\right]}$$

$$\boxed{\mathcal{M}_{matter}=\prod_{s,t}\prod_{i,\alpha}\prod_{J=|s-t|/2}^{\infty}\left(\frac{(2J+\frac{3}{2})^2+(\sigma_{si}-\rho_{t\alpha})^2}{(2J+\frac{1}{2})^2+(\sigma_{si}-\rho_{t\alpha})^2}\right)^2}$$

Saddle point方程式に基づく議論

Saddle point方程式を比較。(注:Large-N limitではsaddle point による評価がexactに.)

ABJM MM

固有値密度 $\rho(x) \equiv \frac{1}{N_1}\sum_{i=1}^{N_1}\delta(x-\sigma_i)$ $\tilde{\rho}(x) \equiv \frac{1}{N_2}\sum_{\alpha=1}^{N_2}\delta(x-\rho_\alpha)$

$$0 = \frac{1}{\pi\lambda_1}x - P\int dy\,\coth\{\pi(x-y)\}\ \rho(y) + \cdots$$

解 $(\rho,\tilde{\rho}) = (\rho_0,\tilde{\rho}_0)$

$$\frac{1}{\pi}\sum_{n=-\infty}^{\infty}\frac{x-y}{n^2+(x-y)^2} = \coth\ \pi(x-y)$$

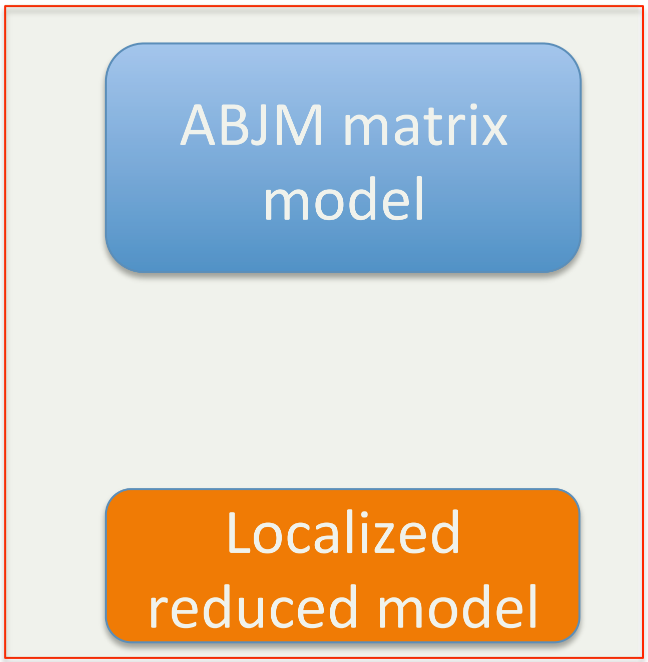
Localized

固有値 $\sum_{\alpha=1}^{N_2}\delta(x-\rho_{s\alpha})$

$$0 = \frac{1}{t_1}x - P\int dy\frac{1}{x-y}\boxed{\rho_0(y)} - \sum_{t(\neq s)}\int dy\frac{x-y}{(s-t)^2+(x-y)^2}\boxed{\rho_0(y)} + \cdots$$

$(\rho^{[s]},\tilde{\rho}^{[s]}) = (\rho_0,\tilde{\rho}_0)$ (for $^\forall s$) は reduced model の解に

Large-N equivalenceの証明



両理論で同一の固有値分布
であることがわかった

Reduced modelの自由エネルギーは
ABJM theoryに一致にすることがわかる

$$F_{red} = \frac{1}{\Lambda+1} \sum_{s=-\Lambda/2}^{\Lambda/2} \int dx \frac{x^2}{t} \rho^{[s]}(x) + \cdots \bigg|_{(\rho^{[s]},\tilde{\rho}^{[s]})=(\rho_0,\tilde{\rho}_0)}$$

$$= \int dx \frac{x^2}{t} \rho_0(x) + \cdots$$

$$= F_{ABJM}$$

(証明終わり)

Conclusion

- 曲がった時空のlarge-N reductionを（超対称な物理量に関して）非摂動的に証明

- 一般のN=2 quiver CS theoryに対しても証明できる。

- Wilson loopに関しても証明できる。

- ほかの物理量 (indexなど)。

- ほかの多様体。