COMMENTS ON KNOTTED

大阪大学

本ポスターの概念図

(結論)

非自明な結び目(knot)構造を持つ
1/2 BPS (SUSYを半分保つ) Wilson loop
を発見した。

Cには「結び目」
を代入できる

$$\frac{\int \mathcal{D}A\ W(R,C,A)\ e^{ikS_{CS}(A)}}{Z} = \text{Jones多項式}\ J_C(q)$$

(CS)ゲージ理論

結び目理論

SUSY化

1.Consistency check
2.直接の経路積分から結び目の
情報を得る(結び目の行列模型)

今回の結果

超対称ゲージ理論 → 変形 → 行列模型

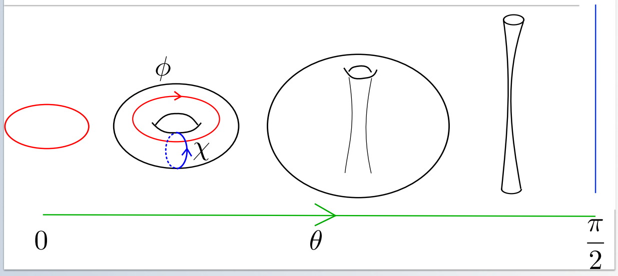
$$\frac{\int \mathcal{D}V\ W_S(R,C,A,\sigma)e^{ikS_{SCS}(V)}}{Z_S} = \text{有限次元の積分}$$

SUSY化

Ellipsoid-like squashed S^3

$$ds^2 = (l^2\sin^2\theta + \tilde{l}^2\cos^2\theta)d\theta^2 + l^2\cos^2\theta d\phi^2 + \tilde{l}^2\sin^2\theta d\chi^2$$

θ : an angle of elevation $\quad \phi,\chi$: torus coordinates



SUSY化

$\mathcal{N}=2$ Vector multiplet

Field (V)	A_μ	σ	λ	$\bar{\lambda}$	D
spin	1	0	1/2	1/2	0

$\mathcal{N}=2$ SUSY変換

$$\delta_\epsilon A_\mu = +\frac{i}{2}\bar{\lambda}\gamma_\mu\epsilon, \quad \delta_{\bar{\epsilon}}A_\mu = -\frac{i}{2}\bar{\epsilon}\gamma_\mu\lambda$$

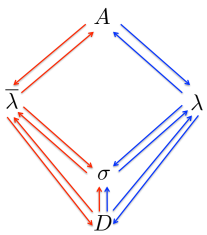
$$\delta_\epsilon\sigma = -\frac{1}{2}\bar{\lambda}\epsilon, \quad \delta_{\bar{\epsilon}}\sigma = +\frac{1}{2}\bar{\epsilon}\lambda$$

$$\delta_\epsilon\lambda = \frac{1}{2}\gamma^{\mu\nu}\epsilon F_{\mu\nu} - D\epsilon + i\gamma^\mu\epsilon\mathcal{D}_\mu\sigma + \frac{2i}{3}\sigma\gamma^\mu\mathcal{D}_\mu\epsilon, \quad \delta_{\bar{\epsilon}}\lambda = 0$$

$$\delta_\epsilon\bar{\lambda} = 0, \quad \delta_{\bar{\epsilon}}\bar{\lambda} = \frac{1}{2}\gamma^{\mu\nu}\bar{\epsilon}F_{\mu\nu} + D\bar{\epsilon} - i\gamma^\mu\bar{\epsilon}\mathcal{D}_\mu\sigma - \frac{2i}{3}\sigma\gamma^\mu\mathcal{D}_\mu\bar{\epsilon}$$

$$\delta_\epsilon D = -\frac{i}{2}\mathcal{D}_\mu\bar{\lambda}\gamma^\mu\epsilon + \frac{i}{2}[\bar{\lambda}\epsilon,\sigma] - \frac{i}{6}\bar{\lambda}\gamma^\mu\mathcal{D}_\mu\epsilon,$$

$$\delta_{\bar{\epsilon}}D = -\frac{i}{2}\bar{\epsilon}\gamma^\mu\mathcal{D}_\mu\lambda + \frac{i}{2}[\bar{\epsilon}\lambda,\sigma] - \frac{i}{6}\mathcal{D}_\mu\bar{\epsilon}\gamma^\mu\lambda$$



$\mathcal{D}_\mu$ はこのメトリックでの共変微分＋補正項

SUSY不変な作用とWilson loop

$$S_{\mathcal{N}=2\ SCS}(V) \equiv \frac{1}{4\pi}\int \mathrm{Tr}(AdA - \frac{2i}{3}A^3) - \frac{1}{4\pi}\int d^3x\sqrt{g}\mathrm{Tr}(\bar{\lambda}\lambda - 2D\sigma)$$

$$\begin{aligned} S_{\mathcal{N}=2\ SYM}(V) &= \int d^3x\ \sqrt{g}\ \mathrm{Tr}\Big(\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\mathcal{D}_\mu\sigma\mathcal{D}^\mu\sigma + \frac{1}{2}(D+\frac{\sigma}{f})^2 \\ &\qquad + \frac{i}{2}\bar{\lambda}\gamma^\mu\mathcal{D}_\mu\lambda + \frac{i}{2}\bar{\lambda}[\sigma,\lambda] - \frac{1}{4f}\bar{\lambda}\lambda\Big) \\ &= \delta_{\bar{\epsilon}}\underbrace{\delta_\epsilon\int d^3x\ \sqrt{g}\ \mathrm{Tr}(\frac{1}{2}\bar{\lambda}\lambda - 2D\sigma)}_{\mathcal{V}(V)} \\ &= \delta_{\bar{\epsilon}}\mathcal{V}(V) \end{aligned}$$

$$W_S(R,C,A,\sigma) \equiv \mathrm{Tr}_R\mathcal{P}\exp\oint_C d\tau(iA_\mu\dot{x}^\mu + \sigma|\dot{x}|)$$

変形

Localization principle

$$W_S(t) \equiv \int \mathcal{D}V\ W_S(R,C,A,\sigma)e^{ikS_{SCS}(V) - t\delta_{\bar{\epsilon}}\mathcal{V}(V)}$$

は、$\delta_{\bar{\epsilon}}W_S(R,C,A,\sigma) = 0$ のとき t に依らない。

$\because$

$$\begin{aligned} \frac{d}{dt}W_S(t) &= \int \mathcal{D}V\ W_S(R,C,A,\sigma)\frac{d}{dt}e^{ikS_{SCS}(V) - t\delta_{\bar{\epsilon}}\mathcal{V}(V)} \\ &= \int \mathcal{D}V\ W_S(R,C,A,\sigma)(-\delta_{\bar{\epsilon}}\mathcal{V}(V))e^{ikS_{SCS}(V) - t\delta_{\bar{\epsilon}}\mathcal{V}(V)} \\ &= \int \mathcal{D}V\delta_{\bar{\epsilon}}\Big(W_S(R,C,A,\sigma)(-\mathcal{V}(V))e^{ikS_{SCS}(V) - t\delta_{\bar{\epsilon}}\mathcal{V}(V)}\Big) \\ &\quad - \int \mathcal{D}V\Big(\delta_{\bar{\epsilon}}W_S(R,C,A,\sigma)\Big)(-\mathcal{V}(V))e^{ikS_{SCS}(V) - t\delta_{\bar{\epsilon}}\mathcal{V}(V)} \\ &= \int \mathcal{D}V\delta_{\bar{\epsilon}}\Big(W_S(R,C,A,\sigma)(-\mathcal{V}(V))e^{ikS_{SCS}(V) - t\delta_{\bar{\epsilon}}\mathcal{V}(V)}\Big) \\ &= 0 \end{aligned}$$

1/2 BPS WILSON LOOPS

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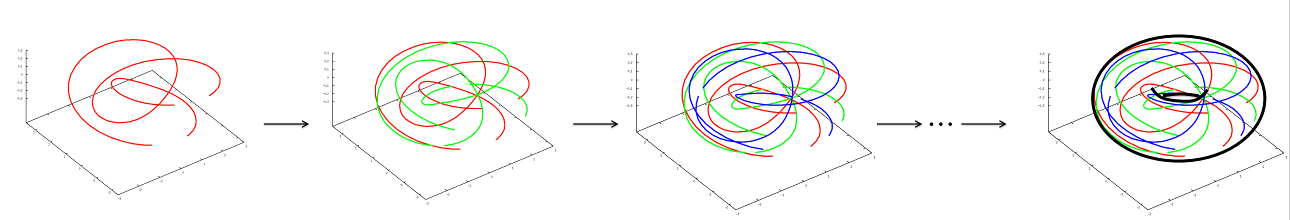
今回の結果

沢山 1/2 BPS Wilson loop を集めると、S^3のSeifert fibrationが得られる。



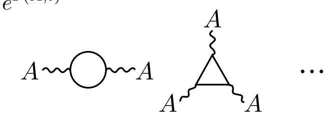
Parity anomaly と level shift

$\lim_{t\to+0} W_S(t) \neq W_S(0)$　である。 なぜなら、

$$\lim_{t\to+0} W_S(t) \ni \int \mathcal{D}\lambda\,\mathcal{D}\overline{\lambda}\; e^{\text{gaugino term in } (ikS_{SCS}(V)-tS_{YM}(V))}$$

$$= e^{\Gamma(A,t)}$$

$$=$$



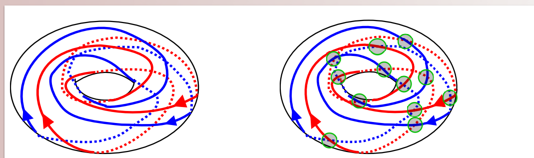
$$\to e^{\frac{-iN}{4\pi}\int d^3x\ \epsilon^{\mu\nu\lambda}\mathrm{Tr}(A_\mu\partial_\nu A_\lambda - \frac{2i}{3}A_\mu A_\nu A_\lambda)}$$

だから、レベルがkから(k-N)にずれる。これはCSの量子補正で更に(+N)されて、
結局SCSではレベルのシフトがない(ゲージーノが打ち消す。)

Phaseの違い＝linking number

$$\frac{W_{12\ldots n}}{Z} = e^{\sum_{i,j=1}^{n}\mathrm{lk}(C_i,C_j)\frac{2\pi i}{k}} \times \text{Jones polynomial of link}(C_1, C_2, \ldots, C_n),$$

と考えると、説明できる。



Ellipsoid-like squashed S^3 でのLocalization を用いたWilson loop 期待値の計算

$\delta_{\overline{\epsilon}} W_S(R,C,A,\sigma) = 0$　のとき　$\lim_{t\to+0} W_S(t) = \lim_{t\to+\infty} W_S(t)$

$$\delta_{\overline{\epsilon}} W_S(R,C,A,\sigma) = 0$$

の条件を満たすWilson loop
(1/2 BPS Wilson loop)

$$\delta_{\overline{\epsilon}} W_S(R,C;A,\sigma) \propto \frac{1}{2}\overline{\epsilon}(\gamma_\mu \dot{x}^\mu + |\dot{x}|)\lambda = 0$$

$$\overline{\epsilon}(\gamma_\mu \dot{x}^\mu + |\dot{x}|) = 0$$

$$\dot{x}^\mu \frac{\partial}{\partial x^\mu} = \begin{cases} \frac{1}{l}\frac{\partial}{\partial\phi} - \frac{1}{\tilde{l}}\frac{\partial}{\partial\chi} & (\theta \neq 0, \frac{\pi}{2}) \\ \frac{1}{l}\frac{\partial}{\partial\phi} & (\theta = 0) \\ -\frac{1}{\tilde{l}}\frac{\partial}{\partial\chi} & (\theta = \frac{\pi}{2}) \end{cases}$$

ここは、tが大きいせいで2次しか効かない
＝鞍点法が厳密

$\delta_{\overline{\epsilon}}\mathcal{V}(V) = S_{\mathcal{N}=2\ SYM}(V)$ の鞍点

$$A = 0,\quad \sigma = \sigma_0(\text{constant}),\quad D = -\frac{\sigma_0}{f}$$

$$\left(\lambda = 0,\quad \overline{\lambda} = 0\right)$$

(N. Hama, K. Hosomichi, and S. Lee)

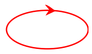
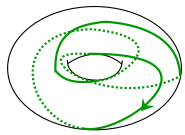
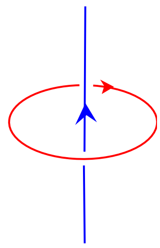
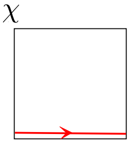
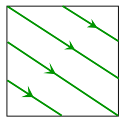
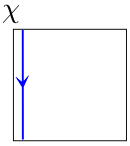
$$\mathcal{Z}^{(HHL)}_{\text{1-loop}}(\sigma_0) = \prod_{\alpha>0}\sinh(l\alpha(\sigma_0))\sinh(\tilde{l}\alpha(\sigma_0))$$

$$Z_S = \int_{\text{Cartan}} d\sigma_0\ e^{i\frac{k}{4\pi}S_{SCS}(0,\sigma_0,0,0,-\frac{\sigma_0}{f})}\mathcal{Z}^{(HHL)}_{\text{1-loop}}(\sigma_0)$$

Wilson loopに拡張した

$$\lim_{t\to+\infty} W_S(t) = \int_{\text{Cartan}} d\sigma_0\ e^{i\frac{k}{4\pi}S_{SCS}(0,\sigma_0,0,0,-\frac{\sigma_0}{f})} W_S(R,C;0,\sigma_0) \times \mathcal{Z}^{(HHL)}_{\text{1-loop}}(\sigma_0)$$

$$= \int_{\text{Cartan}} d\sigma_0\ e^{-ik\pi l\tilde{l}\mathrm{Tr}(\sigma_0{}^2)}\mathrm{Tr}_R(e^{\sigma_0\oint_C d\tau|\dot{x}|}) \times \mathcal{Z}^{(HHL)}_{\text{1-loop}}(\sigma_0)$$

今回の結果	(Knot型のもの)			(Link型のもの)
S^3の中での ½ BPS Wilson loop の形	 $\theta = 0$	 $\theta \neq 0, \frac{\pi}{2}$	 $\theta = \frac{\pi}{2}$	 $\theta = 0$ & $\theta = \frac{\pi}{2}$
ϕ, χでの表示	χ  ϕ	χ  ϕ	χ  ϕ	
※ $\frac{\lim_{t\to+\infty} W_S(t)}{Z}$	$e^{-\frac{l}{\tilde{l}}\frac{2\pi i}{k}} \frac{q-q^{-1}}{q^{1/2}-q^{-1/2}}$	$e^{-l\tilde{l}\frac{2\pi i}{k}} \frac{q^{\frac{1}{2}(l-1)(\tilde{l}-1)}}{1-q^2}(1+q^{l+\tilde{l}}-q^{l+1}-q^{\tilde{l}+1}) \times \frac{q-q^{-1}}{q^{1/2}-q^{-1/2}}$	$e^{-\frac{\tilde{l}}{l}\frac{2\pi i}{k}} \frac{q-q^{-1}}{q^{1/2}-q^{-1/2}}$	$e^{-(2+\frac{l}{\tilde{l}}+\frac{\tilde{l}}{l})\frac{2\pi i}{k}}(q^3+q^2+q+1)$
Jones 多項式	$\frac{q-q^{-1}}{q^{1/2}-q^{-1/2}}$	$\frac{q^{\frac{1}{2}(l-1)(\tilde{l}-1)}}{1-q^2}(1+q^{l+\tilde{l}}-q^{l+1}-q^{\tilde{l}+1}) \times \frac{q-q^{-1}}{q^{1/2}-q^{-1/2}}$	$\frac{q-q^{-1}}{q^{1/2}-q^{-1/2}}$	(q^3+q^2+q+1)

※ただし、U(2)ゲージ理論、R=2で計算。