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Making Confining Strings out of Mesons

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Collaboration with R. Kitano and M. Nakamura

1. Introduction

Quark Confinement in QCD ⬅ Most Challenging Problem in QFT

A PHYSICAL Quantity for Describing Confinement

➡ Potential Energy between (Heavy) Quarks

- From Quarkonium Energy Spectrum in Hadron Experiments
- From Wilson Loop Calc. in Lattice QCD

★ Cornell Potential

σ : String Tension

$$V_{Q\bar{Q}}(r) = -\frac{A}{r} + \sigma r \ (+V_0)$$

Coulomb Potential ($-\frac{A}{r}$)

Linear Potential (σr)

This Potential Strongly Indicates STRING-LIKE Flux between Quarks.

1st Route: (Dual) String Model ➡ AdS/CFT, Holographic QCD.

(Nambu, Nielsen, Susskind ...)

(Maldacena, Son-Stephanov, Sakai-Sugimoto, ...)

2nd Route: Solitonic Vortex for Squeezed Flux (Nielsen-Olesen)

Electric-Magnetic (EM) Duality ↓ (Nambu, 't Hooft, Mandelstam, …)

Vortex in EM-Dual Theory is Confining Flux !

We Have Learned Much About the Dualities from SUSY Gauge Theories :

(Since Mid 90's)

1. Exact Montonen-Olive Duality in N=4 SYM.
2. EM-Duality in N=2 SUSY Gauge Theory (Seiberg-Witten)
3. Seiberg-Duality in N=1 SUSY QCD

From New (?) Understanding of the Duality,

Possible to Obtain A New Effective Description for QCD ?

Today, **We Propose A Possible Dual Theory for QCD Based on Seiberg-Duality and Discuss the Relation to the Hidden Local Sym. & Confining Strings.**

- Guiding Principle : Chiral Symmetry Breaking in QCD

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B$$

$$\longrightarrow SU(N_f)_V \times U(1)_B$$

Construct Seiberg-Dual Theory Realizing the SAME Breaking Pattern.

If the Seiberg-Dual Description for QCD Exists,

Dual Gauge Bosons Must Appear as Eff. Degrees of Freedom.

Hidden Local Symmetry Implies ρ & ω Mesons are $U(N_f)$ Massive Gauge Boson after Higgsing.

(Bando, Kugo, Uehara, Yamawaki, Yanagida)

★ Actually, Soft SUSY Breaking Plays A Crucial Role for the Correct Chiral Sym. Breaking.

2. Seiberg-Dual of Softly-Broken SUSY QCD (arXiv: 1109.6158 by R. Kitano)

Seiberg-Duality : For $N_c + 1 < N_f < 3N_c$

$\mathcal{N} = 1\ SU(N_c)$ QCD with N_f-Flavors

⇕

$\mathcal{N} = 1\ SU(N_f - N_c)$ QCD with N_f Flavors and A Meson $+\ W = qM\tilde{q}$

In Real QCD, $N_c = 3$ and $N_f = 2$ or $N_f = 3$,

Seiberg-Dual Theory Does NOT Exist.

Earlier Analysis on Softly-Broken SUSY QCD with $N_f \leq N_c$

➡ NON QCD-Like Behavior (Aharony et al., Martin et al.)

- A Seiberg-Dual for QCD :

Electric Theory :

$SU(N_c)$ QCD with Massless N_f Quarks + Massive N_c (Quark)'s

& Soft SUSY Breaking Mass for Squarks and Gauginos.

Quantum Numbers in Ele. Theory

Gauged

	$\mathrm{SU}(N_c)$	$\mathrm{SU}(N_f)_L$	$\mathrm{SU}(N_f)_R$	$\mathrm{U}(1)_B$	$\mathrm{SU}(N_c)_V$	$\mathrm{U}(1)_{B'}$	$\mathrm{U}(1)_R$
Q	N_c	N_f	1	1	1	0	$(N_f - N_c)/N_f$
$\overline{Q}$	$\overline{N_c}$	1	$\overline{N_f}$	-1	1	0	$(N_f - N_c)/N_f$
Q'	N_c	1	1	0	$\overline{N_c}$	1	1
$\overline{Q}'$	$\overline{N_c}$	1	1	0	N_c	-1	1

Soft SUSY Breaking :

$$\mathcal{L}_{\rm soft} = -\tilde{m}^2(|Q|^2 + |\bar{Q}|^2 + |Q'|^2 + |\bar{Q}'|^2) - \left(\frac{m_\lambda}{2}\lambda\lambda + \mathrm{h.c.}\right) - (BmQ'\bar{Q}' + \mathrm{h.c.})$$

All Mass Parameters (incl. Soft-Mass) are Large ➡ Massless QCD with N_f Flavor

$N_f < N_c$: Dual Theory is More Weakly-Coupled

★ Seiberg-Dual Theory :

$SU(N_f)$ QCD with Dual quarks and Dual (quark)'s & Singlet Mesons

Fields and Quantum Numbers in Mag. Theory

Gauged

	$\mathrm{SU}(N_f)$	$\mathrm{SU}(N_f)_L$	$\mathrm{SU}(N_f)_R$	$\mathrm{U}(1)_B$	$\mathrm{SU}(N_c)_V$	$\mathrm{U}(1)_{B'}$	$\mathrm{U}(1)_R$
q	N_f	$\overline{N_f}$	1	0	1	N_c/N_f	N_c/N_f
$\overline{q}$	$\overline{N_f}$	1	N_f	0	1	$-N_c/N_f$	N_c/N_f
Φ	1	N_f	$\overline{N_f}$	0	1	0	$2(N_f-N_c)/N_f$
q'	N_f	1	1	1	N_c	$-(N_f-N_c)/N_f$	0
$\overline{q}'$	$\overline{N_f}$	1	1	-1	$\overline{N_c}$	$(N_f-N_c)/N_f$	0
Y	1	1	1	0	$1+\mathrm{Adj.}$	0	2
Z	1	1	$\overline{N_f}$	-1	$\overline{N_c}$	1	$(2N_f-N_c)/N_f$
$\overline{Z}$	1	N_f	1	1	N_c	-1	$(2N_f-N_c)/N_f$

Soft SUSY Breaking :

$$\mathcal{L}_{\rm soft} = -\tilde{m}_q^2(|q|^2+|\bar{q}|^2+|q'|^2+|\bar{q}'|^2) - \tilde{m}_M^2(|Y|^2+|Z|^2+|\bar{Z}|^2+|\Phi|^2) - \left(\frac{m_{\tilde{\lambda}}}{2}\tilde{\lambda}\tilde{\lambda} + \tilde{B}m\Lambda Y + Ah\left(q'Y\bar{q}' + q'Z\bar{q} + q\bar{Z}\bar{q}' + q\Phi\bar{q}\right) + \text{h.c.}\right).$$

- Dynamics of Dual Theory

Add the Soft-Masses for Squarks $(\tilde{m}_Q^2 > 0)$ (Arkani-Hamed-Rattazzi, ...)

➡ Soft-Mass for Dual Squarks Become Tachyonic, $\tilde{m}_q^2 < 0$

The Vacuum of Dual (Magnetic) Theory

$$q = \bar{q} = v\mathbf{1} \neq 0, \quad \Phi = v_\Phi \mathbf{1} \neq 0$$

Chiral Sym. Breaking & Color-Flavor Locking with Higgsing

$$SU(N_f)_L \times \boxed{SU(N_f)_g} \times SU(N_f)_R \longrightarrow SU(N_f)_V$$

★ Hidden Local Symmetry is Realized as Dual Gauge Symmetry

Note:

- All other Matter Fields (q', Y, Z) Become Massive and Decouple.
- Fermionic Partners of (q, Φ) Also Become Massive.
- Extra Gauged $U(1)_{B'}$ is Also Broken and Higgsed.

3. Confining Strings in the Dual Theory (arXiv: 1202.3260)

The Dual Theory at Low-Energy by $U(N_f)$ Gauge Theory with N_f Squarks

Low-Energy Eff. Action :

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F^{(\omega)}_{\mu\nu}F^{(\omega)\mu\nu} - \frac{1}{4}F^{(\rho)a}_{\mu\nu}F^{(\rho)\mu\nu a} \\ &\quad + \frac{f_\pi^2}{2}\mathrm{Tr}\left[|D_\mu H_L|^2 + |D_\mu H_R|^2\right] \\ &\quad - V(H_L, H_R).\end{aligned}$$

$$\begin{aligned}V(H_L, H_R) &= f_\pi^4\Bigg[\frac{\lambda_0 - \lambda_A}{8N_f}\left(\mathrm{Tr}(H_L H_L^\dagger) + \mathrm{Tr}(H_R^\dagger H_R) - 2N_f\right)^2 \\ &\quad + \frac{\lambda_A}{8}\left\{\mathrm{Tr}\left[(H_L^\dagger H_L + H_R H_R^\dagger)^2\right] - 4\left(\mathrm{Tr}(H_L H_L^\dagger) + \mathrm{Tr}(H_R^\dagger H_R)\right)\right\} \\ &\quad + \frac{\lambda' - \lambda''}{8N_f}\left(\mathrm{Tr}(H_L H_L^\dagger) - \mathrm{Tr}(H_R^\dagger H_R)\right)^2 \\ &\quad + \frac{\lambda''}{8}\mathrm{Tr}\left[(H_L^\dagger H_L - H_R H_R^\dagger)^2\right]\Bigg].\end{aligned}$$

- This Action is Nothing But A Linear-Realization of

ρ and ω Effective Theory with Hidden Local Symmetry.

$$H_L : (N_f, \bar{N}_f, 1), \;\; H_R : (\bar{N}_f, 1, N_f) \quad \text{under} \quad SU(N_f)^3$$

Working Hypothesis :

We Identify the Dual Theory and $\rho - \omega$ Effective Theory

Immediate Consequence:

★ VEV of the Higgs Fields $\langle H_L \rangle = \langle H_R \rangle = \mathbf{1}$

→ Chiral Sym. Breaking AND Dual Meissner Effect !

∴ Higgs Fields in Magnetic Dual Theory is "Monopole"s in Electric Theory.

$\langle \bar{m} m \rangle \neq 0$ → Confinement & Chiral Sym. Breaking

(Konishi-Marmorini-NY, ...)

Under This Hypothesis, Particle Spectrum and Parameters Can Be Identified :

singlet vector (ω): $m_\omega^2 = g_1^2 f_\pi^2$ ⟶ $\omega(782)$

adjoint vector (ρ): $m_\rho^2 = g_2^2 f_\pi^2$ ⟶ $\rho(770)$

singlet scalar (f_0): $m_S^2 = 2\lambda_0 f_\pi^2$ ⟶ $f_0(980)$

adjoint scalar (a_0): $m_A^2 = 2\lambda_A f_\pi^2$ ⟶ $a_0(980)$

From Vector Meson Data : $g \equiv g_1 = g_2$

$$g = \frac{m_\rho^2}{g_\rho} = 5.0, \qquad f_\pi = \frac{g_\rho}{m_\rho} = 150 \text{ MeV}$$

From Scalar Meson Data :

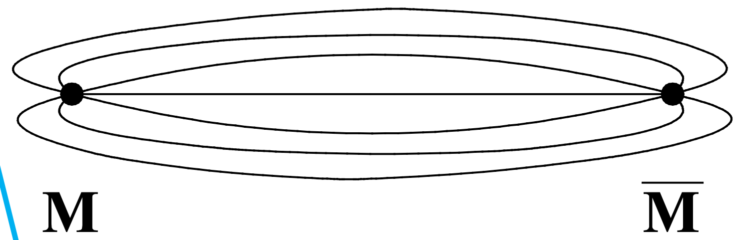
$$\sqrt{\lambda_0} = \sqrt{\lambda_A} = 4.6$$

- Confining Potential from Vortex Solution

Confining Flux Between Quarks in QCD

EM-Duality ↕

Vortex Soln. with Ends of Magnetic Monopoles



- Topological Properties of Vortex Solution

Breaking of Extra $U(1)_{B'}$ Gauge Sym. ➡ $\pi_1(U(1)) = \mathbf{Z}$

Existence of ω-Meson is Responsible for STABLE Confining String !

Confining String out of Mesons

Our Dual Theory is $U(N_f)$ Gauge Theory with N_f Fund. Higgs

➡ Non-Abelian Vortex for Minimum Flux (Hanany-Tong, Auzzi et. al.)

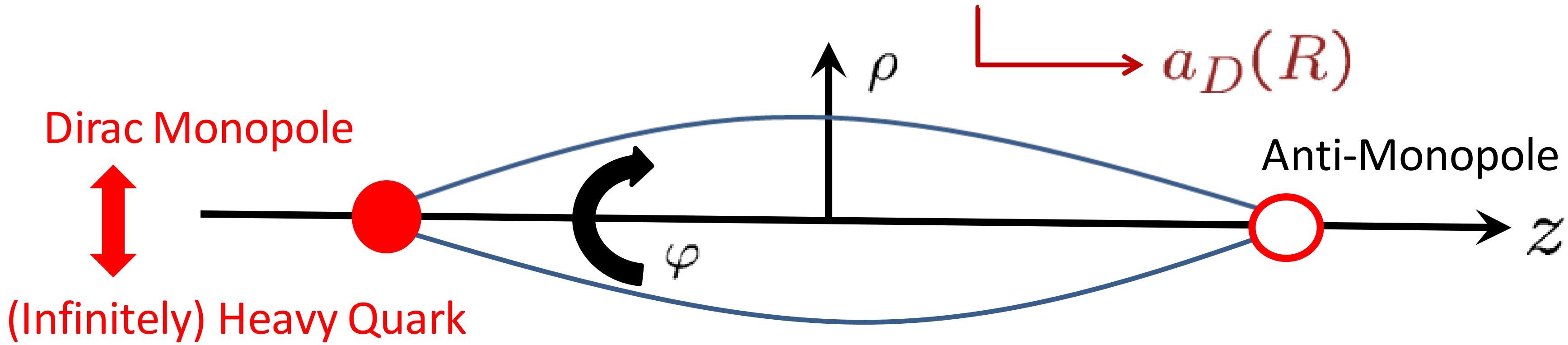
We Construct Non-Abelian Vortex with the Ends of Non-Abelian Monopoles.

● Numerical Construction of Vortex Solution

(Ball-Caticha, Phys. Rev. D37 ('88))

In Our Case, Partial Differential Eq. Should be Solved Numerically.

1. Ansatz for Gauge Field with Dirac-Type Monopoles at $z = \pm R/2$:



2. Vortex Eq. with $\sqrt{2} f_\pi = 1$ and $\lambda_0 = \lambda_A$:

$$\nabla^2 \phi_1 - \frac{g^2}{2}(a^1 + a_D)^2 \phi_1 = \frac{\lambda_0}{2}\left(\phi_1^2 - 1\right)\phi_1,$$

$$\left(\nabla^2 - \frac{1}{\rho^2}\right) a^1 = \frac{g^2}{2}(a^1 + a_D)\phi_1^2.$$

For $A_\mu^{11} = a^1(\rho, z) + a_D(R), \ \phi_{11} = \phi_1(\rho, z),$

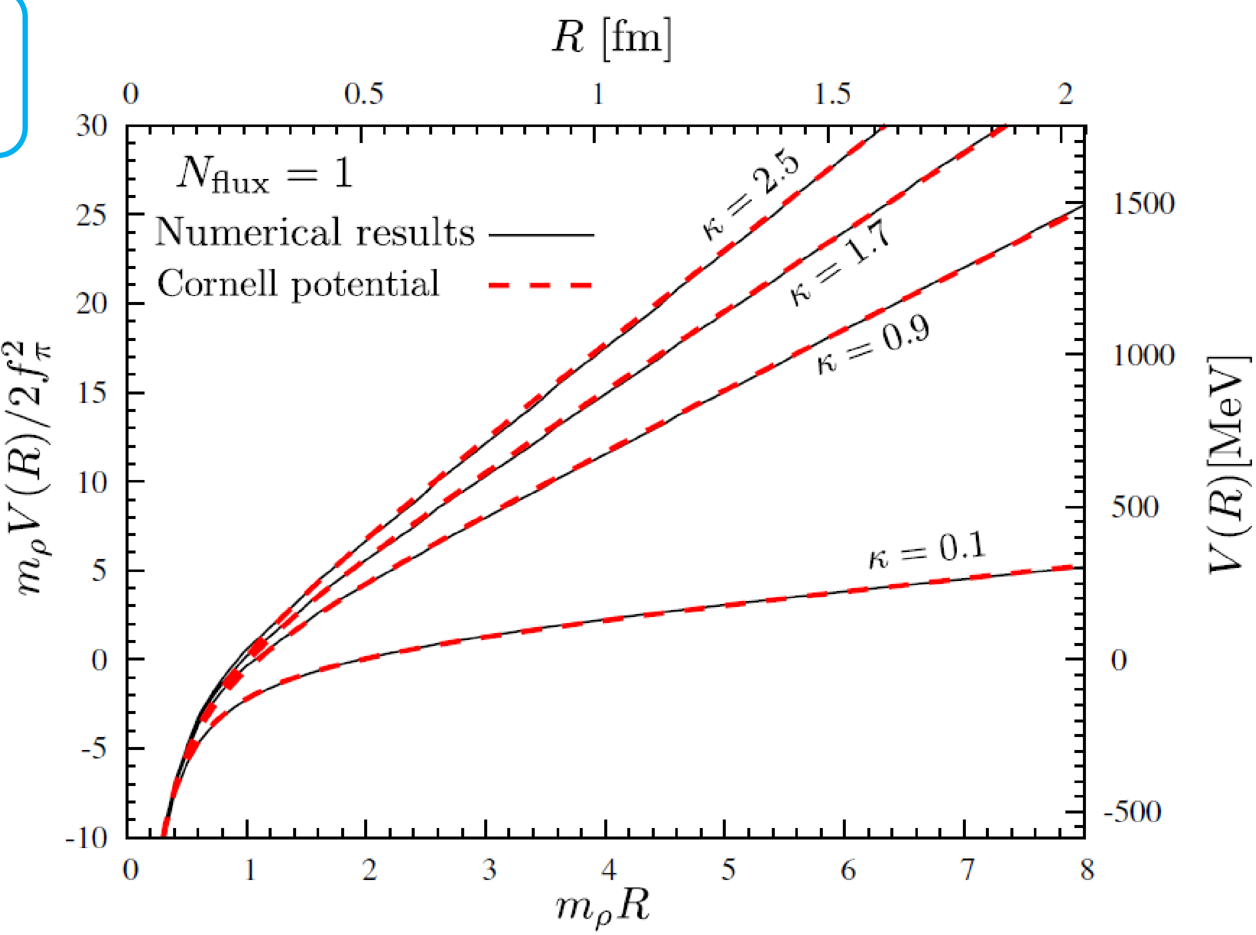
$A_\mu^{ij} \equiv 0, \ \phi_{ii} \equiv 1 \qquad (i, j \neq 1)$

- Numerical Results for $V(R)$:

GL Parameter $\kappa = \frac{m_S}{\sqrt{2}m_\rho} = \frac{\sqrt{\lambda_0}}{g}$

$$V(R) = -\frac{2\pi N_{\rm flux}^2}{g^2 R} + \int d^3x \left[-\frac{g^2}{4}\phi_1^2 (a^1 + a_D) a^1 - \frac{\lambda_0}{8}(\phi_1^4 - 1) \right].$$

Potential Formula is Indep. of N_f



★ Cornell Potential Fits Very Well

- Comparison with Lattice Results:

Cornell Potential

$$V(R) = -\frac{A}{R} + \sigma R$$

1. Coulomb Force Coefficient

From Lattice QCD, $A \sim 0.25 - 0.5$

This is Effect of One-Gluon Exchange $\sim$ $A = \frac{N_c^2 - 1}{2N_c} \frac{g_s^2}{4\pi}$

Our Calculation from Dirac Monopoles :

$$A = \frac{q_m^2}{4\pi} = \frac{2\pi}{g^2} \quad \longrightarrow \quad A \sim 0.25$$

Dirac Condition

Consistent !

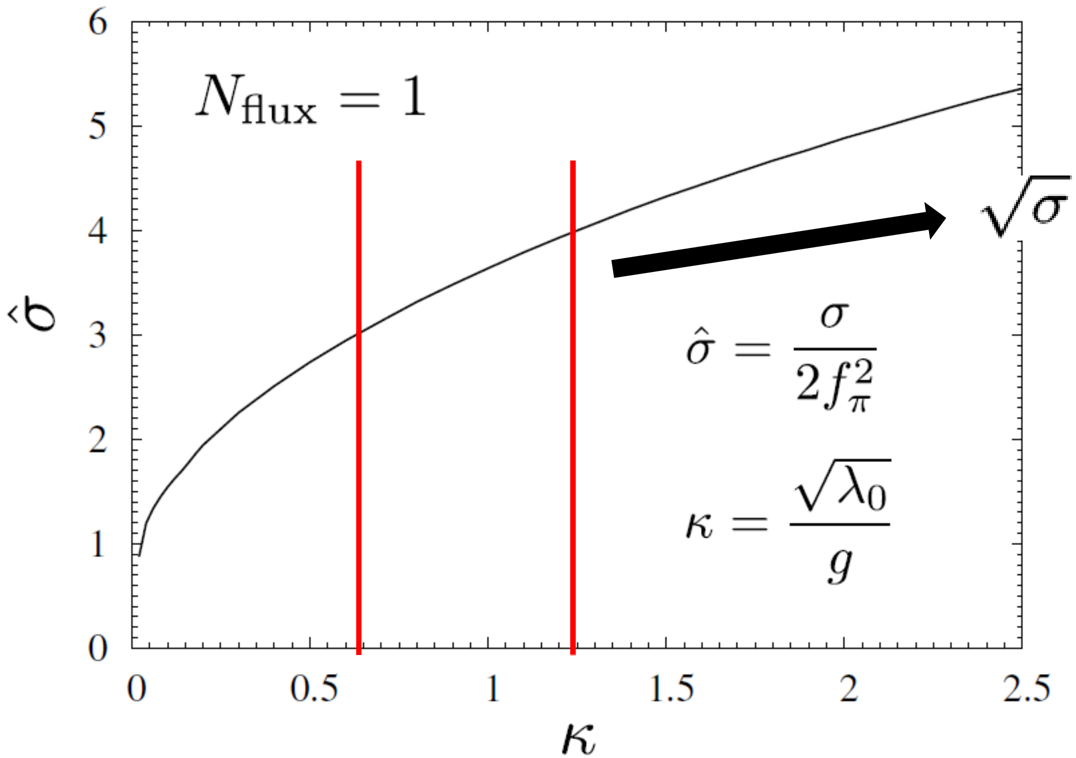
Tension of Confining String :

Note: Loop Corrections $\sim \dfrac{g^2 N_f}{16\pi^2}$ Always Exist.

From Quarkonium Spectra and Lattice QCD, $\sqrt{\sigma} \sim 430$ MeV.

Our Calculation from Vortex Soln.

→ $\sqrt{\sigma} = 400$ MeV. $(\kappa = 0.9)$



$\sqrt{\sigma} \sim 360 - 420$ MeV

$(f_\pi = 150$ MeV$)$

- Another Trial : QCD Phase Transition

Our Effective Theory is A Kind of Standard Higgs Model.
&
Our Dual Theory is Weakly-Coupled .

→ Analysis of Chiral Sym. Restoration in Finite Temp. is Possible.

(1-Loop) Finite Temp. Effect for Mass-Squared Can be Evaluated. (Dolan-Jackiw)

Temp. Corrected Mass-Squared is Vanishing at Transition Temp.

$$T_c = \sqrt{\frac{8}{\eta N_f}} f_\pi, \qquad \eta = 1 + \frac{m_\rho^2}{m_S^2} + \frac{2m_{PS}^2 + m_S^2}{3m_S^2}$$

Numerically,

$$T_c = \begin{cases} 170 \text{ MeV} \times \left(\frac{\eta}{3}\right)^{-1/2}, & (N_f = 2) \\ 140 \text{ MeV} \times \left(\frac{\eta}{3}\right)^{-1/2}, & (N_f = 3) \end{cases}$$

- These Numerical Values are (Roughly) Consistent with Lattice QCD Results.

4. Summary and Discussions

- We Have Discussed A Possible (Seiberg) Dual Description for Real QCD. Our Dual Theory is Closely Related to $\rho - \omega$ Meson Effective Theory with Hidden Local Symmetry.

- Using the Identification between Our Dual Theory and Meson Eff. Theory, Inter-Quark Potential (incl. String Tension) is Calculated from the Vortex Soln. in the Dual Theory.

- Trans. Temp. of Chiral Sym. Restoration is Also Estimated in the Dual Theory.

So Far, Our Numerical Results are Consistent with Lattice QCD and Hadron Data !

● Discussions

1. Confining String in Pure Yang-Mills **?**

2. Is Seiberg-Duality Really Electric-Magnetic Duality **?**

   ➡ Analysis Embedded into *N*=2 SUSY QCD or *N*=4 SYM

3. Relation to Dynamics of NA-Vortex and NA-Monopole

   - Kink on the Vortex as Monopole ➡ Light Quark ?
   - Luscher Term on the Vortex as Coulomb Coefficient ?

4. EW-Higgs as Dual Squark **?**