On supersymmetric interfaces in string theory

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Plan

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3. Susy b.d. states for GS strings
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1. Introduction

Conformal (world-sheet) interface:

natural extension of conformal boundary

\[
\begin{align*}
\text{CFT} & | \hspace{1cm} \text{CFT1} | \hspace{1cm} \text{CFT2} \\
\text{boundary} & | \hspace{1cm} \text{interface (defect)} \\
\text{cond. mat. w/ boundary} & | \hspace{1cm} \text{cond. mat. w/defect} \\
\text{D-brane} & | \hspace{1cm} \text{??}
\end{align*}
\]
may play an interesting role in

- condensed matter phys.
- CFT
- string theory
In fact,

- transform set of D–branes to another  
  [Graham–Watts ’03]

- “generator” of RG flow  
  [cf. Gaiotto ’12]  
  [Graham–Watts ’03]

- “generator” of symmetry (duality) of RCFT  
  [Frohlich–Fuchs–Runkel–Schweigert ’04, ’07]

- target–space interpretation as “bi–brane”  
  in G x G (WZW model)  
  [Fuchs–Schweigert–Waldorf ’07]

- fusion of interfaces  
  ≈ spectrum generating “algebra” of string theory?  
  [cf. Geroch group, U–duality group]  

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<th>CFT1</th>
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However,

- not fully understood, especially, in string theory

- generally, only 1 Virasoro is preserved
  \[ \implies \text{ghost problem if embedded in string theory?} \]

  [Bachas–de Boer–Dijkgraaf–Ooguri ’01]

- conformal interface in full string
  world-sheet had not been discussed
Motivation of this work is simple:

- to construct interface in string theory [fixed genus]
- to study its basic properties
- to address issue of ghost, if possible

To avoid subtleties concerning ghosts, work w/ Green–Schwarz formulation in light–cone gauge
2. Conformal interfaces

World-sheet conformal interface

- consider 1 dim. defect/interface in 2dim. world-sheet
- condition to keep conformal Ward id.

\[
T_1(z) - \tilde{T}_1(\bar{z}) \approx T_2(z) - \tilde{T}_2(\bar{z})
\]

[along interface]

- when \( T_1(z) \approx T_2(z) , \tilde{T}_1(\bar{z}) \approx \tilde{T}_2(\bar{z}) \)

\[ \Rightarrow \]
- interface : freely deformed
- called topological interface
**(Un)folding trick**

- a way to construct interface: unfold conformal boundary

- take b.d. state in $\text{CFT1} \otimes \text{CFT2}$

\[
|\mathcal{B}\rangle = \sum_{i,j} c_{ij} |\mathcal{B}_i\rangle_1 \otimes |\mathcal{B}_j\rangle_2
\]

\[
\text{s.t. } (L^1_n + L^2_n - \tilde{L}^1_{-n} - \tilde{L}^2_{-n})|\mathcal{B}\rangle = 0
\]

then,

\[
\mathcal{I} = \sum_{i,j} c_{ij} |\mathcal{B}_i\rangle_1 \cdot 2\langle\overline{\mathcal{B}_j}|
\]

satisfies

\[
(L^1_n - \tilde{L}^1_{-n})\mathcal{I} = \mathcal{I}(L^2_n - \tilde{L}^2_{-n}) \quad \iff \text{interface}
\]
3. Susy boundary states for GS string

**Type II GS string**

- notation (IIB):

\[ \alpha^I_n, \tilde{\alpha}^I_n \ (I = 1, ..., 8) : \text{right, left bosonic modes} \]
\[ S^a_n, \tilde{S}^a_n \ (a = 1, ..., 8) : \text{so(8) spinor modes} \]
\[ \gamma^I = \begin{pmatrix} 0 & \sigma^I \\ \bar{\sigma}^I & 0 \end{pmatrix} : \text{8-dim. gamma matrices} \]

- supercharges

**linear susy** \[ Q^a := \sqrt{2p^+} S^a_0 \]

**non-linear susy** \[ \dot{Q}^a := \frac{1}{\sqrt{p^+}} \sigma^I_{a\dot{a}} \sum_n S^a_{-n} \alpha^I_n \]
D–branes  [Green–Gutperle ’ 96]

- GS string in l.c.–gauge: not a CFT
- Conformal boundary states

⇒ space–time susy boundary states s.t.

\[(Q^a + iM_{ab}\tilde{Q}^b)|\mathcal{B}\rangle = (\tilde{Q}^\dot{a} + iM_{\dot{a}\dot{b}}\tilde{Q}^{\dot{b}})|\mathcal{B}\rangle = 0\]

⇒ \[|\mathcal{B}(M)\rangle = \mathcal{C} \prod_{n=1} \exp\left[\frac{1}{n}M_{IJ}\alpha^I_n\tilde{\alpha}^J_n - iM_{ab}S^a_n\tilde{S}^b_n\right]|\mathcal{B}\rangle_{b0}|\mathcal{B}\rangle_{f0}\] [zero mode]

\[M_{KL} = (e^{\omega_{IJ}\Sigma^{IJ}})_{KL}, \quad M_{\alpha\beta} = \left(e^{\frac{1}{2}\omega_{IJ}\gamma^{IJ}}\right)_{\alpha\beta} = \left(\begin{array}{cc} M_{ab} & 0 \\ 0 & M_{\dot{a}\dot{b}} \end{array}\right)\]

- b.d. cond. \[(\alpha^I_n - M_{IJ}\tilde{\alpha}^J_n)|\mathcal{B}\rangle = (S^a_n + iM_{ab}\tilde{S}^b_n)|\mathcal{B}\rangle = 0\]
- In GS, “conformal” $\Rightarrow$ “space–time supersymmetric”

Conformal Interface $\Rightarrow$ Susy Interface
4. Susy interface for GS string

- we would like to find susy interface defined by

\[
(Q_1^a + iR_{ab}^1 \tilde{Q}_1^b)\mathcal{I} = \mathcal{I}(Q_2^a + iR_{ab}^2 \tilde{Q}_2^b)
\]

\[
(Q_1^a + iR_{\dot{a}b}^1 \tilde{Q}_1^\dot{b})\mathcal{I} = \mathcal{I}(Q_2^\dot{a} + iR_{\dot{a}b}^2 \tilde{Q}_2^\dot{b})
\]

for some \( R_{ab}^A, R_{\dot{a}b}^\dot{A} \) (\( A = 1, 2 \)) \text{ [IIB case]}

- for this,
  - prepare boundary states w/ “doubled” fields

\[
(\alpha_n^{AI}, \tilde{\alpha}_n^{AI}), (S_n^{Aa}, \tilde{S}_n^{A\dot{a}}) \quad \text{[just as an intermediate step]}
\]

  - unfold this to interface
  - impose susy cond.
(analog of) topological interface

- after some analysis, obtain, e.g.,

\[
\mathcal{I}(M^1, M^2) = C \prod_{n=1}^{\infty} e^{\frac{1}{n} \left( \alpha_{-n}^1 M_{KJ}^1 M_{KIJ}^1 \tilde{\alpha}_{-n}^1 \tilde{\alpha}_{n}^2 J \right)} \times e^{S_{-n}^1 S_{n}^2 + M_{ca}^1 M_{cb}^2 \tilde{S}_{-n}^1 \tilde{S}_{n}^2 b} \cdot \mathcal{I}_{b0} \mathcal{I}_{f0}
\]

\[
\mathcal{I}_{b0} = \sum |k^{1I}, \tilde{k}^{1J}\rangle \langle \tilde{k}^{2K}, k^{2L} |
\]

\[
k^{1I} = k^{2I}, M^1_{IJ} \tilde{k}^{1J} = M^2_{IJ} \tilde{k}^{2J}
\]

\[
\mathcal{I}_{f0} = |I\rangle T_I \langle I| + |\dot{a}\rangle T_I \langle \dot{a}|
\]

\[
T_I = M^1_{PJK} M^2_{PK} J \langle K| + M^1_{pb} M^2_{p\dot{c}} |\dot{b}\rangle \langle \dot{c}|
\]

\[
\Rightarrow \quad Q^a_1 \approx Q^a_2, \quad M^1_{ab} \tilde{Q}^b_1 \approx M^2_{ab} \tilde{Q}^b_2
\]

\[
Q_{\dot{a}}^1 \approx Q_{\dot{a}}^2, \quad M^1_{\dot{a}b} \tilde{Q}^b_1 \approx M^2_{\dot{a}b} \tilde{Q}^b_2
\]

\[
S_{1a}^1 \approx S_{2a}^2, \quad M^1_{ab} \tilde{S}^{1b}_1 \approx M^2_{ab} \tilde{S}^{1b}_2
\]

\[
\partial_{-} X_{1I} \approx \partial_{-} X_{2I}, \quad M^1_{IJ} \partial_{+} X_{1J} \approx M^2_{IJ} \partial_{+} X_{2J}
\]

\[
T_1 \approx T_2, \quad \tilde{T}_1 \approx \tilde{T}_2
\]

“topological”
5. Properties

- in the following, we
  - concentrate on un-compactified case
  - set for simplicity

\[
M_{IJ}^A = \begin{pmatrix}
 -1_{p_A+1} & 0 \\
 0 & 1_{7-p_A}
\end{pmatrix} \quad (p_1 < p_2)
\]
**Target space geometry**

- target space geometry is probed by localized states

\[ |x\rangle = \int d^8 k \, e^{-i k \cdot x} |k\rangle \]

- then,

\[ \langle x | \mathcal{I}(Y) | x' \rangle \sim \prod_{I=p_2+2}^8 \delta(x_I - x'_I - Y^I) \]

⇒ Interface: localized in a submanifold (bi-brane)

\[ x = x' + Y \]

in doubled (transverse) target space \( \mathbb{R}^8 \times \mathbb{R}^8 \ni (x, x') \)
**Coupling through interfaces**

- consider massless NS–NS fields $|\zeta\rangle := \zeta_{IJ}|I\rangle|J\rangle$
  \[\text{[spinor zero mode]}\]
- coupling is read off from $\langle\langle \zeta | \mathcal{I} | \zeta' \rangle \rangle$
- e.g., when $p_1 + 1 = p_2 =: p$ (IIB–IIA), this gives

\[-\zeta^{*}_{p+1} p+1 \zeta'_{p+1} p+1 + \zeta^{*}_{(p+1) I} \zeta'_{(p+1) I} + \zeta^{*}_{(p+1) I} \zeta'_{(p+1) I} + \sum_{I,J \neq p+1} \zeta^{*}_{IJ} \zeta'_{IJ}\]

$\Rightarrow$ nothing but Buscher rules (T–duality)

\[g'_{p+1} p+1 = \frac{1}{g_{p+1} p+1}, \quad g'_{p+1} I = \frac{b_{p+1} I}{g_{p+1} p+1}, \quad b'_{p+1} I = \frac{g_{p+1} I}{g_{p+1} p+1}\]

\[\phi' = \phi - \frac{1}{2} \log g_{p+1} p+1\]
Transformation of D–branes

- interface transforms D–branes as

\[ |\mathcal{B}'(M')\rangle = \lim_{q \to 1} \mathcal{I}(M^1, M^2) q^{L_0^2 + \tilde{L}_0^2} |\mathcal{B}(M)\rangle \]

- in the present case, results in SO(8) trans.

\[ M' = M (M^2)^T M^1 \]

Partition fn. w/ interfaces inserted

- modes coupled to interfaces
- susy breaking, Casimir energy
6. Summary

- we have constructed [fixed genus] susy ($\approx$ conformal) interfaces for type II GS string
  - generate T–duality (Buscher rules)
  - interpreted as a submanifold in doubled target space (bi–brane)
  - transform (rotate) D–branes
  - partition fn. w/ interfaces, Casimir energy

- correspond to those preserving 2 Virasoros $\Rightarrow$ evade ghost problem
Future directions

- NSR formulation? \implies [Bachas–Brunner–Roggenkamp ’12]

- more general interfaces? \implies probably, No [Bachas et al.]

- richer algebras among interfaces when compactified
  \implies monoid (semi–group) extension of $O(d, d | \mathbb{Q})$ [Bachas et al.]

- double field theory?

- symmetry of string theory?
  applications?

\ldots