Exact Result for boundaries (and domain walls) in 2d supersymmetric theory

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Based on the paper arXiv:1308.2217
Collaboration with Takuya Okuda

@YITP workshop on “Field Theory and String Theory” 2013
3 papers about the supersymmetric localization with boundary appeared from Japan!!

Sugishita-Terashima 1308.1973
Honda-Okuda 1308.2217
Hori-Romo 1308.2438

The three groups coordinated the submission to the arXiv.
2d N=(2,2) GLSMs on hemispheres

GLSM

Boundaries

Low energy

NLSM or LG model

D-branes

Target space = vacuum manifold of GLSM
What we did

• Construction of N=(2,2) GLSMs on hemispheres
• Supersymmetric localization
• Derivation of hemisphere partition functions and their various properties
• Domain walls (non-dynamical)
Motivations

• String theoretic (boundaries)
  • D-branes in Calabi-Yau manifold
  • Mirror symmetry

• Gauge theoretic (domain walls)
  • Line + Surface operators in 4d theory
  • Integrable structure
Plan of this talk

• Construction of $\mathcal{N}=(2,2)$ GLSMs on hemispheres

• Hemisphere partition functions and their properties
N=(2,2) GLSMs on hemispheres
Bulk data of 2d $\mathbf{N}=(2,2)$ GLSM

Gauge group $G$

Vector multiplet $(A_\mu, \sigma_1, \sigma_2, \lambda, \bar{\lambda}, D)$

$a$-th chiral multiplet $(\phi_a, \psi_a, F_a)$ in irreducible rep. $R_a$

Complexified FI parameter $t = r - i\theta$

(Defined for each abelian factor)

Fl parameter / theta angle

Superpotential $W: \text{holomorphic function of } \phi = (\phi_a)$

Complexified twisted masses $m = (m_a)$

R-charge and real twisted masses for flavor symmetry
Supersymmetry

We choose the supercharge constructed by Gomis-Lee (2013). Deformation of the sphere does not change the partition functions.

semi-infinite cylinder with a cap at infinity

B-type supersymmetry

R-symmetry flux

A-type twist

Periodic around circle $\rightarrow$ Ramond-Ramond sector
Long propagation through cylinder $\rightarrow$ Zero energy state

boundary state $\langle B|1 \rangle$ state without any insertion

Setting of Cecotti-Vafa (1991)
Boundary conditions

For vector multiplet:
Gauge symmetry preserving condition
\[ \sigma_1 = D_1 \sigma_2 = A_1 = F_{12} = \bar{\epsilon} \lambda = \epsilon \bar{\lambda} = \cdots = 0 \]

(Gauge symmetry broken condition? )

For chiral multiplets:
Neumann condition
\[ D_1 \phi = D_1 \bar{\phi} = \bar{\epsilon} \gamma_3 \psi = \epsilon \gamma_3 \bar{\psi} = \cdots = 0 \]

Dirichlet condition
\[ \phi = \bar{\phi} = \bar{\epsilon} \psi = \epsilon \bar{\psi} = \cdots = 0 \]

These conditions determine the submanifolds on which D-branes are wrapped.
Boundary interactions

\[ \mathbb{Z}_2 \text{-graded Chan-Paton space} \quad \mathcal{V} = \mathcal{V}^e \oplus \mathcal{V}^o \]

brane / anti-brane

Inclusion of the Wilson loop at the boundary

\[
\text{Str}_\mathcal{V} \left[ P \exp \left( i \int d\varphi A_\varphi \right) \right]
\]

\[
A_\hat{\varphi} = \rho_\ast (A_\hat{\varphi} + i\sigma_2) + \rho_\ast (m) - \frac{i}{2} \{Q, \bar{Q}\} + \ldots
\]

Tachyon profile \( Q(\phi), \bar{Q}(\bar{\phi}) \) odd operators on \( \mathcal{V} \)

Matrix factorization \( Q^2 = W \cdot 1_\mathcal{V}, \bar{Q}^2 = \bar{W} \cdot 1_\mathcal{V} \)

\( \rightarrow \) supersymmetry preserved
Low energy behavior is not changed by

1. boundary D-term deformation (deformation of fibre metric)
2. brane anti-brane annihilation

Brane / anti-brane bound state

\[ D \text{-brane wrapped on the zero locus of } U = \{ Q, \bar{Q} \} \]

IR equivalence of UV descriptions
= Quasi-isomorphism in the derived category of the coherent sheaves

Herbst-Hori-Page (0803.2045)

Any B-brane is obtained as (quasi-isomorphic to) the bound state (complex) of space filling branes.
Hemisphere partition functions and their properties
Hemisphere partition functions

\[ Z_{\text{hem}}(\mathcal{B}; t; m) = \frac{1}{|W(G)|} \int_{\sigma \in \text{it}} d^{\text{rk}(G)} \sigma \]
\[ \times \text{Str}_\mathcal{Y} \left[ e^{-2\pi i (\sigma + m)} \right] e^t \cdot \sigma Z_{\text{1-loop}}(\mathcal{B}; \sigma; m) \]

\[ Z_{\text{1-loop}}(\mathcal{B}; \sigma; m) = \prod_{\alpha > 0} \alpha \cdot \sigma \frac{\sin(\pi \alpha \cdot \sigma)}{-\pi} \prod_{a \in \text{Neu}} \prod_{w \in R_a} \Gamma(w \cdot \sigma + m_a) \]

Boundary interaction

Classical action

Vector multiplet

Chiral multiplets (Neumann)

Chiral multiplets (Dirichlet)

\[ \prod_{a \in \text{Dir}} \prod_{w \in R_a} \frac{-2\pi i e^{\pi i (w \cdot \sigma + m_a)}}{\Gamma(1 - w \cdot \sigma - m_a)} \]
Hemisphere partition function
= B-brane central charge

D-brane central charge = central charge of the SUSY algebra for non-compact dimensions in Calabi-Yau compactification

\[ \langle \mathcal{B} \mid \Phi \rangle \mid 1 \rangle = \text{central charge of the D-brane} \quad \text{Ooguri-Oz-Yin (1996)} \]

Comparison with the large volume formula obtained by anomaly inflow argument \( \text{Minasian-Moore (1997) Aspinwall (hep-th/0403166)} \)

\[ Z_{\text{hem}}(\mathcal{B}, t, m = 0) \simeq \int_M \text{ch}(\mathcal{E}) e^{B+i\omega} \sqrt{\hat{A}(TM)} \quad \text{Re } t \to \infty \]

up to overall factor, higher derivative corrections and (worldsheet) instanton corrections.
Example: Quintic Calabi-Yau

A hypersurface in $\mathbb{P}^4$ determined by a degree 5 polynomial

$$Z_{\text{hem}}[O_M(n)] = \int_{i\mathbb{R}} \frac{d\sigma}{2\pi i} e^{-2\pi i n \sigma} (e^{-5\pi i \sigma} - e^{5\pi i \sigma}) e^{t \sigma} \Gamma(\sigma)^5 \Gamma(1 - 5\sigma)$$

$$= -\frac{20}{3} \pi^4 \left( \frac{t}{2\pi i} - n \right) \left( 2 \left( \frac{t}{2\pi i} - n \right)^2 + 5 \right) - 400\pi i \zeta(3) + \mathcal{O}(e^{-t})$$

higher derivative corrections

instanton corrections

$$\int_M \text{ch}(O_M(n)) e^{B+i\omega} \sqrt{\hat{A}(TM)} = -\frac{5}{12} \left( \frac{t}{2\pi i} - n \right) \left( 2 \left( \frac{t}{2\pi i} - n \right)^2 + 5 \right) + \mathcal{O}(e^{-t})$$

Identification of Kähler parameter in large volume limit

$$B + i\omega = \frac{it}{2\pi} + \mathcal{O}(e^{-t})$$
Hilbert space interpretation

\[\langle g \mid \mathcal{B}_a \rangle \quad \chi_{ab} = \langle \mathcal{B}_a \mid \mathcal{B}_b \rangle \quad \langle \mathcal{B}_b \mid f \rangle\]

\[\langle g \mid f \rangle = \langle g \mid \mathcal{B}_a \rangle \chi^{ab} \langle \mathcal{B}_b \mid f \rangle\]

Comparison with the large volume formula

→ Fixing overall factor of the hemisphere partition function

Cylinder partition function = index → No ambiguity

We can fix the ambiguity of the sphere partition function!
Seiberg-like dualities

\[ U(N) \] gauge group \[ N_F \] fundamentals

Duality map: \( (N, N_F, t, m) \rightarrow (N_F - N, N_F, t, -m) \)

\[ \text{Gr}(N, N_F) \simeq \text{Gr}(N_F - N, N_F) \]

\[ Z_{\text{hem}}[\text{Gr}(N, N_F); \mathcal{B}; t; m] = Z_{\text{hem}}[\text{Gr}(N_F - N, N_F); \mathcal{B}^\vee; t; -m] \]

Note that \( \sum_f m_f = 0 \)

appropriate “dual” brane wrapped on the same submanifold

\( N_F \) fundamental / anti-fundamental matters, 1 adjoint matter

\[ T^* \text{Gr}(N, N_F) \simeq T^* \text{Gr}(N_F - N, N_F) \]

Nontrivial duality relation

cf: Kapustin-Willett-Yaakov (1012.4021)
    Kim-Kim-Kim-Lee (1204.3895)
    Ito-Maruyoshi-Okuda (2013)
Conclusion

• Constructed 2d N=(2,2) GLSMs on hemispheres with general B-type boundary conditions and boundary interactions.

• Determined properties of hemisphere partition functions.
  • D-brane central charge
  • Hilbert space interpretation
  • Seiberg-like Dualities

Strong tests of our results!
Comments on domain walls

- Domain walls are boundaries in folded theories.
- Line operators on surface operators and AGT correspondence (open Verlinde operators)
- Affine Hecke algebra from monodromy domain wall algebra (Integrability suggests the presence of quantum group symmetry.)
- Relations between domain walls and geometric representation theory