Lepton flavor structure from point interactions in an extra dimension

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The origin of generations, the fermion’s mass hierarchy, and the flavor structure are key ingredients of the standard model. In the context of five-dimensional gauge theories on a circle with point interactions, we can produce the three generations, the mass hierarchy, the flavor structure and the CP phase from a single generation fermions with a gauge singlet scalar field. Tiny neutrino masses and the large mixing structure of the leptons are connected with each other.\(^1\)

We consider a five-dimensional (5d) gauge theory on a circle with point interactions. The action for a fermion is given by \(^2\)

\[
S = \int d^4x \int_0^L dy \, \Psi(x, y)(i\Gamma^M \partial_M + M_F)\Psi(x, y).
\]  

\(1\)

By introducing three point interactions, which are additions boundary points, we can impose boundary conditions for the fermion.

\[
\Psi_L(x, y) = 0 \quad \text{at positions of the point interactions} \tag{2}
\]

or

\[
\Psi_R(x, y) = 0 \quad \text{at positions of the point interactions} \tag{3}
\]

Under the boundary conditions, we can obtain three chiral zero modes,

\[
\Psi(x, y) = \sum_{i=1}^3 \psi_R^{(i)}(x) f_0^{(i)}(y) + (\text{KK-particles}), \quad \text{for } \Psi_L = 0, \tag{4}
\]

\[
\Psi(x, y) = \sum_{i=1}^3 \psi_L^{(i)}(x) g_0^{(i)}(y) + (\text{KK-particles}), \quad \text{for } \Psi_R = 0. \tag{5}
\]

The schematic figure of the mode functions \(\{f_0^{(i)}(y)\}\) and \(\{g_0^{(i)}(y)\}\) are depicted in Fig.1. Mode functions live in the different segment of the extra dimension.

\(1\)The collaborators of this work are Kenji Nisiwaki (Harish-Chandra Inst.), Makoto Sakamoto (Kobe Univ.) and Ryo Takahashi (Shimane Univ.).

\(2\)We adopt \(\eta_{MN} = \text{diag}(-1, 1, 1, 1)\), \(\Gamma^y = -i\gamma_5\) and \(\{\Gamma^M, \Gamma^N\} = -2\eta^{MN}1_4\).
In general, we can change the position of point interactions with respect to fermions, and then off diagonal components appears in a mass matrix. The mass matrix elements including off diagonal components can be expressed as

$$m_{ij} = \lambda^{(5)} \int_0^L dy \langle H(y) \rangle \langle \Phi(y) \rangle \mathcal{F}_0^{(i)}(y) f^{(j)}_0(y),$$

where $\langle H(y) \rangle$, $\langle \Phi(y) \rangle$ are the vacuum expectation value (VEV) of the Higgs doublet and the gauge singlet, and $\mathcal{F}_0^{(i)}(y)$ and $f^{(i)}_0(y)$ are mode functions of SU(2) doublet fermion and singlet fermion, respectively. It comes from the geometry of the extra dimension that the large value of the bulk mass $M_F$ makes diagonal elements small and off diagonal elements large. This implies that tiny neutrino masses (large quark masses) may be realized with a large (small) flavor mixing.

We should note that in this set up, the system consists of single-generation fermions so that we need a new source of a CP phase. It was also found that under the twisted boundary conditions, the Higgs VEV possesses an extra-dimension coordinate-dependent phase,

$$\langle H(y) \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} e^{i \frac{\theta}{L} y},$$

which provide us a CP phase[1]. The Robin boundary condition for the gauge singlet

$$\begin{cases} 
\Phi(0) + L_+ \partial_y \Phi(0) = 0, \\
\Phi(L) - L_- \partial_y \Phi(L) = 0,
\end{cases} \quad (-\infty \leq L_+ \leq +\infty),$$

will lead a extra-dimensional coordinate-dependent VEV[2, 3],

$$\langle \Phi(y) \rangle \sim e^{\alpha(y-L)},$$

which may provide a large mass hierarchy to us through the overlap integral eq.(6).
References

