Dark energy function in modified gravity and supergravity

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We propose new theoretical constraints on the dynamical dark energy function in $f(R)$ gravity and $F(\mathcal{R})$ supergravity theories by demanding the effective scalar potential to be (i) renormalisable and (ii) supersymmetric.
A model of the hidden sector responsible for spontaneous supersymmetry breaking is also proposed.

Introduction

Einstein gravity

$$S_{\text{E.H.}} = -\frac{1}{2}\int d^4x\sqrt{-g}\ R$$

notation

$c = \hbar = M_{\text{Pl}} = 1$

$\eta_{\mu\nu} = (+,-,-,-)$

→ arbitrary function $R \to f(R)$ →

$f(R)$ gravity

$$S_f = \int d^4x\sqrt{-g}\ f(R)$$

→ SUSY →

$F(\mathcal{R})$ supergravity

$$S_F = \int d^4x d^2\theta \mathcal{E} F(\mathcal{R}) + \text{h.c.}$$

(Gates and Ketov, Phys. Lett. **B674** (2009) 59)

$f(R)$ gravity

features

- new degree of freedom
  $f'(R) = \omega \neq \text{const.} \longrightarrow$ *scalaron*
- equivalence to scalar-tensor gravity
- both inflationary model and DE model
- *stability conditions*
  - $f'(R) < 0$
    classical stability
    (graviton is not a ghost)
  - $f''(R) > 0$
    quantum stability
    (scalaron is not a tachyon)

scalar potential in $f(R)$ gravity

$f(R)$ gravity action can be transformed to

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}\tilde{R} + \frac{3}{4(f')^2}\tilde{g}^{\mu\nu}\partial_\mu f'(\phi)\partial_\nu f'(\phi) - V(f'(\phi))\right]$$

where $\tilde{g}_{\mu\nu} = g_{\mu\nu}f'(R),\ \phi = R.$
Then the scalar potential is given by

$$V(\phi) = -\frac{1}{2}\left[\frac{\phi f'(\phi) + f(\phi)}{f'(\phi)^2}\right]$$

This is a quadratic equation with respect to $f'(R)$, so it can be rewritten to the form (*inverse problem*)

$$f'(R) = \frac{-R \pm \sqrt{R^2 - 8Vf}}{4V}$$

$F(\mathcal{R})$ supergravity

features

- supersymmetric extension of $f(R)$ gravity
- classical equivalence to the standard $\mathcal{N} = 1$ Poincare supergravity coupled to a dynamical chiral superfield
- classical stability is replaced by a stronger condition:
  $F'(X) < 0,\ X = \bar{X}$ (X: auxiliary field)
  (※no fermions)
- It may unify inflation, dark energy and dark matter

scalar potential in $F(\mathcal{R})$ gravity

$F(\mathcal{R})$ supergravity action can be rewritten to

$$S = \int d^4x d^2\theta \mathcal{E}\left[-\mathcal{Y}\mathcal{R} + Z(\mathcal{Y})\right] + \text{h.c.}$$

$\mathcal{R}$: scalar supercurvature
$\mathcal{Y} = e^{\sqrt{2/3}\Phi}$: chiral scalar superfield
$Z(\mathcal{Y})$: holomorphic function

It yields the chiral superpotential

$$W(\mathcal{Y}) = \sqrt{\frac{21}{2}}Z(\mathcal{Y})$$

The scalar potential is

$$V = \frac{21}{2}|Z'(Y)|^2 = \frac{21}{2}|\mathcal{R}(Y)|^2$$

Dark energy function in modified gravity and supergravity

Proposal

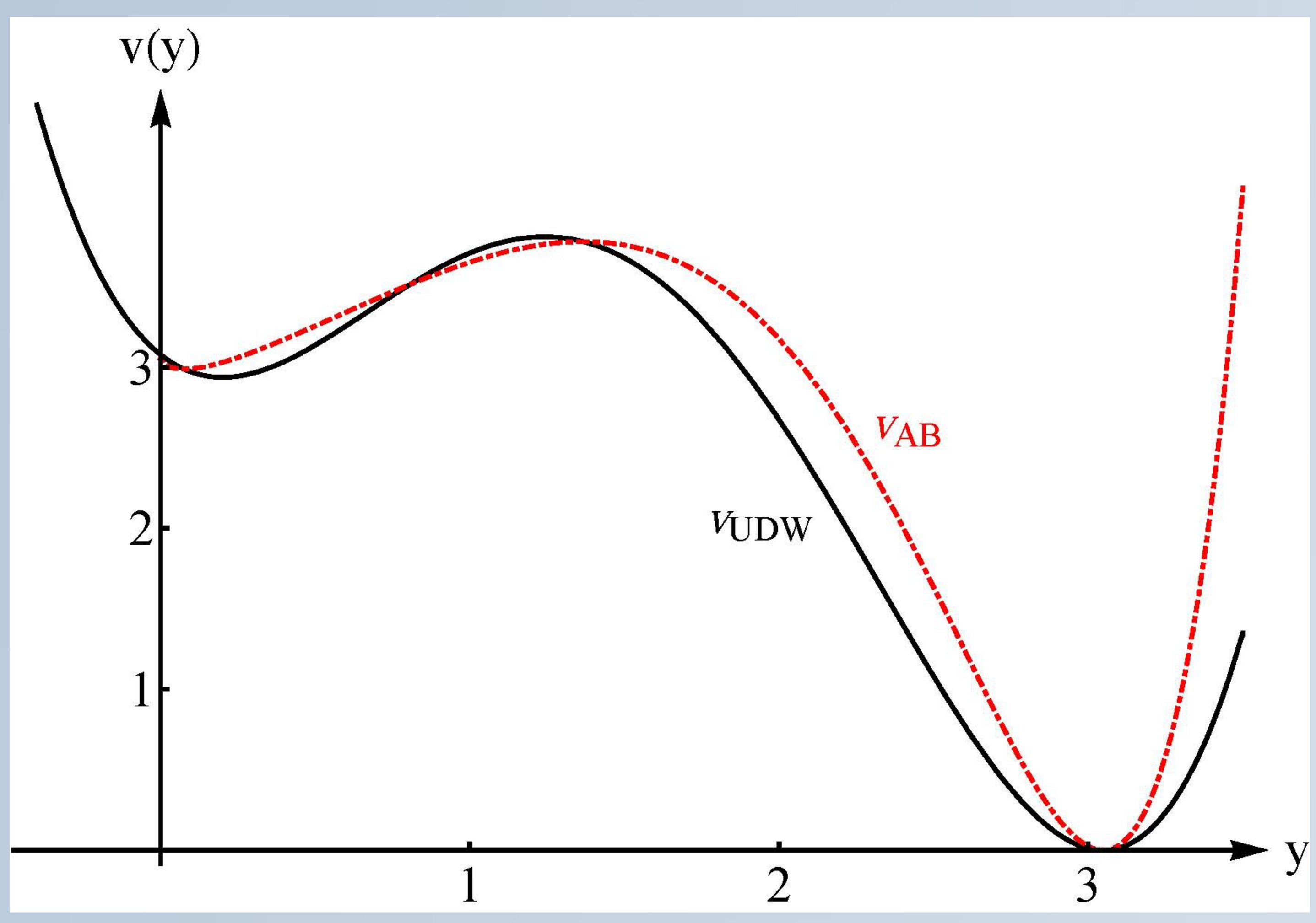
We approximate AB potential $v_{\rm AB}$ by the *Uplifted-Double-Well* (UDW) potential:

$$v_{\rm UDW}(y) = \frac{1}{4}\left[(y-y_0)^2 - v^2\right]^2 + \frac{\mu^2}{2}\left[(y-y_0)-v\right]^2$$

$$y = -\ln f'(\phi) \qquad (y_0, v, \mu\text{: positive parameters})$$

Our parameters can be written for large b

$$\mu^2 = b^{-1} - \frac{1}{2}b^{-4}, \quad y_0 = b - \frac{1}{2}b^{-2}, \quad v = b + \frac{1}{2}b^{-2}$$



AB potential and UDW potential

Appleby-Battye (AB) model

AB function

$$f_{\rm AB}(R) = -\frac{1}{2}R + \frac{1}{2a}\ln\left[\cosh(aR) + \tanh(b)\sinh(aR)\right]$$

$(a, b\text{: positive parameters})$

Newtonian limit

$$f_{\rm AB}(R) \approx -\frac{1}{2}R + \frac{1}{4a}\ln\left(1+e^{2b}\right) + \mathcal{O}\left(e^{-2a|R|+2b}\right)$$

cosmological constant

$$\Lambda \approx \frac{b}{2a} \approx |R_0| = 12H_0^2$$

AB scalar potential

$$V_{\rm AB}(y) = \frac{1}{4a}e^{2y}\left[\ln\left(1-e^{-y}\right) - e^{-y}\ln\left(e^y - 1\right) + 2be^{-y} + C\right]$$

$$C = \ln\left(e^b + e^{-b}\right) - b$$

$$V_0 = \frac{1}{4a} \quad \rightarrow \quad v_{\rm AB}(y) = \frac{V_{\rm AB}(y)}{V_0}$$

AB model:

- is the *ad hoc* model to describe dark energy
- obeys all known theoretical consistency constraints

⇨ To meet observations, $b \geq 30$

UDW potential:

- is renormalisable
- is always positive
- has two minima

$v = 0$: stable = Minkowski vacuum

$v > 0$: meta-stable = de Sitter vacuum ➡ accelerating universe

They are separated by the high potential barrier $(\sim e^{2(b-1)})$

⇩

de Sitter vacuum can live longer than our universe

$v_{\rm UDW}$ can be easily extended to spontaneous SUSY breaking model, using three chiral superfields, Φ_1, Φ_2, Φ_3, and choosing the chiral superpotentials of Φ_1, Φ_2,

$$W_1(\Phi_1) = l^{1/2}\left(\frac{1}{6}\Phi_1^3 - \frac{1}{2}v^2\Phi_1\right), \quad W_2(\Phi_2) = \frac{\mu}{\sqrt{2}}\left(\frac{1}{2}\Phi_2^2 - u\Phi_2\right),$$

$(l, v, \mu, u\text{: real positive parameters})$

then whole chiral superpotential

$$W(\Phi_1, \Phi_2, \Phi_3) = W_1(\Phi_1) + W_2(\Phi_2) + \Phi_3(\Phi_2 - \Phi_1)$$

gives rise to the scalar potential

$$V(\phi) = \frac{l}{4}\left|\phi^2 - v^2\right|^2 + \frac{\mu^2}{2}|\phi - u|^2 \qquad \phi = \Phi|, \quad \Phi = \Phi_1 = \Phi_2$$

t is the complex extension of the UDW potential, where ${\rm Re}(\phi) = y - y_0$ and $u = v$.

Conclusion

By using the inverse relation we replaced the effective scalar potential associated with the *ad hoc* AB function by the Higgs-type scalar potential which gives rise to a meta-stable accelerating universe. We proposed the specific (O'Raifertaigh-type) model of the hidden sector leading to spontaneous SUSY breaking and the UDW scalar potential, in terms of three chiral scalar superfields with the chiral superpotential. In our appproach the chiral scalaron superfield is the universal messenger of the gravitational mediation of SUSY breaking to the visible sector (Standard Model) of elementary particles.

References:

A. Appleby and R. Battye, Phys. Lett. **B654** (2007) 7

S. V Ketov and N. Watanabe, arXiv:1206.0416 [hep-th]