Exact Result for boundaries (and domain walls) in 2d supersymmetric theory

Daigo Honda（本田）

University of Tokyo, Komaba

Based on the paper arXiv:1308.2217

Collaboration with Takuya Okuda

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3 papers about the supersymmetric localization with boundary appeared from Japan!!!

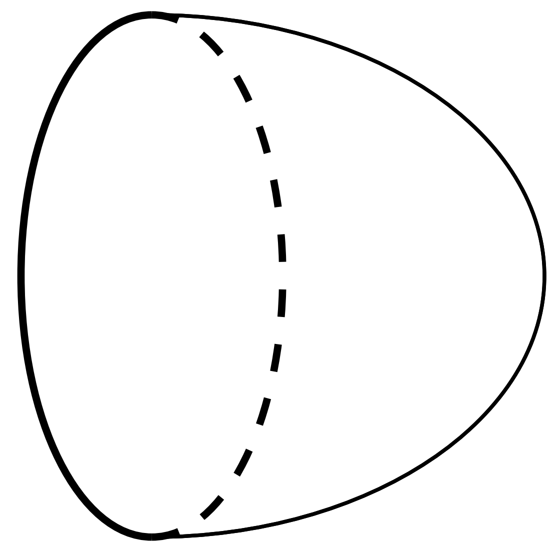
Sugishita-Terashima 1308.1973

Honda-Okuda 1308.2217

Hori-Romo 1308.2438

The three groups coordinated the submission to the arXiv.

2d N=(2,2) GLSMs on hemispheres



GLSM

Boundaries

Low energy

NLSM or LG model

D-branes

Target space = vacuum manifold of GLSM

What we did

- Construction of N=(2,2) GLSMs on hemispheres
- Supersymmetric localization
- Derivation of hemisphere partition functions and their various properties
- Domain walls (non-dynamical)

Motivations

- String theoretic (boundaries)
  - D-branes in Calabi-Yau manifold
  - Mirror symmetry

- Gauge theoretic (domain walls)
  - Line + Surface operators in 4d theory
  - Integrable structure

Plan of this talk

- Construction of N=(2,2) GLSMs on hemispheres
- Hemisphere partition functions and their properties

N=(2,2) GLSMs on hemispheres

Bulk data of 2d N=(2,2) GLSM

Gauge group G

Vector multiplet $(A_\mu, \sigma_1, \sigma_2, \lambda, \bar{\lambda}, \mathrm{D})$

a-th chiral multiplet $(\phi_a, \psi_a, \mathrm{F}_a)$ in irreducible rep. R_a

Complexified FI parameter $t = r - i\theta$

FI parameter / theta angle

(Defined for each abelian factor)

Superpotential W : holomorphic function of $\phi = (\phi_a)$

Complexified twisted masses $m = (m_a)$

R-charge and real twisted masses for flavor symmetry

Supersymmetry

We choose the supercharge constructed by Gomis-Lee (2013).
Deformation of the sphere does not change the partition functions.

semi-infinite cylinder with a cap at infinity

B-type supersymmetry R-symmetry flux A-type twist

Periodic around circle → Ramond-Ramond sector
Long propagation through cylinder → Zero energy state

boundary state $\langle \mathcal{B} | 1 \rangle$ state without any insertion

Setting of Cecotti-Vafa (1991)

Boundary conditions

For vector multiplet:
Gauge symmetry preserving condition

$$\sigma_1 = D_1\sigma_2 = A_1 = F_{12} = \bar{\epsilon}\lambda = \epsilon\bar{\lambda} = \cdots = 0$$

(Gauge symmetry broken condition?)

For chiral multiplets:
Neumann condition

$$D_1\phi = D_1\bar{\phi} = \bar{\epsilon}\gamma_3\psi = \epsilon\gamma_3\bar{\psi} = \cdots = 0$$

Dirichlet condition

$$\phi = \bar{\phi} = \bar{\epsilon}\psi = \epsilon\bar{\psi} = \cdots = 0$$

These conditions determine the submanifolds on which D-branes are wrapped.

Boundary interactions

$\mathbb{Z}_2$-graded Chan-Paton space $\quad \mathcal{V} = \mathcal{V}^e \oplus \mathcal{V}^o$

brane / anti-brane

Inclusion of the Wilson loop at the boundary

$$\mathrm{Str}_{\mathcal{V}} \left[\mathrm{P} \exp \left(i \oint d\varphi \mathcal{A}_\varphi \right) \right]$$

$$\mathcal{A}_{\hat{\varphi}} = \rho_*(A_{\hat{\varphi}} + i\sigma_2) + \rho_*(m) - \frac{i}{2}\{\mathcal{Q}, \bar{\mathcal{Q}}\} + \ldots$$

Tachyon profile $\mathcal{Q}(\phi),\ \bar{\mathcal{Q}}(\bar{\phi})$ odd operators on $\mathcal{V}$

Matrix factorization $\quad \mathcal{Q}^2 = W \cdot \mathbf{1}_{\mathcal{V}},\ \bar{\mathcal{Q}}^2 = \bar{W} \cdot \mathbf{1}_{\mathcal{V}}$

→ supersymmetry preserved

Low energy behavior is not changed by

(1) boundary D-term deformation (deformation of fibre metric)
(2) brane anti-brane annihilation

Brane / anti-brane bound state

Tachyon condensation

D-brane wrapped on the zero locus of $U = \{\mathcal{Q}, \bar{\mathcal{Q}}\}$

IR equivalence of UV descriptions
= Quasi-isomorphism in the derived category of the coherent sheaves

Herbst-Hori-Page (0803.2045)

Any B-brane is obtained as (quasi-isomorphic to) the bound state (complex) of space filling branes.

Hemisphere partition functions and their properties

Hemisphere partition functions

$$Z_{\mathrm{hem}}(\mathcal{B};t;m) = \frac{1}{|W(G)|}\int_{\sigma\in i\mathfrak{t}} \frac{d^{\mathrm{rk}(G)}\sigma}{(2\pi i)^{\mathrm{rk}(G)}}$$

$$\times \mathrm{Str}_{\mathcal{V}}[e^{-2\pi i(\sigma+m)}]e^{t\cdot\sigma} Z_{1\text{-loop}}(\mathcal{B};\sigma;m)$$

Boundary interaction

Classical action

Chiral multiplets (Neumann)

Vector multiplet

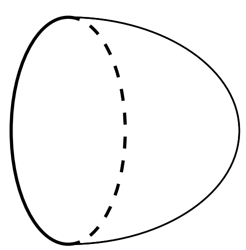
$$Z_{1\text{-loop}}(\mathcal{B};\sigma;m) = \prod_{\alpha>0}\alpha\cdot\sigma\frac{\sin(\pi\alpha\cdot\sigma)}{-\pi}\prod_{a\in\mathbf{Neu}}\prod_{w\in R_a}\Gamma(w\cdot\sigma+m_a)$$

Chiral multiplets (Dirichlet)

$$\times\prod_{a\in\mathbf{Dir}}\prod_{w\in R_a}\frac{-2\pi i e^{\pi i(w\cdot\sigma+m_a)}}{\Gamma(1-w\cdot\sigma-m_a)}$$

Hemisphere partition function = B-brane central charge

D-brane central charge = central charge of the SUSY algebra for non-compact dimensions in Calabi-Yau compactification

$\langle \mathcal{B} |$  $| 1 \rangle$ = central charge of the D-brane Ooguri-Oz-Yin (1996)

Comparison with the large volume formula obtained by anomaly inflow argument Minasian-Moore (1997) Aspinwall (hep-th/0403166)

$$Z_{\rm hem}(\mathcal{B}, t, m=0) \simeq \int_M {\rm ch}(\mathcal{E}) e^{B+i\omega} \sqrt{\hat{A}(TM)} \quad {\rm Re}\; t \to \infty$$

up to overall factor, higher derivative corrections and (worldsheet) instanton corrections.

Example: Quintic Calabi-Yau

A hypersurface in $\mathbb{P}^4$ determined by a degree 5 polynomial

$$Z_{\text{hem}}[\mathcal{O}_M(n)] = \int_{i\mathbb{R}} \frac{d\sigma}{2\pi i} e^{-2\pi i n \sigma} (e^{-5\pi i \sigma} - e^{5\pi i \sigma}) e^{t\sigma} \Gamma(\sigma)^5 \Gamma(1-5\sigma)$$

$$= -\frac{20}{3}\pi^4 \underline{\left(\frac{t}{2\pi i} - n\right)\left(2\left(\frac{t}{2\pi i} - n\right)^2 + 5\right)} - 400\pi i \zeta(3) + \mathcal{O}(e^{-t})$$

higher derivative corrections

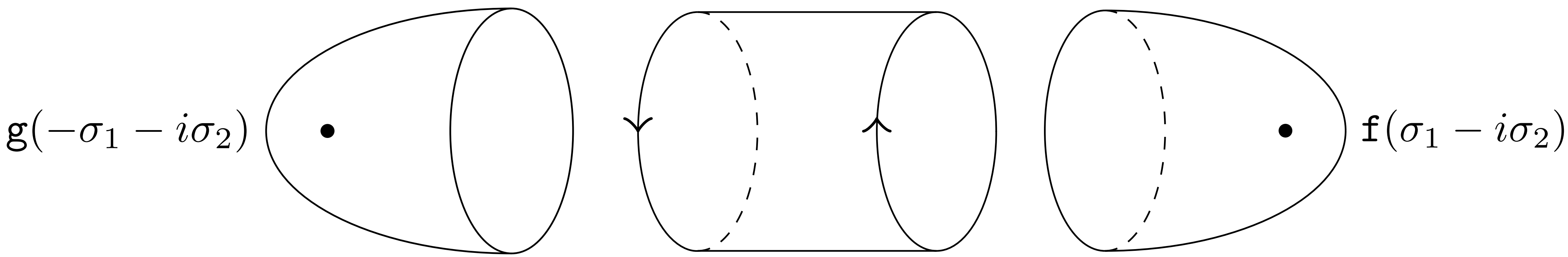
instanton corrections

$$\int_M \mathrm{ch}(\mathcal{O}_M(n)) e^{B+i\omega} \sqrt{\hat{A}(TM)} = -\frac{5}{12} \underline{\left(\frac{t}{2\pi i} - n\right)\left(2\left(\frac{t}{2\pi i} - n\right)^2 + 5\right)} + \mathcal{O}(e^{e^{-t}})$$

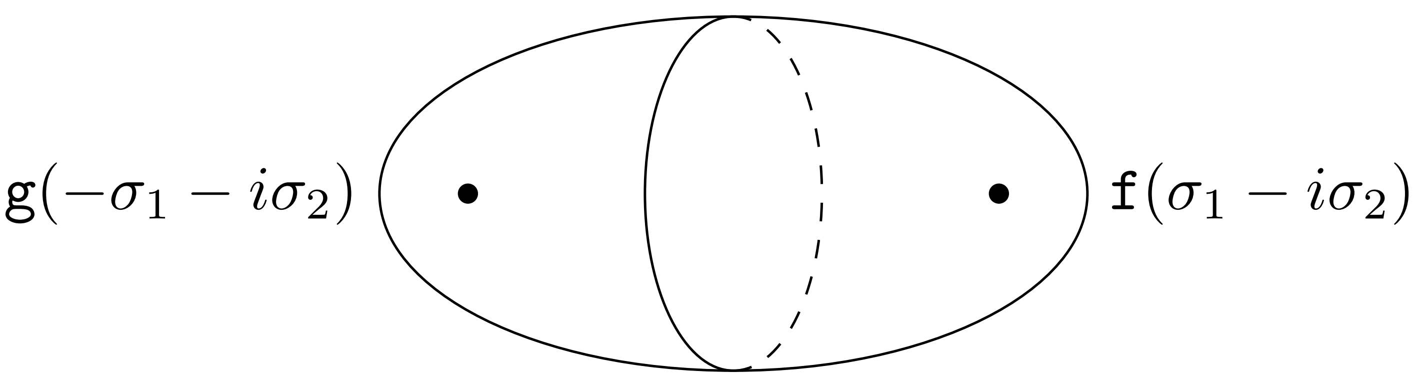
Identification of Kähler parameter in large volume limit

$$B + i\omega = \frac{it}{2\pi} + \mathcal{O}(e^{-t})$$

Hilbert space interpretation



$$\langle \mathtt{g} | \mathcal{B}_a \rangle \qquad \chi_{ab} = \langle \mathcal{B}_a | \mathcal{B}_b \rangle \qquad \langle \mathcal{B}_b | \mathtt{f} \rangle$$



$$\langle \mathtt{g} | \mathtt{f} \rangle = \langle \mathtt{g} | \mathcal{B}_a \rangle \, \chi^{ab} \, \langle \mathcal{B}_b | \mathtt{f} \rangle$$

Comparison with the large volume formula

→ Fixing overall factor of the hemisphere partition function

Cylinder partition function = index → No ambiguity

We can fix the ambiguity of the sphere partition function!

Seiberg-like dualities

$U(N)$ gauge group $\quad N_{\rm F}$ fundamentals

Duality map: $(N, N_{\rm F}, t, m) \to (N_{\rm F} - N, N_{\rm F}, t, -m)$

$$\mathrm{Gr}(N, N_{\rm F}) \simeq \mathrm{Gr}(N_{\rm F} - N, N_{\rm F})$$

$$Z_{\rm hem}[\mathrm{Gr}(N, N_{\rm F}); \mathcal{B}; t; m] = Z_{\rm hem}[\mathrm{Gr}(N_{\rm F} - N, N_{\rm F}); \mathcal{B}^{\vee}; t; -m]$$

Note that $\sum_f m_f = 0$

appropriate "dual" brane
wrapped on the same submanifold

$N_{\rm F}$ fundamental / anti-fundamental matters, 1 adjoint matter

$$T^*\mathrm{Gr}(N, N_{\rm F}) \simeq T^*\mathrm{Gr}(N_{\rm F} - N, N_{\rm F})$$

Nontrivial duality relation

cf: Kapustin-Willett-Yaakov (1012.4021)
Kim-Kim-Kim-Lee (1204.3895)
Ito-Maruyoshi-Okuda (2013)

Conclusion

- Constructed 2d N=(2,2) GLSMs on hemispheres with general B-type boundary conditions and boundary interactions.
- Determined properties of hemisphere partition functions.
    - D-brane central charge
    - Hilbert space interpretation
    - Seiberg-like Dualities

Stong tests of our results!

Comments on domain walls

- Domain walls are boundaries in folded theories.
- Line operators on surface operators and AGT correspondence (open Verlinde operators)
- Affine Hecke algebra from monodromy domain wall algebra (Integrability suggests the presence of quantum group symmetry.)
- Relations between domain walls and geometric representation theory