基研研究会：場の理論と弦理論

YITP Workshop : Field Theory and String Theory

July 24, 2012

Embedding Tensors, Dualities, and Auxiliary Fields in 4D $\mathcal{N} = 2$ Supergravity

木村 哲士 (立教大学)

- 動機
- 設定

    4D $\mathcal{N} = 2$ system

- 解析

    Magnetic gauge fields

    Tensor gauge fields

- まとめと議論

動 機

適度に低い時空次元・超対称性において、

ゲージ化された超重力理論をどれだけ知っているか？

適度に低い時空次元・超対称性において、
ゲージ化された超重力理論をどれだけ知っているか？

●

系統立った議論

Embedding tensor formalism

by B. de Wit, H. Samtleben, M. Trigiante, etc.

最大超重力理論 (maximal-SUGRA) では３次元から９次元まで完成

0212239, 0507289, 0805.4767, 0901.2054;

0209205, 1106.1760 (9D); 0306179, 1110.2886 (8D); 0506237 (7D); 0712.4277 (6D);

0412173 (5D); 0705.2101 (4D); 0103032, 0801.1294 (3D); etc.

(U-)duality 変換前後のゲージ理論が「ゲージ理論のまま」移り合える

ゲージ化する操作：embedding tensor $\Theta_M{}^\alpha$ を導入

$$T_M \equiv \Theta_M{}^\alpha t_\alpha \qquad \begin{cases} t_\alpha \in \text{Lie}\, G_0 & \text{global} \\ T_M \in \text{Lie}\, G & \text{local} \end{cases}$$

$$\partial_\mu \longrightarrow \mathcal{D}_\mu = \partial_\mu - \mathrm{g} W_\mu^M T_M$$

$$[T_M, T_N] = -T_{MN}{}^P T_P \,, \qquad T_{MN}{}^P \equiv \Theta_M{}^\alpha (t_\alpha)_N{}^P$$

$$[\mathcal{D}_\mu, \mathcal{D}_\nu] \equiv -\mathrm{g} \mathcal{F}_{\mu\nu}^M T_M$$

$$\mathcal{F}_{\mu\nu}^M \equiv \partial_\mu W_\nu^M - \partial_\nu W_\mu^M + \mathrm{g} T_{[NP]}{}^M W_\mu^N W_\nu^P$$

closure constraint : $\quad 0 = f_{\alpha\beta}{}^\gamma \Theta_M{}^\alpha \Theta_N{}^\beta + (t_\alpha)_N{}^P \Theta_M{}^\alpha \Theta_P{}^\gamma$

$$T_{(MN)}{}^{P} T_{P} \; = \; 0$$

そして、$T_{(MN)}{}^{P} \; = \; 0$ とは限らない

$$\delta \mathcal{F}_{\mu\nu}^{M} \; = \; 2 \mathcal{D}_{[\mu} \delta W_{\nu]}^{M} - 2 g \, T_{(PQ)}{}^{M} \, W_{[\mu}^{P} \delta W_{\nu]}^{Q}$$

ゲージ変換 $\delta W_{\mu}^{M} = \mathcal{D}_{\mu} \Lambda^{M}$ に対して共変ではない

tensor gauge fields $B_{\mu\nu}^{(NP)}$ を導入して共変化

$$\mathcal{H}_{\mu\nu}^{M} \; \equiv \; \mathcal{F}_{\mu\nu}^{M} + g \, T_{(NP)}{}^{M} B_{\mu\nu}^{(NP)}$$

with

$$\begin{aligned} \delta W_{\mu}^{M} \;&=\; \mathcal{D}_{\mu} \Lambda^{M} - g \, T_{(NP)}{}^{M} \Xi_{\mu}^{(NP)} \\ \delta B_{\mu\nu}^{(NP)} \;&=\; 2 \mathcal{D}_{[\mu} \Xi_{\nu]}^{(NP)} - 2 \Lambda^{(N} \mathcal{H}_{\mu\nu}^{P)} + 2 W_{[\mu}^{(N} \delta W_{\nu]}^{P)} \end{aligned}$$

$$\mathscr{L}(W_\mu^M)$$

gauging ↓ G_0-covariant

$$\mathscr{L}(W_\mu^M, \Theta_M{}^\alpha, B_{\mu\nu}^{(NP)})$$

$\mathscr{L}(W_\mu^M, \Theta_M{}^\alpha, B_{\mu\nu}^{(NP)})$ は 形式的に G_0 共変

各 $\Theta_M{}^\alpha$ の値ごとに、一つの物理的 Lagrangian を与える

それぞれの物理的な Lagrangian 同士は

$\Theta_M{}^\alpha$ を G_0-変換 (双対変換) することでつながる

設定 : 4D $\mathcal{N}=2$

Embedding tensor formalism

Embedding tensor formalism の強みは、超重力理論を形式的に双対共変に書き表すことにある。

この手法が 4 次元 $\mathcal{N}=2$ 超重力理論に適用できたのは 2011 年である。
最大超重力理論と異なり、4 次元 $\mathcal{N}=2$ 超重力理論はベクトル場と物質場が違う多重項に属するため、
非常に多様な性質を持ち、面白い。

B. de Wit and M. van Zalk, arXiv:1107.3305

4D $\mathcal{N}=2$ system が持つ global symmetry group G_0 は

$$G_0 \;=\; G_{\rm symp} \times G_{\rm hyper}$$

vector 多重項から　　　hyper 多重項から

$$\partial_\mu X^\Lambda \;\to\; \mathcal{D}_\mu X^\Lambda \;=\; \partial_\mu X^\Lambda + {\rm g} W_\mu^N T_{NP}{}^\Lambda X^P$$ vector 多重項のスカラー場

$$\partial_\mu \phi^A \;\to\; \mathcal{D}_\mu \phi^A \;=\; \partial_\mu \phi^A - {\rm g} W_\mu^M k^A{}_M$$ hyper 多重項のスカラー場

$$T_{MN}{}^P \;=\; \Theta_M{}^{\sf a} (t_{\sf a})_N{}^P \qquad [T_M, T_N] = -T_{MN}{}^P T_P$$

$$k^A{}_M \;=\; \Theta_M{}^{\sf m} k^A{}_{\sf m} \qquad k^B{}_M \, \partial_B k^A{}_N - k^B{}_N \, \partial_B k^A{}_M \;=\; T_{MN}{}^P k^A{}_P$$

$$T_{M[N}{}^Q \Omega_{P]Q} \;=\; 0 \,, \qquad T_{(MN}{}^Q \Omega_{P)Q} \;=\; 0$$

$$\mathscr{L}(\Theta_M{}^\alpha) \;=\; \mathscr{L}_{\text{vector}} + \mathscr{L}_{\text{top}} + \mathscr{L}_{\text{hyper}} + \mathscr{L}_{\text{conf}} + \ldots$$

$$\begin{aligned}
e^{-1}\mathscr{L}_{\text{vector}} &= -\frac{1}{4}\mathcal{I}_{\Lambda\Sigma}\,\mathcal{H}^{\Lambda}_{\mu\nu}\mathcal{H}^{\mu\nu\Sigma} - \frac{\mathrm{i}}{8}\,e^{-1}\varepsilon^{\mu\nu\rho\sigma}\,\mathcal{R}_{\Lambda\Sigma}\,\mathcal{H}^{\Lambda}_{\mu\nu}\mathcal{H}^{\Sigma}_{\rho\sigma} + \ldots \\
e^{-1}\mathscr{L}_{\text{top}} &= \frac{\mathrm{i}}{8}\,\mathrm{g}\,e^{-1}\varepsilon^{\mu\nu\rho\sigma}(\Theta^{\Lambda\mathsf{a}}B_{\mu\nu\mathsf{a}} + \Theta^{\Lambda\mathsf{m}}B_{\mu\nu\mathsf{m}})\Big[\mathcal{F}_{\rho\sigma\Lambda} - \frac{\mathrm{g}}{4}(\Theta_{\Lambda}{}^{\mathsf{b}}B_{\rho\sigma\mathsf{b}} + \Theta_{\Lambda}{}^{\mathsf{n}}B_{\rho\sigma\mathsf{n}})\Big] + \ldots \\
e^{-1}\mathscr{L}_{\text{hyper}} &= -\frac{1}{2}\,g_{AB}\,\mathcal{D}_{\mu}\phi^{A}\,\mathcal{D}^{\mu}\phi^{B} + \ldots \\
e^{-1}\mathscr{L}_{\text{conf}} &= \frac{1}{6}(K+\chi)R - \frac{1}{2}(2K-\chi)D + \ldots
\end{aligned}$$

$$\mathcal{H}^{\Lambda}_{\mu\nu} \;=\; \mathcal{F}^{\Lambda}_{\mu\nu} + \frac{\mathrm{g}}{2}(\Theta^{\Lambda\mathsf{a}}B_{\mu\nu\mathsf{a}} + \Theta^{\Lambda\mathsf{m}}B_{\mu\nu\mathsf{m}})$$

以後、解析を簡単にするために **abelian gauging** を考える

それでも $T_{(MN)}{}^{P}$ は自明とは限らない

fields	before	after	#
W_{μ}^{Λ} electric	dynamical		$n_{\rm V}+1$
$W_{\mu\Lambda}$ magnetic	auxiliary		$n_{\rm V}+1$
$B_{\mu\nu\mathsf{a}}$	auxiliary		$n_{\mathsf{a}} \equiv \dim G_{\rm symp}$
$B_{\mu\nu\mathsf{m}}$	auxiliary		$n_{\mathsf{m}} \equiv \dim G_{\rm hyper}$
ϕ^{A} hyper	dynamical		$4(n_{\rm H}+1)$

$\Theta_{M}{}^{\mathsf{a}}, \Theta_{M}{}^{\mathsf{m}}$ の選び方次第で、

dynamical fields が選定される (dualize される) ことを確認する

解析

Magnetic gauge fields

Tensor gauge fields*

Embedding tensor formalism 特有のテンソル補助場の扱いに注目する。

電気的配位では補助場に過ぎない道具が、
磁気的配位に双対変換する時にきちんと力学的場になることを追跡する。
双対変換においてスカラー場がテンソル場に変換されることを、この formalism でも確認すべきである。

Embedding tensors $\Theta_M{}^{\mathsf{a}}$, $\Theta_M{}^{\mathsf{m}}$ の選び方で、適当な frame が決まる

Purely electric configuration (電気的結合のみの「通常の」ゲージ化)：

$$\Theta_M{}^{\mathsf{a}} = \left(\Theta_\Lambda{}^{\mathsf{a}} , \Theta^{\Lambda \mathsf{a}} \right) = \left(\Theta_\Lambda{}^{\mathsf{a}} , 0 \right)$$

$$\Theta_M{}^{\mathsf{m}} = \left(\Theta_\Lambda{}^{\mathsf{m}} , \Theta^{\Lambda \mathsf{m}} \right) = \left(\Theta_\Lambda{}^{\mathsf{m}} , 0 \right)$$

$$\mathsf{a} = 1, \ldots, \dim G_{\text{symp}} , \qquad \mathsf{m} = 1, \ldots, \dim G_{\text{hyper}}$$

通常の electric gauging ($T_{(MN)}{}^P = 0$) に戻る

電磁双対変換され、テンソル場が登場する系

R. D'Auria, et. al., hep-th/0312210, hep-th/0409097, etc.

次の embedding tensors の状況を考える：

$$\Lambda \ = \ (I \, , \, U) \, , \qquad \mathsf{a} \ = \ (\mathsf{a}' \, , \, \widehat{\mathsf{a}}) \, , \qquad \mathsf{m} \ = \ (\mathsf{m}' \, , \, \widehat{\mathsf{m}})$$

$$\Theta^{\Lambda \mathsf{a}} \ = \ \begin{pmatrix} \Theta^{I \mathsf{a}'} & \color{gray}{\Theta^{I \widehat{\mathsf{a}}}} \\ \color{gray}{\Theta^{U \mathsf{a}'}} & \color{gray}{\Theta^{U \widehat{\mathsf{a}}}} \end{pmatrix} , \qquad \Theta_{\Lambda}{}^{\mathsf{a}} \ = \ \begin{pmatrix} \Theta_I{}^{\mathsf{a}'} & \xcancel{\Theta_I{}^{\widehat{\mathsf{a}}}} \\ \Theta_U{}^{\mathsf{a}'} & \Theta_U{}^{\widehat{\mathsf{a}}} \end{pmatrix} , \qquad \exists \ (\Theta^{-1})_{\mathsf{a}' I}$$

$$\Theta_{\Lambda}{}^{\mathsf{m}} \ = \ \begin{pmatrix} \Theta_I{}^{\mathsf{m}'} & \color{gray}{\Theta_I{}^{\widehat{\mathsf{m}}}} \\ \Theta_U{}^{\mathsf{m}'} & \color{gray}{\Theta_U{}^{\widehat{\mathsf{m}}}} \end{pmatrix} , \qquad \Theta^{\Lambda \mathsf{m}} \ = \ \begin{pmatrix} \xcancel{\Theta^{I \mathsf{m}'}} & \xcancel{\Theta^{I \widehat{\mathsf{m}}}} \\ \Theta^{U \mathsf{m}'} & \xcancel{\Theta^{U \widehat{\mathsf{m}}}} \end{pmatrix} , \qquad \exists \ (\Theta^{-1})_{\mathsf{m}'}{}^{I}$$

$\Theta^{I \mathsf{m}'}, \Theta^{U \widehat{\mathsf{m}}}$ が自明になるのは物理的要請

fields		before	after	#
W_μ^Λ electric	W_μ^I	dynamical		$n_{\rm T}$
	W_μ^U	dynamical		$(n_{\rm V}+1)-n_{\rm T}$
$W_{\mu\Lambda}$ magnetic	$W_{\mu I}$	auxiliary		$n_{\rm T}$
	$W_{\mu U}$	auxiliary		$(n_{\rm V}+1)-n_{\rm T}$
$B_{\mu\nu\mathsf{a}}$	$B_{\mu\nu\mathsf{a}'}$	auxiliary		$n_{\rm T}$
	$B_{\mu\nu\widehat{\mathsf{a}}}$	auxiliary		$n_{\mathsf{a}}-n_{\rm T}$
$B_{\mu\nu\mathsf{m}}$	$B_{\mu\nu\mathsf{m}'}$	auxiliary		$n_{\rm T}$ (def.)
	$B_{\mu\nu\widehat{\mathsf{m}}}$	auxiliary		$n_{\mathsf{m}}-n_{\rm T}$
ϕ^A hyper	ϕ^A	dynamical		$4(n_{\rm H}+1)$

vectors $W_\mu^\Lambda, W_{\mu\Lambda}$ と tensors $B_{\mu\nu\mathsf{a}}, B_{\mu\nu\mathsf{m}}$ が分岐

$\mathrm{g}\Theta^{I\mathsf{a}'} B_{\mu\nu\mathsf{a}'} \equiv B^I_{\mu\nu}$ の運動方程式を解く：

$$\begin{aligned} B^I_{\mu\nu} &= \widehat{\mathcal{I}}^{IJ} \Big\{ \mathrm{i}\, e\, \varepsilon_{\mu\nu\rho\sigma} \mathcal{J}^{\rho\sigma}_J + 2 (r \mathcal{I}^{-1})_J{}^K \mathcal{J}_{\mu\nu K} \Big\} \,, \\ \mathcal{J}_{\mu\nu I} &= 2 \partial_{[\mu} W_{\nu] I} + \ldots \,. \end{aligned}$$

これを $\mathscr{L}_{\rm vector}$, $\mathscr{L}_{\rm top}$ に代入して $B_{\mu\nu\mathsf{a}'}$ を追い出し、$W_{\mu I}$ の運動項を獲得する：

$$\begin{aligned} & e^{-1} (\mathscr{L}_{\rm vector} + \mathscr{L}_{\rm top}) \\ &\quad = -\frac{1}{4} \Big[\widehat{\mathcal{I}}^{IJ} \widehat{\mathcal{F}}_{\mu\nu I} \widehat{\mathcal{F}}^{\mu\nu}_J + 2 \widehat{\mathcal{I}}^I{}_V \widehat{\mathcal{F}}_{\mu\nu I} \widehat{\mathcal{F}}^{\mu\nu V} + \widehat{\mathcal{I}}_{UV} \widehat{\mathcal{F}}^U_{\mu\nu} \widehat{\mathcal{F}}^{\mu\nu V} \Big] \\ &\qquad - \frac{\mathrm{i}}{8} e^{-1} \varepsilon^{\mu\nu\rho\sigma} \Big[\widehat{\mathcal{R}}^{IJ} \widehat{\mathcal{F}}_{\mu\nu I} \widehat{\mathcal{F}}_{\rho\sigma J} + 2 \widehat{\mathcal{R}}^I{}_V \widehat{\mathcal{F}}_{\mu\nu I} \widehat{\mathcal{F}}^V_{\rho\sigma} + \widehat{\mathcal{R}}_{UV} \widehat{\mathcal{F}}^U_{\mu\nu} \widehat{\mathcal{F}}^V_{\rho\sigma} \Big] \\ &\qquad + \frac{\mathrm{i}}{8} \mathrm{g}\, e^{-1} \varepsilon^{\mu\nu\rho\sigma} \Theta^{U\mathsf{m}'} B_{\mu\nu\mathsf{m}'} \mathcal{F}_{\rho\sigma U} - \frac{\mathrm{i}}{32} \mathrm{g}^2 e^{-1} \varepsilon^{\mu\nu\rho\sigma} \Theta^{U\mathsf{m}'} \Theta_U{}^{\mathsf{n}'} B_{\mu\nu\mathsf{m}'} B_{\rho\sigma\mathsf{n}'} + \ldots \,. \end{aligned}$$

$$\widehat{\mathcal{F}}_{\mu\nu I} \equiv 2 \partial_{[\mu} W_{\nu] I} - \frac{\mathrm{g}}{2} \Theta_I{}^{\mathsf{m}'} B_{\mu\nu\mathsf{m}'} \,, \qquad \widehat{\mathcal{F}}^U_{\mu\nu} \equiv 2 \partial_{[\mu} W^U_{\nu]} + \frac{\mathrm{g}}{2} \Theta^{U\mathsf{m}'} B_{\mu\nu\mathsf{m}'}$$

テンソル場の結合 (Stückelberg-type) をきちんと再現

$\mathscr{L}_{\rm top}$ に入っている $B_{\mu\nu \mathsf{m}'}\mathcal{F}_{\rho\sigma U}$ 項のおかげで、

$W_{\mu U}$ の運動方程式を解くと、元来は補助場だった $B_{\mu\nu \mathsf{m}'}$ が運動項を獲得する：

$$0 \;=\; \frac{\delta(e^{-1}\mathscr{L})}{\delta W_{\mu}^{\mathsf{m}'}} \;=\; J^{\mu}_{\mathsf{m}'} + \frac{\mathrm{i}}{12} e^{-1} \varepsilon^{\mu\nu\rho\sigma} H_{\nu\rho\sigma \mathsf{m}'} - \mathcal{M}_{\mathsf{m}'\mathsf{n}'} W^{\mu \mathsf{n}'} \,,$$

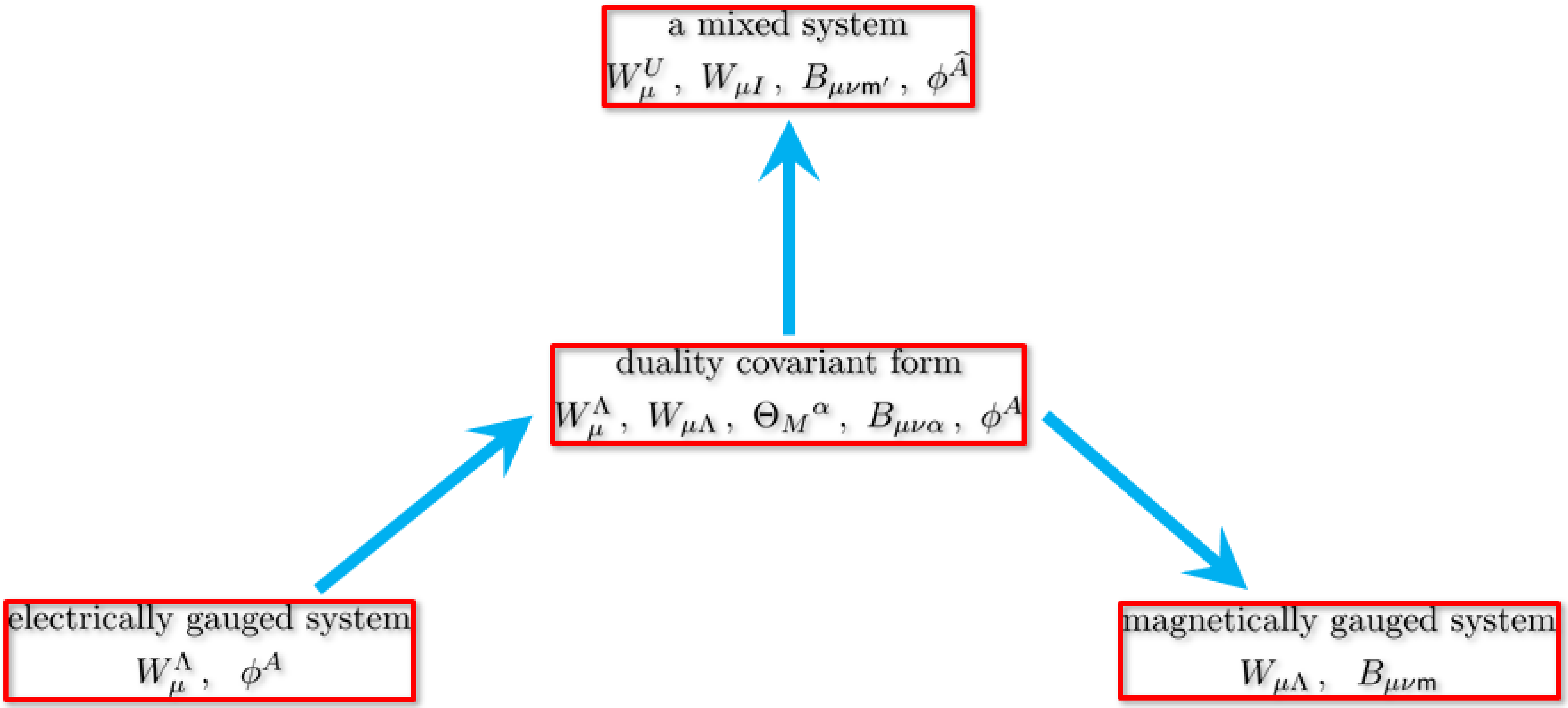
$$\therefore \quad W^{\mu \mathsf{m}'} \;\equiv\; \mathrm{g} W^{\mu}_{U} \Theta^{U \mathsf{m}'} \;=\; \mathcal{M}^{\mathsf{m}'\mathsf{n}'} \Big[\frac{\mathrm{i}}{12} e^{-1} \varepsilon^{\mu\nu\rho\sigma} H_{\nu\rho\sigma \mathsf{n}'} + J^{\mu}_{\mathsf{n}'} \Big] \,.$$

$$H_{\mu\nu\rho \mathsf{m}'} \;\equiv\; 3 \partial_{[\mu} B_{\nu\rho] \mathsf{m}'} \,, \qquad \mathcal{M}_{\mathsf{m}'\mathsf{n}'} \;\equiv\; g_{AB}\, k^{A}{}_{\mathsf{m}'}\, k^{B}{}_{\mathsf{n}'} \,, \qquad J^{\mu}_{\mathsf{m}'} \;\equiv\; (g_{AB} k^{A}{}_{\mathsf{m}'}) \widehat{\mathcal{D}}^{\mu} \phi^{B} + \ldots$$

$$\begin{aligned} e^{-1}\mathscr{L} \;&=\; e^{-1}\mathscr{L}_0 + J^{\mu}_{\mathsf{m}'} W_{\mu}^{\mathsf{m}'} + \frac{\mathrm{i}}{12} e^{-1} \varepsilon^{\mu\nu\rho\sigma} H_{\nu\rho\sigma \mathsf{m}'} W_{\mu}^{\mathsf{m}'} - \frac{1}{2} W_{\mu}^{\mathsf{m}'} \mathcal{M}_{\mathsf{m}'\mathsf{n}'} W^{\mu \mathsf{n}'} \\ &=\; e^{-1}\mathscr{L}_0 - \frac{1}{2\cdot 4!} H_{\mu\nu\rho \mathsf{m}'} \mathcal{M}^{\mathsf{m}'\mathsf{n}'} H^{\mu\nu\rho}_{\mathsf{n}'} \\ &\quad + \frac{\mathrm{i}}{12} e^{-1} \varepsilon^{\mu\nu\rho\sigma} J_{\mu \mathsf{m}'} \mathcal{M}^{\mathsf{m}'\mathsf{n}'} H_{\nu\rho\sigma \mathsf{n}'} + \frac{1}{2} J_{\mu \mathsf{m}'} \mathcal{M}^{\mathsf{m}'\mathsf{n}'} J^{\mu}_{\mathsf{n}'} \,. \end{aligned}$$

$\mathcal{M}^{\mathsf{m}'\mathsf{n}'}$ を評価するときに $k^{A}{}_{\mathsf{m}'}$ に対する制限が課される

fields		before	after	#
W_μ^Λ electric	W_μ^I	dynamical	(gauged away by $\delta B_{\mu\nu\mathsf{a}'}$)	n_T
	W_μ^U	dynamical	dynamical	$(n_\mathrm{V}+1)-n_\mathrm{T}$
$W_{\mu\Lambda}$ magnetic	$W_{\mu I}$	auxiliary	dynamical	n_T
	$W_{\mu U}$	auxiliary	(integrated-out to make $B_{\mu\nu\mathsf{m}'}$ dynamical)	$(n_\mathrm{V}+1)-n_\mathrm{T}$
$B_{\mu\nu\mathsf{a}}$	$B_{\mu\nu\mathsf{a}'}$	auxiliary	(integrated-out to make $W_{\mu I}$ dynamical)	n_T
	$B_{\mu\nu\hat{\mathsf{a}}}$	auxiliary	(decoupled)	$n_\mathsf{a}-n_\mathrm{T}$
$B_{\mu\nu\mathsf{m}}$	$B_{\mu\nu\mathsf{m}'}$	auxiliary	dynamical*	n_T (def.)
	$B_{\mu\nu\hat{\mathsf{m}}}$	auxiliary	(decoupled)	$n_\mathsf{m}-n_\mathrm{T}$
ϕ^A hyper	$\phi^{A'}$	dynamical	(dualized to $B_{\mu\nu\mathsf{m}'}$)	n_T
	$\phi^{\widehat{A}}$	dynamical	dynamical	$4(n_\mathrm{H}+1)-n_\mathrm{T}$


a mixed system
W_μ^U , $W_{\mu I}$, $B_{\mu\nu\mathsf{m}'}$, $\phi^{\widehat{A}}$
duality covariant form
W_μ^Λ , $W_{\mu\Lambda}$, $\Theta_M{}^\alpha$, $B_{\mu\nu\alpha}$, ϕ^A
electrically gauged system
W_μ^Λ , ϕ^A
magnetically gauged system
$W_{\mu\Lambda}$, $B_{\mu\nu\mathsf{m}}$

まとめと議論

- 動機

  ゲージ化された超重力の系統立った議論

- 設定

  Embedding tensor formalism in 4D $\mathcal{N} = 2$

- 解析

  Magnetic gauge fields

  Tensor gauge fields*

- Comment

  Embedding tensor formalism は global SUSY でも使える

  B. de Wit and M. de Vroome, arXiv:0707.2717

- Tensor gauge fields $B_{\mu\nu \mathsf{m}'}$ の超対称変換は既存のものに書ける (はず) ?

- Scalar-tensor 多重項にまとめられる？

  Hyper 多重項のスカラー場 ϕ^A は Hyper-Kähler Cone に値を持つ

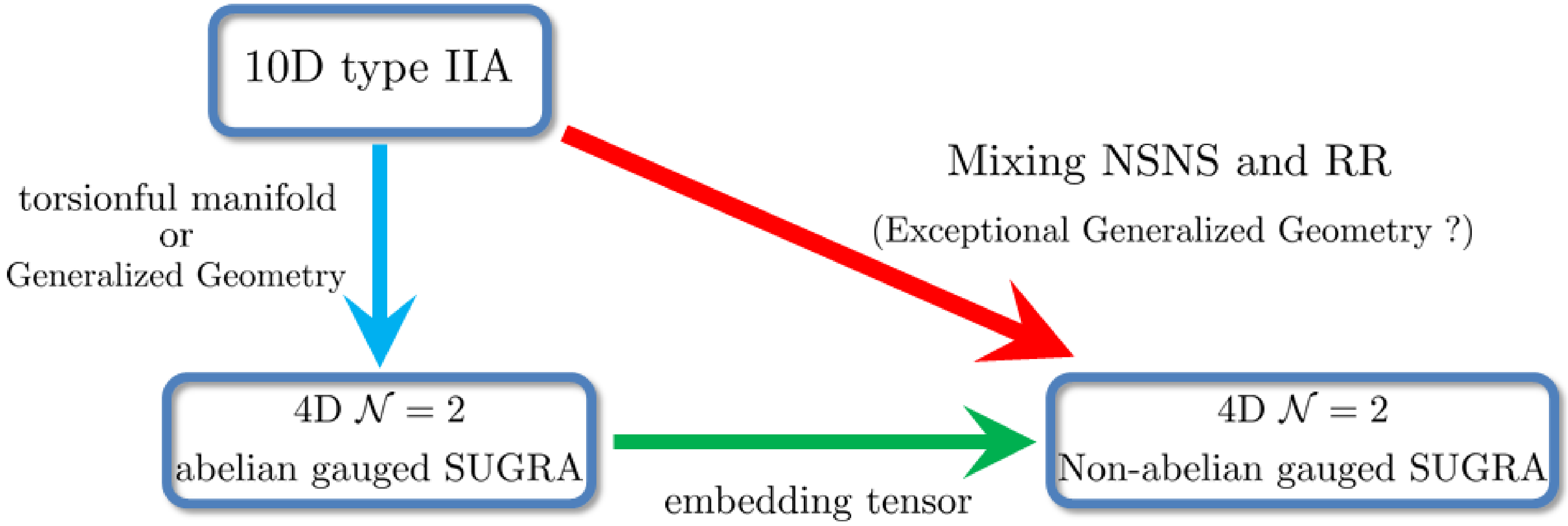
  HKC から綺麗に「切り抜く」条件は？

$$\phi^A: \ 4(n_{\rm H}+1) \text{ scalars (HKC)} \ = \ \begin{cases} \phi^{A'} \ \to \ B_{\mu\nu \mathsf{m}'}: & n_{\rm T} \text{ tensors} \\ \phi^{\widehat{A}} \ \to \ \begin{cases} \phi^{\check{A}}: & (3n_{\rm T}-1)+1 \text{ scalars} \ \to \ L^{ij}_{\mathsf{m}'} \\ \phi^{\widetilde{A}}: & 4(n_{\rm H}+1) - 4n_{\rm T} \text{ scalars (HKC)} \end{cases} \end{cases}$$

- Non-abelian gaugings

- 非可換ゲージ群を実現して、フラックスコンパクト化に組み込む

$$\text{Embedding tensors } \{\Theta_U{}^{\mathsf{m}'}, \Theta^{U\mathsf{m}'}\} \;\sim\; \text{RR-flux charges } \{e_{\mathrm{R}U}, m_{\mathrm{R}}^U\}$$



コメント

CY, $SU(3)$-structure manifolds, $SU(3) \times SU(3)$ generalized geometries では
せいぜい可換ゲージ群しか実現しない

e.g., D. Cassani, arXiv:0804.0595

付 録

Duality groups in (half-)maximal SUGRA

Global symmetry groups in 4D $\mathcal{N}=2$ conformal SUGRA

Duality covariant gauged conformal SUGRA

Gauged conformal SUGRA in a magnetic frame

4D $\mathcal{N}=2$ SUGRA via compactifications scalar field space : $\mathcal{M} = G_0/K$

D	maximal SUGRA G_0	maximal SUGRA K	half-maximal SUGRA G_0	half-maximal SUGRA K
9	$GL(2)$	$SO(2)$	$GL(1) \times SO(1,1+n)$	$SO(1+n)$
8	$SL(2) \times SL(3)$	$SO(2) \times SO(3)$	$GL(1) \times SO(2,2+n)$	$SO(2) \times SO(2+n)$
7	$SL(5)$	$SO(5)$	$GL(1) \times SO(3,3+n)$	$SO(3) \times SO(3+n)$
6	$SO(5,5)$	$SO(5) \times SO(5)$	$GL(1) \times SO(4,4+n)$	$SO(4) \times SO(4+n)$
5	$E_{6(6)}$	$USp(8)$	$GL(1) \times SO(5,5+n)$	$SO(5) \times SO(5+n)$
4	$E_{7(7)}$	$SU(8)$	$SL(2) \times SO(6,6+n)$	$SO(6) \times SO(6+n)$
	U-duality		S-duality $\times$ T-duality	

see, for instance, arXiv:0808.4076

$$G_0 = G_{\text{symp}} \times G_{\text{hyper}}$$

- G_{symp} : invariance group in the vector multiplets sector

$$\begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix} \to \begin{pmatrix} \widetilde{X}^\Lambda \\ \widetilde{F}_\Lambda \end{pmatrix} = \begin{pmatrix} U^\Lambda{}_\Sigma & Z^{\Lambda\Sigma} \\ W_{\Lambda\Sigma} & V_\Lambda{}^\Sigma \end{pmatrix} \begin{pmatrix} X^\Sigma \\ F_\Sigma \end{pmatrix}$$

with $\widetilde{F}(\widetilde{X}) = F(\widetilde{X})$

i.e., $G_{\text{symp}} \subset Sp(2n_{\text{V}} + 2; \mathbb{R})$

- G_{hyper} : the part of the isometry group of the space of hypermultiplets

$$G^{\text{hyper}}_{\text{isometry}} = SU(2) \times G_{\text{hyper}}$$

generated by the tri-holomorphic Killing vectors $k^A{}_{\text{m}}$

space of VM : rigid special Kähler geometry   space of HM : hyper-Kähler cone

B. de Wit and M. van Zalk, arXiv:1107.3305

$$\mathscr{L} = \mathscr{L}_{\text{kin}}^{(1)} + \mathscr{L}_{\text{kin}}^{(2)} + \mathscr{L}_{\text{conf}} + \mathscr{L}_{\text{H,conf}} + \mathscr{L}_{\text{top}} + \mathscr{L}_{\text{aux}} + \mathscr{L}_{\text{g}} + \mathscr{L}_{\text{g}^2} \,,$$

$$\begin{aligned} e^{-1}\mathscr{L}_{\text{kin}}^{(1)} &= -\text{i}\Omega_{MN}\mathcal{D}_\mu X^M \mathcal{D}^\mu \overline{X}^N \\ &\quad + \frac{\text{i}}{4}\Omega_{MN}\left[\overline{\Omega}^{iM} \not{\mathcal{D}} \Omega_i^N - \overline{\Omega}_i^M \not{\mathcal{D}} \Omega^{iN}\right] \\ &\quad - \frac{\text{i}}{2}\Omega_{MN}\left[\overline{\psi}_\mu^i \not{\mathcal{D}} \overline{X}^M \gamma^\mu \Omega_i^N - \overline{\psi}_{\mu i} \not{\mathcal{D}} X^M \gamma^\mu \Omega^{iN}\right] , \end{aligned}$$

$$\begin{aligned} e^{-1}\mathscr{L}_{\text{kin}}^{(2)} &= -\frac{1}{4}\mathcal{I}_{\Lambda\Sigma}\mathcal{H}_{\mu\nu}^{\Lambda}\mathcal{H}^{\mu\nu\Sigma} - \frac{\text{i}}{8}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\mathcal{R}_{\Lambda\Sigma}\mathcal{H}_{\mu\nu}^{\Lambda}\mathcal{H}_{\rho\sigma}^{\Sigma} \\ &\quad + \mathcal{H}_{\mu\nu}^{\Lambda}\mathcal{O}_{\Lambda}^{\mu\nu} - \frac{1}{2}[\mathcal{I}^{-1}]^{\Lambda\Sigma}\mathcal{O}_{\mu\nu\Lambda}\mathcal{O}_{\Sigma}^{\mu\nu} , \end{aligned}$$

$$
\begin{aligned}
e^{-1}\mathscr{L}_{\rm conf} \;=\; & \frac{1}{6}K\Big[R + \Big(e^{-1}\varepsilon^{\mu\nu\rho\sigma}\overline{\psi}{}^{i}_{\mu}\gamma_{\nu}\mathcal{D}_{\rho}\psi_{\sigma i} - \overline{\psi}{}^{i}_{\mu}\psi^{j}_{\nu}T^{\mu\nu}{}_{ij} + (\text{h.c.})\Big)\Big] - K\Big[D + \frac{1}{2}\overline{\psi}{}^{i}_{\mu}\gamma^{\mu}\chi_{i} + \frac{1}{2}\overline{\psi}_{\mu i}\gamma^{\mu}\chi^{i}\Big] \\
& - \Big[K_{\Lambda}\Big(\frac{1}{4}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\overline{\psi}_{\mu i}\gamma_{\nu}\psi^{i}_{\rho}\mathcal{D}_{\sigma}X^{\Lambda} + \frac{1}{48}\overline{\psi}_{\mu i}\gamma^{\mu}\gamma_{\rho\sigma}\Omega^{\Lambda}_{j}T^{\rho\sigma ij}\Big) + (\text{h.c.})\Big] \\
& - \Big[K_{\Lambda}\Big(\frac{1}{3}\overline{\Omega}{}^{\Lambda}_{i}\gamma^{\mu\nu}\mathcal{D}_{\mu}\psi^{i}_{\nu} - \overline{\Omega}{}^{\Lambda}_{i}\chi^{i}\Big) + (\text{h.c.})\Big] \,,
\end{aligned}
$$

$$
\begin{aligned}
e^{-1}\mathscr{L}_{\rm H,conf} \;=\; & \frac{1}{6}\chi\Big[R + \Big(e^{-1}\varepsilon^{\mu\nu\rho\sigma}\overline{\psi}{}^{i}_{\mu}\gamma_{\nu}\mathcal{D}_{\rho}\psi_{\sigma i} - \frac{1}{4}\overline{\psi}{}^{i}_{\mu}\psi^{j}_{\nu}T^{\mu\nu}{}_{ij} + (\text{h.c.})\Big)\Big] + \frac{1}{2}\chi\Big[D + \frac{1}{2}\overline{\psi}{}^{i}_{\mu}\gamma^{\mu}\chi_{i} + \frac{1}{2}\overline{\psi}_{\mu i}\gamma^{\mu}\chi^{i}\Big] \\
& - \frac{1}{2}g_{AB}\mathcal{D}_{\mu}\phi^{A}\mathcal{D}^{\mu}\phi^{B} + \Big[- G_{\overline{\alpha}\beta}\overline{\zeta}{}^{\overline{\alpha}}\not{\mathcal{D}}\zeta^{\beta} + g_{AB}\overline{\gamma}{}^{iB}_{\alpha}\,\overline{\zeta}{}^{\alpha}\gamma^{\mu}\not{\mathcal{D}}\phi^{A}\psi_{\mu i} + (\text{h.c.})\Big] \\
& - \frac{1}{2}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{ik}J^{kj}{}_{AB}(\overline{\psi}{}^{i}_{\mu}\gamma_{\nu}\psi_{\rho j})\chi^{A}\mathcal{D}_{\sigma}\phi^{B} - \frac{1}{4}W_{\overline{\alpha}\beta\overline{\gamma}\delta}\,(\overline{\zeta}{}^{\overline{\alpha}}\gamma_{\mu}\zeta^{\beta})(\overline{\zeta}{}^{\overline{\gamma}}\gamma^{\mu}\zeta^{\delta}) \\
& - \chi_{A}\Big[\gamma^{A}_{i\overline{\alpha}}\Big(\frac{2}{3}\overline{\zeta}{}^{\overline{\alpha}}\gamma^{\mu\nu}\mathcal{D}_{\mu}\psi^{i}_{\nu} + \overline{\zeta}{}^{\overline{\alpha}}\chi^{i} - \frac{1}{6}\overline{\zeta}{}^{\overline{\alpha}}\gamma_{\mu}\psi_{\nu i}T^{\mu\nu ij}\Big) + (\text{h.c.})\Big] \\
& + \Big[\frac{1}{16}\overline{\Omega}_{\alpha\beta}\overline{\zeta}{}^{\alpha}\gamma^{\mu\nu}T_{\mu\nu ij}\varepsilon^{ij}\zeta^{\beta} - \frac{1}{2}\overline{\zeta}{}^{\alpha}\gamma^{\mu}\gamma^{\nu}\psi_{\mu i}(\overline{\psi}{}^{i}_{\nu}G_{\alpha\overline{\beta}}\zeta^{\overline{\beta}} + \varepsilon^{ij}\overline{\Omega}_{\alpha\beta}\overline{\psi}_{\nu j}\zeta^{\beta}) + (\text{h.c.})\Big] \,,
\end{aligned}
$$

$$
\begin{aligned}
e^{-1}\mathscr{L}_{\rm top} \;=\; & \frac{\rm i}{8}{\rm g}\, e^{-1}\varepsilon^{\mu\nu\rho\sigma}(\Theta^{\Lambda\mathsf{a}}B_{\mu\nu\mathsf{a}} + \Theta^{\Lambda\mathsf{m}}B_{\mu\nu\mathsf{m}})\Big(2\partial_{\rho}W_{\sigma\Lambda} + {\rm g}\,T_{[MN]\Lambda}W^{M}_{\rho}W^{N}_{\sigma} - \frac{1}{4}{\rm g}\,\Theta_{\Lambda}{}^{\mathsf{b}}B_{\rho\sigma\mathsf{b}} - \frac{1}{4}{\rm g}\,\Theta_{\Lambda}{}^{\mathsf{n}}B_{\rho\sigma\mathsf{n}}\Big) \\
& + \frac{\rm i}{3}{\rm g}\, e^{-1}\varepsilon^{\mu\nu\rho\sigma}T_{[MN]\Lambda}W^{M}_{\mu}W^{N}_{\nu}\Big(\partial_{\rho}W^{\Lambda}_{\sigma} + \frac{1}{4}{\rm g}\,T_{[PQ]}{}^{\Lambda}W^{P}_{\rho}W^{Q}_{\sigma}\Big) \\
& + \frac{\rm i}{6}{\rm g}\, e^{-1}\varepsilon^{\mu\nu\rho\sigma}T_{[MN]}{}^{\Lambda}W^{M}_{\mu}W^{N}_{\nu}\Big(\partial_{\rho}W_{\sigma\Lambda} + \frac{1}{4}{\rm g}\,T_{[PQ]\Lambda}W^{P}_{\rho}W^{Q}_{\sigma}\Big) \,,
\end{aligned}
$$

$$
\begin{aligned}
e^{-1}\mathscr{L}_{\rm aux} \ &= \ \frac{1}{8}N^{\Lambda\Sigma}\Big[N_{\Lambda\Gamma}Y^{\Gamma}_{ij} + \frac{\rm i}{2}\big(F_{\Lambda\Gamma\Pi}\overline{\Omega}{}^{\Gamma}_i\Omega^{\Pi}_j - \overline{F}_{\Lambda\Gamma\Pi}\overline{\Omega}{}^{k\Gamma}\Omega^{l\Pi}\varepsilon_{ik}\varepsilon_{jl}\big)\Big] \\
&\qquad \times \Big[N_{\Sigma\Xi}Y^{ij\Xi} - \frac{\rm i}{2}\big(\overline{F}_{\Sigma\Xi\Delta}\overline{\Omega}{}^{i\Xi}\Omega^{j\Delta} - F_{\Sigma\Xi\Delta}\overline{\Omega}{}^{\Xi}_m\Omega^{\Delta}_n\varepsilon^{im}\varepsilon^{jn}\big)\Big] \, ,
\end{aligned}
$$

$$
\begin{aligned}
e^{-1}\mathscr{L}_{\rm g} \ &= \ -\frac{1}{2}{\rm g}\Big[{\rm i}\Omega_{MQ}T_{PN}{}^{Q}\varepsilon^{ij}\overline{X}{}^{N}\overline{\Omega}{}^{M}_i\,(\Omega^P_j + \gamma^{\mu}\psi_{\mu j}X^P) + ({\rm h.c.})\Big] \\
&\quad + 2{\rm g}\Big[k_{AM}\gamma^{A}_{i\overline{\alpha}}\varepsilon^{ij}\overline{\zeta}{}^{\overline{\alpha}}(\Omega^M_j + \gamma^{\mu}\psi_{\mu j}X^M) + ({\rm h.c.})\Big] \\
&\quad + {\rm g}\Big[\mu^{ij}{}_{M}\overline{\psi}_{\mu i}(\gamma^{\mu}\Omega^M_j + \gamma^{\mu\nu}\psi_{\nu j}X^M) + ({\rm h.c.})\Big] \\
&\quad + 2{\rm g}\Big[\overline{X}{}^{M}T_M{}^{\gamma}{}_{\alpha}\overline{\Omega}_{\beta\gamma}\overline{\zeta}{}^{\alpha}\zeta^{\beta} + X^M T_M{}^{\overline{\gamma}}{}_{\overline{\alpha}}\Omega_{\overline{\beta}\overline{\gamma}}\overline{\zeta}{}^{\overline{\alpha}}\zeta^{\overline{\beta}}\Big] \\
&\quad - \frac{1}{4}{\rm g}\Big[F_{\Lambda\Sigma\Gamma}\mu^{ij\Lambda}\overline{\Omega}{}^{\Sigma}_i\Omega^{\Gamma}_j + \overline{F}_{\Lambda\Sigma\Gamma}\mu_{ij}{}^{\Lambda}\overline{\Omega}{}^{i\Sigma}\Omega^{j\Gamma}\Big] \\
&\quad + {\rm g}\,Y^{ij\Lambda}\Big[\mu_{ij\Lambda} + \frac{1}{2}(F_{\Lambda\Sigma} + \overline{F}_{\Lambda\Sigma})\mu_{ij}{}^{\Sigma}\Big] \, ,
\end{aligned}
$$

$$
\begin{aligned}
e^{-1}\mathscr{L}_{{\rm g}^2} \ &= \ {\rm i}{\rm g}^2\Omega_{MN}\big(T_{PQ}{}^{M}X^P\overline{X}{}^{Q}\big)\big(T_{RS}{}^{N}\overline{X}{}^{R}X^S\big) - 2{\rm g}^2k^A{}_Mk^B{}_Ng_{AB}X^M\overline{X}{}^N \\
&\quad - \frac{1}{2}{\rm g}^2N_{\Lambda\Sigma}\,\mu_{ij}{}^{\Lambda}\,\mu^{ij\Sigma} \, .
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{L}_{\text{magnetic}} \;&=\; \mathscr{L}_{\text{kin}}^{(1)} + \mathscr{L}_{\text{aux}} + \mathscr{L}_{\text{conf}} + \mathscr{L}_{\text{H,conf}}^{(1)} + \mathscr{L}_{\text{g}} + \mathscr{L}_{\text{g}^2} \\
&\quad + \mathscr{L}_{\text{V1}} + \mathscr{L}_{\text{H,conf},J}^{(2)} + \mathscr{L}_{B} + \mathscr{L}_{J2} + \mathscr{L}_{\text{V2}-W} \, ,
\end{aligned}
$$

$$
\begin{aligned}
e^{-1} \mathscr{L}_{\text{V1}} \;&=\; -\frac{1}{4} \Big[\widehat{\mathcal{I}}^{IJ} \, \widehat{\mathcal{F}}_{\mu\nu I} \widehat{\mathcal{F}}^{\mu\nu}_J + 2 \widehat{\mathcal{I}}^I{}_V \, \widehat{\mathcal{F}}_{\mu\nu I} \widehat{\mathcal{F}}^{\mu\nu V} + \widehat{\mathcal{I}}_{UV} \, \widehat{\mathcal{F}}^U_{\mu\nu} \widehat{\mathcal{F}}^{\mu\nu V} \Big] \\
&\quad - \frac{\mathrm{i}}{8} e^{-1} \varepsilon^{\mu\nu\rho\sigma} \Big[\widehat{\mathcal{R}}^{IJ} \, \widehat{\mathcal{F}}_{\mu\nu I} \widehat{\mathcal{F}}_{\rho\sigma J} + 2 \widehat{\mathcal{R}}^I{}_V \, \widehat{\mathcal{F}}_{\mu\nu I} \widehat{\mathcal{F}}^V_{\rho\sigma} + \widehat{\mathcal{R}}_{UV} \, \widehat{\mathcal{F}}^U_{\mu\nu} \widehat{\mathcal{F}}^V_{\rho\sigma} \Big] \, , \\
e^{-1} \mathscr{L}_{\text{H,conf},J}^{(2)} \;&=\; -\frac{1}{2} \mathcal{G}_{AB} \widehat{\mathcal{D}}_{\mu} \phi^A \widehat{\mathcal{D}}^{\mu} \phi^B + \Big[- G_{\overline{\alpha}\beta} \overline{\zeta}^{\overline{\alpha}} \widetilde{\not{\mathcal{D}}} \zeta^{\beta} + \mathcal{G}_{AB} \overline{\gamma}^{iB}_{\alpha} \big(\overline{\zeta}^{\alpha} \gamma^{\rho} \gamma^{\mu} \psi_{\rho i} \big) \widehat{\mathcal{D}}_{\mu} \phi^A + (\text{h.c.}) \Big] \\
&\quad - \frac{1}{2} e^{-1} \varepsilon^{\mu\nu\rho\sigma} \varepsilon_{ik} \, \mathcal{J}^{kj}{}_{AB} \big(\overline{\psi}^i_{\mu} \gamma_{\nu} \psi_{\rho j} \big) \chi^A \widehat{\mathcal{D}}_{\sigma} \phi^B \, , \\
e^{-1} \mathscr{L}_{B} \;&=\; -\frac{1}{2 \cdot 4!} H_{\mu\nu\rho \mathsf{m}'} \mathcal{M}^{\mathsf{m}'\mathsf{n}'} H^{\mu\nu\rho}_{\mathsf{n}'} + \frac{\mathrm{i}}{12} e^{-1} \varepsilon^{\mu\nu\rho\sigma} J_{\mu \mathsf{m}'} \mathcal{M}^{\mathsf{m}'\mathsf{n}'} H_{\nu\rho\sigma \mathsf{n}'} \, , \\
e^{-1} \mathscr{L}_{J2} \;&=\; \frac{1}{2} \Big[\big(J^{(2)}_{\mu \mathsf{m}'} + J^{(3)}_{\mu \mathsf{m}'} + (\text{h.c.}) \big) + J^{(4)}_{\mu \mathsf{m}'} \Big] \mathcal{M}^{\mathsf{m}'\mathsf{n}'} \Big[\big(J^{\mu(2)}_{\mathsf{n}'} + J^{\mu(3)}_{\mathsf{n}'} + (\text{h.c.}) \big) + J^{\mu(4)}_{\mathsf{n}'} \Big] \, , \\
e^{-1} \mathscr{L}_{\text{V2}-W} \;&=\; \widehat{\mathcal{F}}^U_{\mu\nu} \mathcal{O}^{\mu\nu}_U - \frac{1}{2} [\mathcal{I}^{-1}]^{\Lambda\Sigma} \mathcal{O}_{\mu\nu\Lambda} \mathcal{O}^{\mu\nu}_{\Sigma} + \widehat{\mathcal{I}}^{IJ} \mathcal{O}_{\mu\nu I} \Big[\big(\mathcal{O}^{\mu\nu}_J - \mathcal{I}_{JU} \widehat{\mathcal{F}}^{\mu\nu U} \big) + (r \mathcal{I}^{-1})_J{}^K \big(\widehat{\mathcal{F}}^{\mu\nu}_K - \mathcal{R}_{KU} \widehat{\mathcal{F}}^{\mu\nu U} \big) \Big] \\
&\quad + \frac{\mathrm{i}}{2} e^{-1} \varepsilon^{\mu\nu\rho\sigma} \, \widehat{\mathcal{I}}^{IJ} \mathcal{O}_{\mu\nu I} \Big[\big(\widehat{\mathcal{F}}_{\rho\sigma J} - \mathcal{R}_{JU} \widehat{\mathcal{F}}^U_{\rho\sigma} \big) - (r \mathcal{I}^{-1})_J{}^K \big(\mathcal{O}_{\rho\sigma K} - \mathcal{I}_{KU} \widehat{\mathcal{F}}^U_{\rho\sigma} \big) \Big] \\
&\quad - \frac{\mathrm{i}}{32} \mathrm{g}^2 e^{-1} \varepsilon^{\mu\nu\rho\sigma} \Theta^{U \mathsf{m}'} \Theta_U{}^{\mathsf{n}'} B_{\mu\nu \mathsf{m}'} B_{\rho\sigma \mathsf{n}'} \, .
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_{\mathsf{m}'\mathsf{n}'} &= g_{AB} k^{A}{}_{\mathsf{m}'} k^{B}{}_{\mathsf{n}'} \,, \\
J^{\mu}_{\mathsf{m}'} &= J^{\mu(1)}_{\mathsf{m}'} + \big[J^{\mu(2)}_{\mathsf{m}'} + J^{\mu(3)}_{\mathsf{m}'} + (\text{h.c.}) \big] + J^{\mu(4)}_{\mathsf{m}'} \,, \\
J^{\mu(1)}_{\mathsf{m}'} &= g_{AB} k^{A}{}_{\mathsf{m}'} \widehat{\mathcal{D}}^{\mu} \phi^{B} \,, \\
J^{\mu(2)}_{\mathsf{m}'} &= G_{\overline{\alpha}\beta} (t_{\mathsf{m}'})^{\beta}{}_{\gamma} \big(\overline{\zeta}^{\overline{\alpha}} \gamma^{\mu} \zeta^{\gamma} \big) \,, \\
J^{\mu(3)}_{\mathsf{m}'} &= - g_{AB} k^{A}{}_{\mathsf{m}'} \overline{\gamma}^{iB}_{\alpha} \big(\overline{\zeta}^{\alpha} \gamma^{\rho} \gamma^{\mu} \psi_{\rho i} \big) \,, \\
J^{\mu(4)}_{\mathsf{m}'} &= - \frac{1}{2} e^{-1} \varepsilon^{\mu\nu\rho\sigma} \varepsilon_{ik} J^{kj}{}_{AB} \big(\overline{\psi}^{i}_{\nu} \gamma_{\rho} \psi_{\sigma j} \big) \chi^{A} k^{B}{}_{\mathsf{m}'} \,.
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{G}_{AB} &= g_{AB} - g_{A\mathsf{m}'} \mathcal{M}^{\mathsf{m}'\mathsf{n}'} g_{\mathsf{n}'B} \,, \\
\mathcal{J}^{kj}{}_{AB} &= J^{kj}{}_{AB} - J^{kj}{}_{A\mathsf{m}'} \mathcal{M}^{\mathsf{m}'\mathsf{n}'} g_{\mathsf{n}'B} \,.
\end{aligned}
$$

TK, to appear.

$$0 = -\Theta^{I\mathsf{a}'}\widehat{\mathcal{J}}_{\mu\nu\Lambda} + \frac{1}{2}\Theta^{I\mathsf{a}'}\Big[\mathcal{R}_{\Lambda\Sigma}B^{\Sigma}_{\mu\nu} - \frac{\mathrm{i}}{2}e\,\varepsilon_{\mu\nu\rho\sigma}\mathcal{I}_{\Lambda\Sigma}B^{\rho\sigma\Sigma}\Big] + \frac{1}{2}\Theta_I{}^{\mathsf{a}'}B^I_{\mu\nu}\,,$$

$$\widehat{\mathcal{J}}_{\mu\nu\Lambda} \equiv \widehat{\mathcal{F}}_{\mu\nu\Lambda} - \Big[\mathcal{R}_{\Lambda\Sigma}\widehat{\mathcal{F}}^{\Sigma}_{\mu\nu} - \frac{\mathrm{i}}{2}e\,\varepsilon_{\mu\nu\rho\sigma}\mathcal{I}_{\Lambda\Sigma}\widehat{\mathcal{F}}^{\rho\sigma\Sigma}\Big] - \mathrm{i}\,e\,\varepsilon_{\mu\nu\rho\sigma}\mathcal{O}^{\rho\sigma}_{\Lambda}\,,$$

$$\widehat{\mathcal{F}}_{\mu\nu\Lambda} \equiv \mathcal{F}_{\mu\nu\Lambda} - \frac{1}{2}\widehat{B}_{\mu\nu\Lambda}\,, \qquad \widehat{\mathcal{F}}^{\Lambda}_{\mu\nu} \equiv \mathcal{F}^{\Lambda}_{\mu\nu} + \frac{1}{2}\widehat{B}^{\Lambda}_{\mu\nu}\,,$$

$$B^{\Lambda}_{\mu\nu} \equiv \mathrm{g}\Theta^{\Lambda\mathsf{a}}B_{\mu\nu\mathsf{a}}\,, \qquad \widehat{B}^{\Lambda}_{\mu\nu} \equiv \mathrm{g}\Theta^{\Lambda\mathsf{m}}B_{\mu\nu\mathsf{m}}\,, \qquad \widehat{B}_{\mu\nu\Lambda} \equiv \mathrm{g}\Theta_{\Lambda}{}^{\mathsf{m}}B_{\mu\nu\mathsf{m}}\,,$$

$$\downarrow$$

$$B^I_{\mu\nu} = \widehat{\mathcal{I}}^{IJ}\Big\{\mathrm{i}\,e\,\varepsilon_{\mu\nu\rho\sigma}\widehat{\mathcal{J}}^{\rho\sigma}_{J} + 2(r\mathcal{I}^{-1})_J{}^K\widehat{\mathcal{J}}_{\mu\nu K}\Big\}\,,$$

$$\widehat{\mathcal{I}}^{IJ} \equiv [(\mathcal{I} + r\mathcal{I}^{-1}r)^{-1}]^{IJ}\,, \qquad r_{IJ} \equiv \mathcal{R}_{IJ} + (\Theta^{-1})_{\mathsf{a}'I}\Theta_J{}^{\mathsf{a}'}\,.$$

10D type IIA action $S^{(10\mathrm{D})}_{\mathrm{IIA}} = S_{\mathrm{NS}} + \widetilde{S}_{\mathrm{R}} = S_{\mathrm{NS}} + S_{\mathrm{R}} + S_{\mathrm{CS}}$: (democratic form)

$$S_{\mathrm{NS}} = \frac{1}{2}\int \mathrm{e}^{-2\phi}\Big\{R * \mathbb{1} + 4\mathrm{d}\phi \wedge *\mathrm{d}\phi - \frac{1}{2}H_3 \wedge *H_3\Big\}, \quad \widetilde{S}_{\mathrm{R}} = -\frac{1}{8}\int \big[\mathbf{F} \wedge *\mathbf{F}\big]_{10}$$

with "constraint $\mathbf{F} = \lambda(*\mathbf{F})$" and "EoM (Bianchi) $(\mathrm{d} + H\wedge) * \mathbf{F} = 0 \Leftrightarrow (\mathrm{d} - H\wedge)\mathbf{F} = 0$"

↓ $SU(3)$-structure with $m^{\Lambda}_{\mathrm{R}} = 0$

4D $\mathcal{N} = 2$ abelian gauged SUGRA (with $\xi^{\boldsymbol{I}} \equiv (\xi^I, \widetilde{\xi}_I)^{\mathrm{T}}$):

$$S^{(4\mathrm{D})} = \int \mathrm{d}^4 x \sqrt{-g}\,\Big[\frac{1}{2}R + \frac{1}{4}\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma\mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}}\mathrm{Re}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma}_{\rho\sigma} - g_{a\bar{b}}\,\partial_\mu \mathfrak{t}^a \partial^\mu \bar{\mathfrak{t}}^{\bar{b}} - g_{i\bar{\jmath}}\,\partial_\mu z^i \partial^\mu \bar{z}^{\bar{\jmath}}$$
$$-\partial_\mu\varphi\partial^\mu\varphi + \frac{\mathrm{e}^{2\varphi}}{2}(\mathbb{M}_{\mathrm{H}})_{\boldsymbol{IJ}} D_\mu \xi^{\boldsymbol{I}} D^\mu \xi^{\boldsymbol{J}} - \frac{\mathrm{e}^{2\varphi}}{4}\big(D_\mu a - \xi^{\boldsymbol{I}}(\mathbb{C}_{\mathrm{H}})_{\boldsymbol{IJ}} D_\mu \xi^{\boldsymbol{J}}\big)^2 - V(\mathfrak{t}, \bar{\mathfrak{t}}, q)\Big]$$

- $(e_{\Lambda}{}^I, e_{\Lambda I})$: geometric flux charges & $e_{\mathrm{R}\Lambda}$: RR-flux charges ⟵ non-CY data
  (with constraints $e_{\Lambda}{}^I e_{\Sigma I} - e_{\Lambda I} e_{\Sigma}{}^I = 0$)
- $\mathfrak{t}^a \in \mathrm{SKG}_{\mathrm{V}}$ and $z^i \in \mathrm{SKG}_{\mathrm{H}} \subset \mathcal{HM}$ are ungauged (in general)
- $D_\mu \xi^I = \partial_\mu \xi^I - e_{\Lambda}{}^I A^{\Lambda}_\mu$ & $D_\mu \widetilde{\xi}_I = \partial_\mu \widetilde{\xi}_I - e_{\Lambda I} A^{\Lambda}_\mu$
- $D_\mu a = \partial_\mu a - (2e_{\mathrm{R}\Lambda} - \xi^I e_{\Lambda I} + \widetilde{\xi}_I e_{\Lambda}{}^I) A^{\Lambda}_\mu$
- $V(\mathfrak{t}, \bar{\mathfrak{t}}, q)$: scalar potential

D. Cassani, arXiv:0804.0595

Non-vanishing $m_{\rm R}^{\Lambda}$ dualizes the axion field a in standard SUGRA to B-field.

4D gauged action is different from the standard one:

$$\begin{aligned} S^{(4{\rm D})} \ = \ \int \Big[& \frac{1}{2} R(*\mathbb{1}) + \frac{1}{2} {\rm Im}\mathcal{N}_{\Lambda\Sigma} F_2^{\Lambda} \wedge *F_2^{\Sigma} + \frac{1}{2} {\rm Re}\mathcal{N}_{\Lambda\Sigma} F_2^{\Lambda} \wedge F_2^{\Sigma} - g_{a\bar{b}}\, {\rm d}\mathfrak{t}^a \wedge *{\rm d}\bar{\mathfrak{t}}^{\bar{b}} - g_{i\bar{\jmath}}\, {\rm d}z^i \wedge *{\rm d}\bar{z}^{\bar{\jmath}} \\ & - {\rm d}\varphi \wedge *{\rm d}\varphi - \frac{{\rm e}^{-4\varphi}}{4} H_3 \wedge *H_3 - \frac{{\rm e}^{2\varphi}}{2} (\mathbb{M}_{\rm H})_{\boldsymbol{IJ}} D\xi^{\boldsymbol{I}} \wedge *D\xi^{\boldsymbol{J}} - V(*\mathbb{1}) \\ & + \frac{1}{2} {\rm d}B \wedge \Big[\xi^{\boldsymbol{I}} (\mathbb{C}_{\rm H})_{\boldsymbol{IJ}} D\xi^{\boldsymbol{J}} + \big(2 e_{{\rm R}\Lambda} - \xi^I e_{\Lambda I} + \widetilde{\xi}_I e_{\Lambda}{}^I \big) A_1^{\Lambda} \Big] - \frac{1}{2} m_{\rm R}^{\Lambda} e_{{\rm R}\Lambda} B_2 \wedge B_2 \Big] \end{aligned}$$

Constraints among flux charges:

$$e_{\Lambda}{}^I e_{\Sigma I} - e_{\Lambda I} e_{\Sigma}{}^I \ = \ 0, \quad m_{\rm R}^{\Lambda} e_{\Lambda}{}^I \ = \ 0 \ = \ m_{\rm R}^{\Lambda} e_{\Lambda I}$$

e.g., D. Cassani, arXiv:0804.0595

$$S_{\rm GG}^{(4{\rm D})} = \int {\rm vol}_4 \Big[\frac{1}{2} R - \partial_{\mu} \varphi \partial^{\mu} \varphi - \frac{1}{24} {\rm e}^{-4\varphi} H_{\mu\nu\rho} H^{\mu\nu\rho} - g_{a\bar{b}} \, \partial_{\mu} {\rm t}^a \partial^{\mu} \bar{\rm t}^{\bar{b}} - g_{i\bar{\jmath}} \, \partial_{\mu} z^i \partial^{\mu} \bar{z}^{\bar{\jmath}} - V \Big]$$

$$+ \int \Big[\frac{1}{2} {\rm Im} \mathcal{N}_{\Lambda\Sigma} F^{\Lambda} \wedge * F^{\Sigma} + \frac{1}{2} {\rm Re} \mathcal{N}_{\Lambda\Sigma} F^{\Lambda} \wedge F^{\Sigma} + \frac{1}{2} \widetilde{\Delta}_{\boldsymbol{IJ}} \, {\rm d}\widehat{\xi}^{\boldsymbol{I}} \wedge * {\rm d}\widehat{\xi}^{\boldsymbol{J}}$$

$$+ \frac{1}{2} (\Delta^{-1})^{\Lambda\Sigma} \big({\rm d}\check{C}_{2\Lambda} + \zeta_{\Lambda} {\rm d}B \big) \wedge * \big({\rm d}\check{C}_{2\Sigma} + \zeta_{\Sigma} {\rm d}B \big)$$

$$+ \big({\rm d}\check{C}_{2\Lambda} + \zeta_{\Lambda} {\rm d}B \big) \wedge \big({\rm e}^{2\varphi} \Delta^{-1} U^{\rm T} \mathbb{M}_{\rm H} \big)^{\Lambda}{}_{\boldsymbol{I}} {\rm d}\widehat{\xi}^{\boldsymbol{I}} + \frac{1}{2} {\rm d}B \wedge \widehat{\xi}^{\boldsymbol{I}} (\mathbb{C}_{\rm H})_{\boldsymbol{IJ}} {\rm d}\widehat{\xi}^{\boldsymbol{J}}$$

$$+ \big(\check{C}_{2\Lambda} - e_{{\rm R}\Lambda} B \big) \wedge \Big\{ {\rm d}A_1^{\Lambda} + \frac{1}{2} (\widetilde{U} V)^{\Lambda\Sigma} \check{C}_{2\Sigma} + \frac{1}{2} m_{\rm R}^{\Lambda} B \Big\} \Big] \, ,$$

$$F^{\Lambda} \equiv G_2^{\Lambda} + G_0^{\Lambda} B = {\rm d}A_1^{\Lambda} + (\widetilde{U} V)^{\Lambda\Sigma} \check{C}_{2\Sigma} + m_{\rm R}^{\Lambda} B \, ,$$

$$\widehat{\xi}^{\boldsymbol{I}} \equiv (\mathbb{P}_0)^{\boldsymbol{I}}{}_{\boldsymbol{J}} \xi^{\boldsymbol{J}} \, , \quad 0 = (\mathbb{P}_{\neq 0})^{\boldsymbol{I}}{}_{\boldsymbol{J}} \widehat{\xi}^{\boldsymbol{J}} \, , \quad \zeta_{\Lambda} \equiv (U^{\rm T} \mathbb{C}_{\rm H})_{\Lambda \boldsymbol{I}} \widehat{\xi}^{\boldsymbol{I}} \, ,$$

$$\Delta_{\Lambda\Sigma} \equiv {\rm e}^{2\varphi} (U^{\rm T})_{\Lambda}{}^{\boldsymbol{I}} (\mathbb{M}_{\rm H})_{\boldsymbol{IJ}} U^{\boldsymbol{J}}{}_{\Sigma} \, , \quad \widetilde{\Delta}_{\boldsymbol{IJ}} \equiv {\rm e}^{2\varphi} \big(\mathbb{M}_{\rm H} - {\rm e}^{2\varphi} \mathbb{M}_{\rm H} U \Delta^{-1} U^{\rm T} \mathbb{M}_{\rm H} \big)_{\boldsymbol{IJ}} \, ,$$

$$\check{G}_1^{\Lambda} \equiv -(\Delta^{-1})^{\Lambda\Sigma} \Big[* \, {\rm d}\check{C}_{2\Sigma} + \zeta_{\Sigma} * {\rm d}B + {\rm e}^{2\varphi} \big(U^{\rm T} \mathbb{M}_{\rm H} \big)_{\Sigma \boldsymbol{I}} {\rm d}\widehat{\xi}^{\boldsymbol{I}} \Big] \, , \quad G_0^{\boldsymbol{\Lambda}} = c^{\boldsymbol{\Lambda}} + \widetilde{Q}^{\boldsymbol{\Lambda}}{}_{\boldsymbol{I}} \widehat{\xi}^{\boldsymbol{I}} \, .$$