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A double-well SUSY matrix model as 2D type IIA superstrings in RR background

Fumihiko Sugino
(Okayama Inst. Quantum Phys.)

Based on two papers in preparation with T. Kuroki (Rikkyo U.)

1 Introduction

◇ We previously considerd a simple SUSY matrix model: [Kuroki-F.S. 2009]

$$S = N\mathrm{tr}\left[\frac{1}{2}B^2 + iB(\phi^2 - \mu^2) + \bar{\psi}(\phi\psi + \psi\phi)\right],$$

where

$$\left.\begin{array}{ll} B, \phi & : \text{Bosonic} \\ \psi, \bar{\psi} & : \text{Fermionic} \end{array}\right\} N \times N \text{ hermitian matrices.}$$

- SUSY:

$$\begin{aligned} &Q\phi = \psi, \quad Q\psi = 0, \quad Q\bar{\psi} = -iB, \quad QB = 0, \\ &\bar{Q}\phi = -\bar{\psi}, \quad \bar{Q}\bar{\psi} = 0, \quad \bar{Q}\psi = -iB, \quad \bar{Q}B = 0. \end{aligned}$$

$\Rightarrow Q^2 = \bar{Q}^2 = 0$ (nilpotent)

- Double-well scalar potential : $\frac{1}{2}(\phi^2 - \mu^2)^2$

$\diamondsuit$ (SUSY preserving) large-N solution with filling fraction (ν_+, ν_-):

[Kuroki-F.S. 2009]

$$\begin{aligned}\rho(x) &\equiv \frac{1}{N}\mathrm{tr}\,\delta(x-\phi)\\ &= \begin{cases} \frac{\nu_+}{\pi}\, x\sqrt{(x^2-a^2)(b^2-x^2)} & (a<x<b)\\ \frac{\nu_-}{\pi}\, |x|\sqrt{(x^2-a^2)(b^2-x^2)} & (-b<x<-a)\end{cases}\end{aligned}$$

with $a=\sqrt{\mu^2-2}$, $b=\sqrt{\mu^2+2}$.

- Exists for $\mu^2 > 2$.
- a and b are independent of $\nu_\pm$!
- SUSY is preserved from
  (large-N free energy) $= 0$, $\langle \frac{1}{N}\mathrm{tr}\, B^n\rangle = 0$ $(n=1,2,\cdots)$.
- The SUSY minima are infinitely degenerate, parametrized by (ν_+, ν_-).

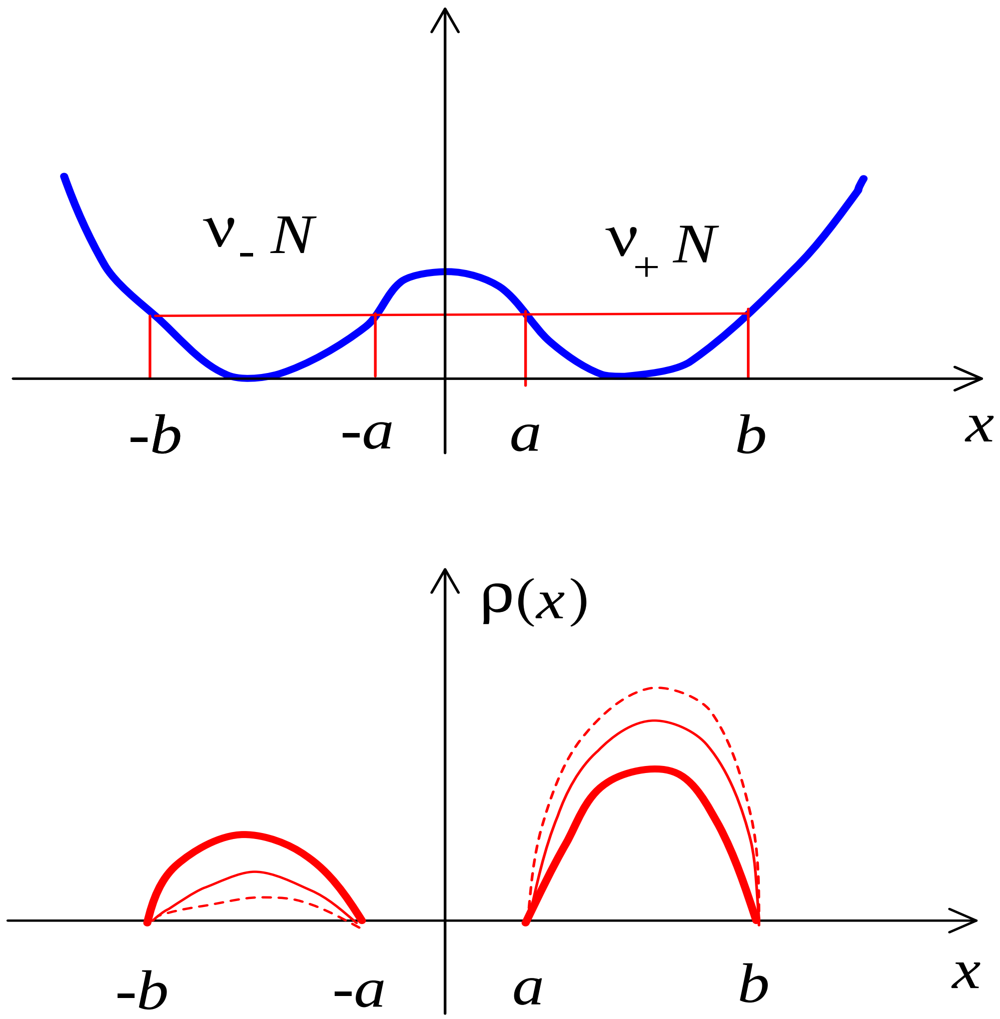

Figure 1: Double-well scalar potential (upper panel) and the eigenvalue distribution for $\nu_+ > \nu_-$ (lower panel).

$\diamondsuit$ In this talk,

- we compute correlation functions of this matrix model.

(→ Logarithmic critical behavior)

- discuss a correspondence between the matrix model and 2D type IIA superstring theory.

$\diamondsuit$ Logarithmic critical behavior is reminiscent of
the $\boldsymbol{c = 1}$ matrix model (matrix quantum mechanics) [Kazakov-Migdal 1988]
or the Penner model (zero-dimenisonal matrix model). [Distler-Vafa 1991]

$\Rightarrow$ Our matrix model $\sim$ a SUSY version of the Penner model
$\sim$ 2D superstring with target-space SUSY.

Note:

- This matrix model is equivalent to the $O(n=-2)$ model on a random surface [Kostov 1989].
- Its critical behavior is described by $c=-2$ topological gravity or (2,1) minimal string theory. [Kostov-Staudacher 1992]
- It is easily seen by the Nicolai mapping $H=\phi^2$. [Gaiotto-Rastelli-Takayanagi 2004]

For $\mathrm{tr}\,\phi^{2n}$ or $\mathrm{tr}\,B^n$, this approach is effective in $\frac{1}{N}$-expansion.

However, $\mathrm{tr}\,\phi^{2n+1}$, $\mathrm{tr}\,\psi^{2n+1}$, $\mathrm{tr}\,\bar{\psi}^{2n+1}$, ... are not observables in the topological gravity. $(\mathrm{tr}\,\psi^{2n}=\mathrm{tr}\,\bar{\psi}^{2n}=0.)$

Actually, we see nontrivial logarithmic critical behavior for these operators.

Interesting observation:

◇ Suppose that ψ and $\bar{\psi}$ correspond to target-space fermions in some superstring theory.

$$\psi \Leftrightarrow \text{(NS,R) sector}, \qquad \bar{\psi} \Leftrightarrow \text{(R,NS) sector}.$$

Then,

$$\begin{aligned}
(-1)^{\mathrm{F}_L} &: \quad \psi \to \psi, \qquad \bar{\psi} \to -\bar{\psi}, \\
(-1)^{\mathrm{F}_R} &: \quad \psi \to -\psi, \qquad \bar{\psi} \to \bar{\psi}.
\end{aligned}$$

In order for the matrix model action to be invariant under $(-1)^{\mathrm{F}_L}$ and $(-1)^{\mathrm{F}_R}$,

$$\begin{aligned}
(-1)^{\mathrm{F}_L} &: \quad B \to B, \qquad \phi \to -\phi, \\
(-1)^{\mathrm{F}_R} &: \quad B \to B, \qquad \phi \to -\phi.
\end{aligned}$$

This means

$$B \Leftrightarrow \text{(NS,NS) sector}, \qquad \phi \Leftrightarrow \text{(R,R) sector}.$$

2 Planar one-point functions

$$\left\langle \frac{1}{N}\,\mathrm{tr}\,\phi^n \right\rangle_0 = \int dx\, x^n \rho(x)$$
$$= (\nu_+ + (-1)^n \nu_-)\,(2+\mu^2)^{n/2}\, F\left(-\frac{n}{2}, \frac{3}{2}, 3; \frac{4}{2+\mu^2}\right)$$

- reduces to a polynomial of μ^2 for n even:

$$\left\langle \frac{1}{N}\mathrm{tr}\,\phi^2 \right\rangle_0 = \mu^2, \qquad \left\langle \frac{1}{N}\mathrm{tr}\,\phi^4 \right\rangle_0 = 1 + \mu^4, \qquad \cdots.$$

- exhibits logarithimic singular behavior as $\mu^2 \to 2$ for n odd:

$$\omega \equiv \tfrac{1}{4}(\mu^2 - 2)$$

$$\left\langle \frac{1}{N}\mathrm{tr}\,\phi^{2k+1} \right\rangle_0 = (\nu_+ - \nu_-)\left[(\text{const.})\,\omega^{k+2}\ln\omega + (\text{less singluar})\right].$$

3 Planar two-point functions (Bosons)

- "Even-even" correlators:

$$\Big\langle \frac{1}{N}\mathrm{tr}\,\phi^{2k}\,\frac{1}{N}\mathrm{tr}\,\phi^{2\ell}\Big\rangle_{C,0} = (\text{polynomial of } \mu^2 \text{ indep. of } (\nu_+ - \nu_-))$$

- "Even-odd" correlators:

$$\Big\langle \Phi_{2k+1}\,\frac{1}{N}\mathrm{tr}\,\phi^{2\ell}\Big\rangle_{C,0} = (\nu_+ - \nu_-)(\text{const.})\,\omega^{k+1}\ln\omega + (\text{less singular})$$

- "Odd-odd" correlators:

$$\langle \Phi_{2k+1}\,\Phi_{2\ell+1}\rangle_{C,0} = (\nu_+ - \nu_-)^2(\text{const.})\,\omega^{k+\ell+1}(\ln\omega)^2 + (\text{less singular}),$$

where it is convenient to change a basis of the "odd" operators:

$$\Phi_{2k+1} = \frac{1}{N}\mathrm{tr}\,\phi^{2k+1} + \sum_{i=1}^{k}\alpha_{2k+1,2i}(\omega)(\nu_+ - \nu_-)\frac{1}{N}\mathrm{tr}\,\phi^{2i}$$

with $\alpha_{2k+1,2i}(\omega)$ regular at $\omega = 0$.

4 Planar three-point functions (Bosons)

We obtain

$$\langle \Phi_1 \Phi_1 \Phi_1 \rangle_{C,0} = (\nu_+ - \nu_-)^3 \left[\frac{1}{16\pi^3} (\ln \omega)^3 + \mathcal{O}((\ln \omega)^2) \right],$$

$$\langle \Phi_1 \Phi_1 \Phi_3 \rangle_{C,0} = (\nu_+ - \nu_-)^3 \left[\frac{2}{\pi^3} + \frac{3}{8\pi^3} \omega (\ln \omega)^3 + \mathcal{O}(\omega (\ln \omega)^2) \right].$$

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- The results so far suggest

  $$\langle \Phi_{2k_1+1} \cdots \Phi_{2k_n+1} \rangle_{C,0} = (\nu_+ - \nu_-)^n (\text{const.})\, \omega^{2-\gamma+\sum_{i=1}^{n}(k_i-1)} (\ln \omega)^n + (\text{less singular})$$

  with $\gamma = -1$. $\leftarrow$ string susceptibility of $c = -2$ topological gravity

  Gravitational scaling dimension of Φ_{2k+1} is k.

  But, the logarithmic scaling violation is more severe than the case of the $c = 1$ matrix model.

◇ For fermions,

$$\begin{aligned}\boldsymbol{\Psi}_{2k+1} &= \frac{1}{N}\mathrm{tr}\,\psi^{2k+1} + \cdots,\\ \bar{\boldsymbol{\Psi}}_{2k+1} &= \frac{1}{N}\mathrm{tr}\,\bar{\psi}^{2k+1} + \cdots\end{aligned}$$

have the dimension k same as Φ_{2k+1}.

6 2D type IIA superstring [Kutasov-Seiberg 1990, Ita-Nieder-Oz 2005]

- (Target space) $= (x, \varphi)$,
  where $x \in S^1$ with self-dual radius ($R = 1$) and φ: Liouville.
  ($\nwarrow$ Same as the Penner model!)

- EM tensor (except ghost part):

$$T_m = -\frac{1}{2}(\partial x)^2 - \frac{1}{2}\psi_x \partial \psi_x - \frac{1}{2}(\partial\varphi)^2 + \frac{Q}{2}\partial^2\varphi - \frac{1}{2}\psi_\ell \partial \psi_\ell$$

  with $Q = 2$

- Target-space SUSY is nilpotent. ($\leftarrow$ Same as MM!)

$$q_+(z) = e^{-\frac{1}{2}\phi - \frac{i}{2}H - ix(z)}, \qquad Q_+ = \oint \frac{dz}{2\pi i}\, q_+(z),$$

$$\bar{q}_-(\bar{z}) = e^{-\frac{1}{2}\bar{\phi} + \frac{i}{2}\bar{H} + i\bar{x}}(\bar{z}), \qquad \bar{Q}_- = \oint \frac{d\bar{z}}{2\pi i}\, \bar{q}_-(\bar{z}).$$

  where $\psi_\ell \pm i\psi_x = \sqrt{2} e^{\mp iH}$. $\nwarrow$ Exist only at the self-dual radius!

  $\Rightarrow Q_+^2 = \bar{Q}_-^2 = \{Q_+, \bar{Q}_-\} = 0$.

- Vertex operators:

$$\text{NS sector } (-1)\text{-picture}: \quad T_k(z) = e^{-\phi+ikx+p_\ell\varphi}(z)$$
$$\text{R sector } (-\tfrac{1}{2})\text{-picture}: \quad V_{k,\,\epsilon}(z) = e^{-\frac{1}{2}\phi+\frac{i}{2}\epsilon H+ikx+p_\ell\varphi}(z)$$

with $\epsilon = \pm 1$.

Locality with supercurrents, mutual locality, superconformal inv., level matching

$\Rightarrow$ physical vertex operators with $p_\ell = 1 - |k|$ and $k = \epsilon|k|$

Winding background:

$$\begin{array}{llll}
(\text{NS, NS}): & T_k(z)\,\bar{T}_{-k}(\bar{z}) & (k \in \mathbf{Z} + \frac{1}{2}) & \text{winding "tachyon"} \\
(\text{R}+,\ \text{R}-): & V_{k,\,+1}(z)\,\bar{V}_{-k,\,-1}(\bar{z}) & (k = 1/2,\ 3/2, \cdots) & \text{RR boson} \\
(\text{R}-,\ \text{R}+): & V_{-k,\,-1}(z)\,\bar{V}_{k,\,+1}(\bar{z}) & (k = 0, 1, 2, \cdots) & \\
 & & & \text{RR 2-form field strength} \\
(\text{NS, R}-): & T_{-k}(z)\,\bar{V}_{-k,\,-1}(\bar{z}) & (k = 1/2,\ 3/2, \cdots) & \text{fermion}(-) \\
(\text{R}+,\ \text{NS}): & V_{k,\,+1}(z)\,\bar{T}_k(z) & (k = 1/2,\ 3/2, \cdots) & \text{fermion}(+)
\end{array}$$

Interesting observation:

Let us assume the correspondence of supercharges between the matrix model and the type IIA theory:

$$(Q,\bar{Q}) \Leftrightarrow (Q_+,\bar{Q}_-).$$

$\Rightarrow$ SUSY transformation properties naturally leads to

$$\Phi_1 = \frac{1}{N}\mathrm{tr}\,\phi \Leftrightarrow \int d^2z\, V_{\frac{1}{2},+1}(z)\,\bar{V}_{-\frac{1}{2},-1}(\bar{z}),$$

$$\Psi_1 = \frac{1}{N}\mathrm{tr}\,\psi \Leftrightarrow \int d^2z\, T_{-\frac{1}{2}}(z)\,\bar{V}_{-\frac{1}{2},-1}(\bar{z}),$$

$$\bar{\Psi}_1 = \frac{1}{N}\mathrm{tr}\,\bar{\psi} \Leftrightarrow \int d^2z\, V_{\frac{1}{2},+1}(z)\,\bar{T}_{\frac{1}{2}}(\bar{z}),$$

$$\frac{1}{N}\mathrm{tr}(-iB) \Leftrightarrow \int d^2z\, T_{-\frac{1}{2}}(z)\,\bar{T}_{\frac{1}{2}}(\bar{z}).$$

Furthermore, for $k = 0, 1, 2, \cdots$,

$$\Phi_{2k+1} \;\Leftrightarrow\; \int d^2z\, V_{k+\frac{1}{2},\,+1}(z)\, \bar{V}_{-k-\frac{1}{2},\,-1}(\bar{z}),$$

$$\Psi_{2k+1} \;\Leftrightarrow\; \int d^2z\, T_{-k-\frac{1}{2}}(z)\, \bar{V}_{-k-\frac{1}{2},\,-1}(\bar{z}),$$

$$\bar{\Psi}_{2k+1} \;\Leftrightarrow\; \int d^2z\, V_{k+\frac{1}{2},\,+1}(z)\, \bar{T}_{k+\frac{1}{2}}(\bar{z}),$$

$$\frac{1}{N}\mathrm{tr}(-iB)^{k+1} \;\Leftrightarrow\; \int d^2z\, T_{-k-\frac{1}{2}}(z)\, \bar{T}_{k+\frac{1}{2}}(\bar{z})$$

seems also natural.

(Single trace operators in the matrix model) $\Leftrightarrow$ (Integrated vertex operators in IIA)

(Powers of matrices) $\Leftrightarrow$ (Windings or Momenta)

Note:

- RR 2-form field strength in (R$-$, R$+$) is a singlet under the target-space SUSYs $\boldsymbol{Q}_{+}$, $\bar{\boldsymbol{Q}}_{-}$, and appears to have no matrix-model counterpart.
- $\langle \boldsymbol{\Phi}_{\boldsymbol{2k+1}} \rangle_{\boldsymbol{0}}$, $\left\langle \boldsymbol{\Psi}_{\boldsymbol{2k+1}} \bar{\boldsymbol{\Psi}}_{\boldsymbol{2k+1}} \right\rangle_{\boldsymbol{C,0}}$ are nonvanishing in the matrix model.

$\Rightarrow$ The matrix model is considered to correspond to IIA on a background of the RR 2-form.

Let us check by computing amplitudes in IIA theory.

7 Correspondence between the matrix model and the IIA theory

◇ Correlation functions among integrated vertex operators in IIA on the trivial background:

$$\left\langle \prod_i \mathcal{V}_i \right\rangle = \frac{1}{\text{Vol.(CKV)}} \int \mathcal{D}(x, \varphi, H, \text{ghosts})\, e^{-S_{\rm CFT}} e^{-S_{\rm int}} \prod_i \mathcal{V}_i,$$

$$S_{\rm int} = \omega \int d^2 z\, T^{(0)}_{-\frac{1}{2}}(z) \bar{T}^{(0)}_{\frac{1}{2}}(\bar{z}) \qquad (\leftarrow \text{0-picture (NS, NS) “tachyon”})$$

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◇ Correlation functions in IIA on (R−, R+) background: c.f.[Takayanagi 2004]

$$\left\langle\!\left\langle \prod_i \mathcal{V}_i \right\rangle\!\right\rangle \equiv \left\langle \left(\prod_i \mathcal{V}_i \right) e^{W_{\rm RR}} \right\rangle,$$

where $W_{\rm RR}$ is invariant under the target-space SUSYs:

$$W_{\rm RR} = (\nu_+ - \nu_-) \sum_{k \in \mathbf{Z}} a_k\, \omega^{k+1} \mathcal{V}^{\rm RR}_k, \qquad (a_k : \text{numerical consts.})$$

$$\mathcal{V}^{\rm RR}_k \equiv \begin{cases} \int d^2z\, V_{k,\,-1}(z) \bar{V}_{-k,\,+1}(\bar{z}) & (p_\ell = 1 - |k|,\ k = 0, -1, -2, \cdots) \\ \int d^2z\, V^{(\rm nonlocal)}_{-k,\,-1}(z) \bar{V}^{(\rm nonlocal)}_{k,\,+1}(\bar{z}) & (p_\ell = 1 + |k|,\ k = 1, 2, \cdots). \end{cases}$$

◇ Standard Liouville theory computation for amplitudes leads to:

$$\bullet \left\langle \frac{1}{N} \mathrm{tr}(-iB)\, \Phi_{2k+1} \right\rangle_0 = \frac{1}{4} \partial_\omega \left\langle \Phi_{2k+1} \right\rangle_0 \Leftrightarrow$$

$$-\frac{1}{4}(\nu_+ - \nu_-) \sum_{\ell \in \mathrm{Z}} a_\ell\, \omega^{\ell+1} \left\langle \left(\int T_{-\frac{1}{2}} \bar{T}_{\frac{1}{2}} \right) \left(\int V_{k+\frac{1}{2},\,+1} \bar{V}_{-k-\frac{1}{2},\,-1} \right) \mathcal{V}_\ell^{\mathrm{RR}} \right\rangle$$

$$= -\frac{1}{2}(\nu_+ - \nu_-)\, a_k\, \omega^{k+1} \ln \omega,$$

$$\bullet \left\langle \Phi_{2k_1+1} \Phi_{2k_2+1} \right\rangle_0 \Leftrightarrow$$

$$\frac{1}{2}(\nu_+ - \nu_-)^2 \sum_{\ell_1, \ell_2 \in \mathrm{Z}} a_{\ell_1} a_{\ell_2}\, \omega^{\ell_1+\ell_2+2}$$

$$\times \left\langle \left(\int V_{k_1+\frac{1}{2},\,+1} \bar{V}_{-k_1-\frac{1}{2},\,-1} \right) \left(\int V_{k_2+\frac{1}{2},\,+1} \bar{V}_{-k_2-\frac{1}{2},\,-1} \right) \mathcal{V}_{\ell_1}^{\mathrm{RR}}\, \mathcal{V}_{\ell_2}^{\mathrm{RR}} \right\rangle$$

$$= (\nu_+ - \nu_-)^2\, 2\pi\, a_{k_1+k_2}\, a_{-1} \left(\frac{(k_1 + k_2)!}{k_1! k_2!} \right)^2 \omega^{k_1+k_2+1} (\ln \omega)^2,$$

$\cdots$

with appropriate regularization by the Liouville volume $V_L = -2 \ln \omega$.

- Consistent with the correspondence!
- Higher powers of $\ln\omega$ comes from resonances to the (R−,R+) background.

Regularization:

For example, the amplitude

$$\int d^2z\, z^{\alpha}\bar{z}^{\bar{\alpha}}(1-z)^{\beta}(1-\bar{z})^{\bar{\beta}} = \pi \frac{\Gamma(\bar{\alpha}+1)\Gamma(\bar{\beta}+1)}{\Gamma(\bar{\alpha}+\bar{\beta}+2)} \frac{\Gamma(-\alpha-\beta-1)}{\Gamma(-\alpha)\Gamma(-\beta)}$$

with

$$\alpha = \bar{\alpha} = k_3k_4 - p_{\ell_3}p_{\ell_4} = k_1 + k_2,$$
$$\beta = \bar{\beta} = k_2k_4 - p_{\ell_2}p_{\ell_4} - \frac{1}{2} = -k_1 - 1, \qquad (k_1, k_2 = 0, 1, 2, \cdots)$$

is indefinite.

We regularize it as

$$\alpha \to \alpha + \epsilon, \qquad \bar{\alpha} \to \bar{\alpha} + \epsilon, \qquad \beta \to \beta + \epsilon, \qquad \bar{\beta} \to \bar{\beta} + \epsilon$$

with $\epsilon = \frac{1}{V_L}$, and get the result $\frac{\pi}{2}\left(\frac{(k_1+k_2)!}{k_1!k_2!}\right)^2 V_L$.

- This regularization preserves the mutual locality of vertex operators, i.e. does not change $\alpha - \bar{\alpha}$ and $\beta - \bar{\beta}$.

9 Summary and discussions

$\diamondsuit$ We computed correlation functions in the double-well SUSY matrix model, and discussed its correspondence to 2D type IIA superstring theory on (R$-$,R$+$) background by computing amplitudes in both sides.

This is an interesting example of matrix models for superstrings with target-space SUSY, in which various amplitudes are explicitly calculable.

$\diamondsuit$ MM-counterpart of positive winding "tachyons" $\boldsymbol{T_{k-\frac{1}{2}}\bar{T}_{-k+\frac{1}{2}}}$ $(\boldsymbol{k=1,2,\cdots})$?

Similar to the Kontsevich-Penner MM (introducing an external matrix source)?

[Imbimbo-Mukhi 1995]

$\diamondsuit$ D-brane interpretation of the matrix model?

FZZT?

$\diamondsuit$ Black-hole (cigar) target space?