Cosmological constant problem and lower-dimensional field theory, Liouville theory on de Sitter space

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Content

This work is in progress.

Brief summary

Brief summary in advance :

1) Quantum corrections for cosmo const Λ

- We have computed loop corrections to cosmo const Λ in a few models in low D dS space. $[\phi^4]_{2D}, [\phi^6]_{3D}, [e^{\phi}]_{2D}$
  Potential terms pick up time dependence.
- We have constructed a good 2D toy model (Liouville + matter) for 4D quantum gravity + matter models.

2) Solvability in dS space

We wish to extend Liouville theory and other (S-G) solvable theories from flat space to dS space.
Question : Can we construct "conservation" laws in dS space?

Part I Quantum effects on cosmological constant

1 Introduction

- What is cosmological constant (cosmo const) Λ?

  Positive Λ is responsible for inflation (accelerating expansion). $\Lambda = H^2$
  Matter / dark matter decelerate expansion.
  Vacuum energy = Cosmo const = Dark energy

A long standing problem —— cosmological constant problem

$$\Lambda_{\rm th} \cong (M_{\rm pl})^4 \simeq (10^{19}{\rm GeV})^4$$

$$\updownarrow$$

$$\Lambda_{\rm exp} = (10^{-3}{\rm eV})^4$$

There is a gap of 120figures!

Quantum effects C may be additive or multiplicative.

$$\Lambda_{\rm exp} = \Lambda_{\rm th} - C$$

$$\Lambda_{\rm exp} = \Lambda_{\rm th} \times C$$

The problem may be explained by quantum effects C.

Purpose: Explain the tiny observed value Λ by calculating Infrared screening effects.

2 Cosmol const Λ and de Sitter space

- Freedman equation derived from Einstein equation

$$R_{\mu\nu} + \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

equation for metric $g_{\mu\nu}$.

curvature of space — matter

$R \equiv g^{\alpha\beta} R^{\mu}{}_{\alpha\mu\beta}$ scalar curvature

$\kappa = 8\pi G$ Newton's const G

Λ cosmo const

$T_{\mu\nu}$ energy momentum tensor

$$R_{\mu\nu} + \frac{1}{2} g_{\mu\nu} R + \left(\Lambda - \frac{\kappa}{4} T_{\rho}{}^{\rho}\right) g_{\mu\nu} = 0$$

Λ_{eff} effective cosmo const (observed value)

Large $T_{\rho}{}^{\rho}$ may cancel large Λ explaining tiny cosmo const.

We wish to calculate quantum effects on $T_{\mu\nu}$.

Tsamis et al. 97 Polyakov 82,08
Kitamoto et al. 10 Urakawa et al. 09,10

2 Cosmol const Λ and de Sitter space

- Positive Λ means de Sitter space $\Lambda = H^2\,(4D)$

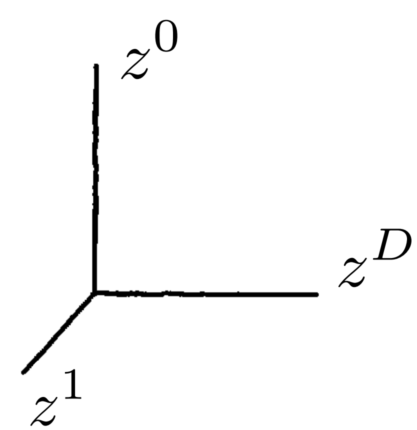
D dim dS space is embedded
in D+1 Minkowski space z^μ.

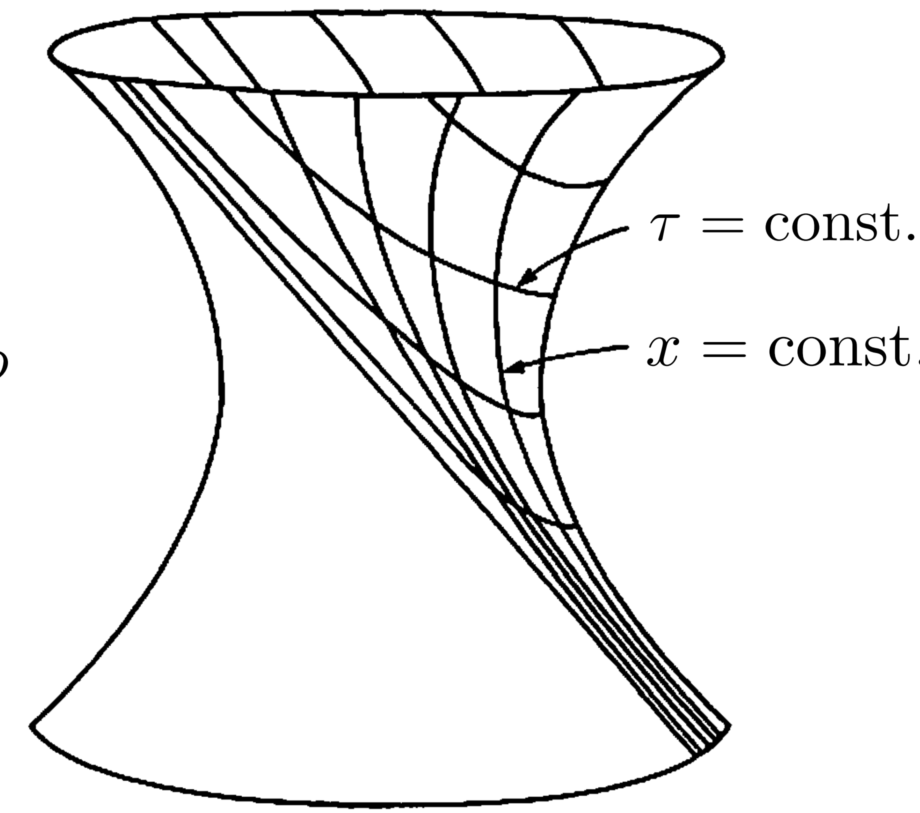
$$(t, \vec{x}) = (z^0, z^i) \quad i = 1, 2, \cdots, D$$

$$(z^0)^2 - (z^1)^2 - (z^2)^2 - \cdots - (z^D)^2 = -\frac{1}{H^2}$$

↓

D dim dS space

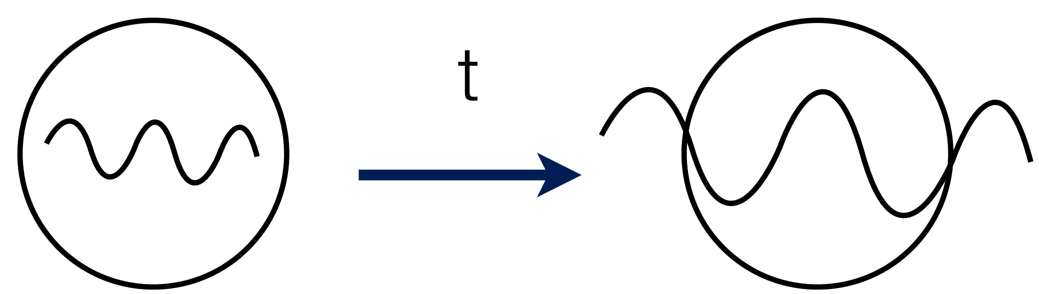




Flat chart (Poincaré coordinate) is convenient to discuss $T_{\mu\nu}$.

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 = a^2(\tau)(-d\tau^2 + d\vec{x}^2) = a^2(\tau) \times (\mathrm{flatgeometry})$$

In dS space distance between two points grows as time passes.



3 Infrared (IR) screening effects

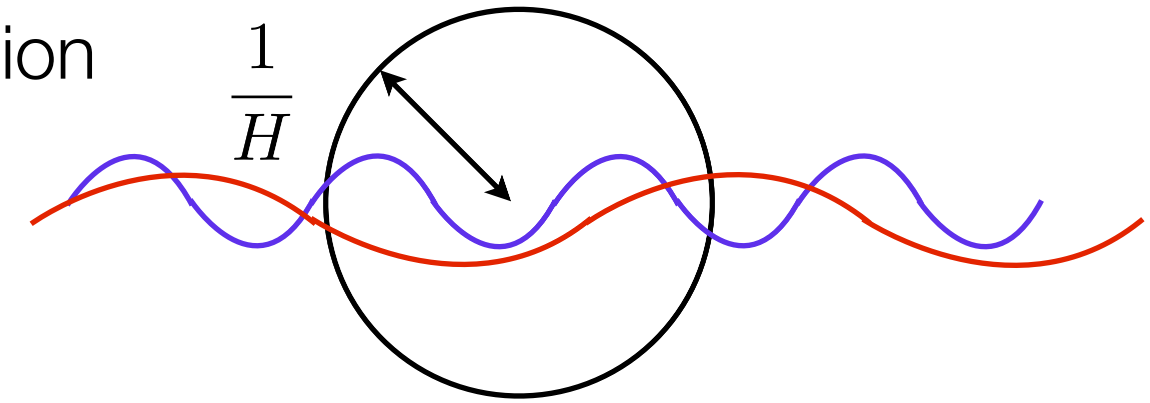
- What is IR screening effect?

  Some quantity may look smaller from very long distances.

- Field theory in de Sitter space

  In dS space, IR divergence is different from in Minkowski space.

Momentum integral → UV region / IR region



$$\int dP = \boxed{\int_H^{\Lambda_{UV}} dP} + \boxed{\int_{\epsilon_0 H|\tau|}^{H} dP}$$

UV IR

This is momentum integral is used to compute scalar vacuum graph.

$$\langle \phi(x)\phi(x) \rangle_{dS} = (UV\ \text{finite}) + \underline{\frac{H^2}{4\pi^2} |\log(H|\tau|)|}$$

IR divergence

$$\tau \equiv -\frac{1}{H} e^{-Ht}$$

$$|\tau| \to 0\ (t \to \infty)$$

$$\boxed{\Lambda_{\text{eff}} = \Lambda_{\text{th}} - \frac{\kappa}{D} T_\rho{}^\rho(\tau)}$$

4 How to compute quantum corrections on dS

To see IR div effects, we consider massless scalar theories.

In lower dimensions

In flat space, IR divergence are stronger than in 4 dimension.

Some models are solvable, so we may study effects in closed form.

cf: 2D Gravity   Callan et al. (92)

1 Perturbative effects

ϕ^6 theory (3 dim)

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{\lambda}{6!} \phi^6$$

ϕ^4 theory (2 dim) (Landau-Ginzburg theory)

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{\lambda}{4!} \phi^4$$

2 Non-perturbative effects

Liouville theory (2 dim)

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - e^{\lambda\phi}$$

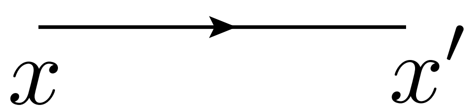
Non linear sigma model (2 dim)

$$\mathcal{L} = \frac{1}{2g^2} g^{\mu\nu}(x) \partial_\mu \phi^a(x) \partial_\nu \phi^b(x) G_{ab}(\phi)$$

4 How to compute quantum corrections on dS

For loop correction, we need propagator Δ_∞ on dS.

We consider massless scalar field in D dim.



$$i\Delta_\infty(x:x') = \cdots\left\{\Gamma\left(\frac{D}{2}-1\right)\left(\frac{4}{y}\right)^{\frac{D}{2}-1} {}_2F_1\left(\frac{1}{2}+\nu,\frac{1}{2}-\nu;2-\frac{D}{2};\frac{y}{4}\right)+\cdots\right\}$$

$$=\cdots\Gamma\left(\frac{D}{2}-1\right)\left\{\left(\frac{4}{y}\right)^{\frac{D}{2}-1}+\frac{\Gamma(2-\frac{D}{2})}{\Gamma(\frac{1}{2}+\nu)\Gamma(\frac{1}{2}-\nu)}\sum_{n=0}^{\infty}\left[\cdots\times\left(\frac{y}{4}\right)^{n-\frac{D}{2}+2}-\cdots\left(\frac{y}{4}\right)^{n}\right]\right\}$$

Janssen et al. (08)

de Sitter invariant distance

$$y \equiv \frac{-(\tau-\tau')^2+(\vec{x}-\vec{x}')^2}{\tau\tau'}$$

$$\nu \equiv \frac{D-1}{2}$$

Massless propagator is UV (y→0) div as well as IR (y→∞) div.

UV div→dim regularization
IR div→cut off

4 How to compute quantum corrections on dS

The energy momentum tensor $\Lambda_{\mathbf{eff}} = \Lambda - \frac{\kappa}{4} T_\rho{}^\rho$

$$T_{\mu\nu} \equiv \frac{-2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}$$

$$= \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} \left\{ \int \sqrt{-g}\, d^4x \, [g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + V(\phi)] \right\}$$

$$= \left(\delta_\mu{}^\rho \delta_\nu{}^\sigma - \frac{1}{2} \eta_{\mu\nu} \eta^{\rho\sigma} \right) \partial_\rho \phi \partial_\sigma \phi - g_{\mu\nu} (V(\phi))$$

leading term

Degree of IR div is weakened by derivative.

↓

IR effects from kinetic term is weaker than potential term.

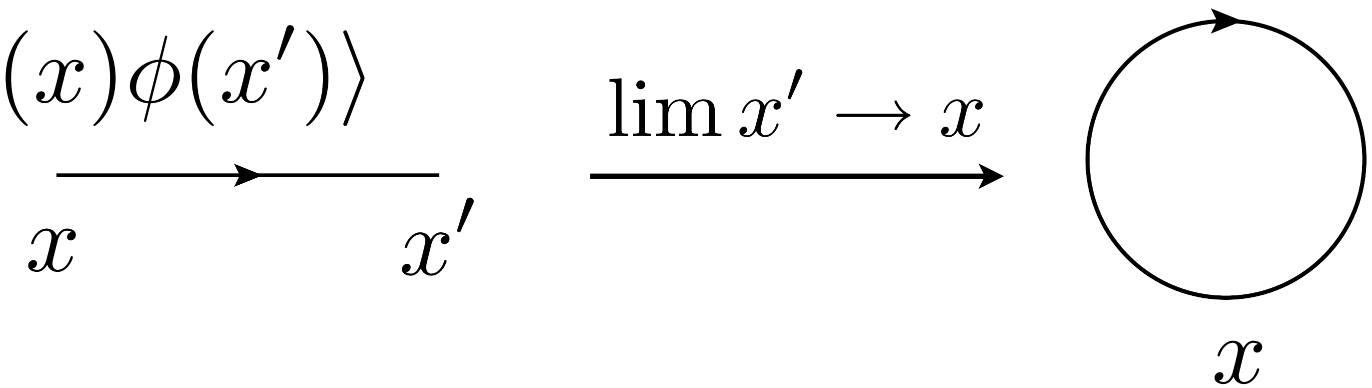
(effects in potential term) > (effects in kinetic term)

5 Field theory in lower-dim dS space

In 2 dim theory

1-loop vacuum energy

$$\langle \phi(x)\phi(x')\rangle$$



We have obtained $T_{\mu\nu}$.

Conformal anomaly calculation (point splitting) using propagator.

$$< T_{\mu\nu} >= -\frac{R}{48\pi} g_{\mu\nu}$$ Davis Fulling (76)

$$\Lambda_{\text{eff}} = \Lambda + \frac{R}{48\pi}$$ 2 D

$$\Lambda_{\mathbf{eff}} = \Lambda - \kappa \frac{3H^4}{32\pi^2}$$ 4 D (KK) (11)

5 Field theory in lower-dim dS space

Connections to Potential term's effect

ϕ^4 theory (2D)

$$\langle\phi(x)\phi(x')\rangle = \alpha\{\gamma(y) + \beta\mathbf{log}(\mathbf{a}(\tau)\mathbf{a}(\tau'))\}$$

Propagator

$$y \equiv \frac{-(\tau-\tau')^2 + (\overrightarrow{x} - \overrightarrow{x}')^2}{\tau\tau'}$$

de Sitter invariant distance

$$a(\tau) = -\frac{1}{H\tau}$$

α, β : constant

$$\langle T_{\mu\nu}\rangle = (\textbf{kinetic term}) - \underline{g_{\mu\nu}\frac{\lambda}{4!}\langle\phi^4\rangle}$$

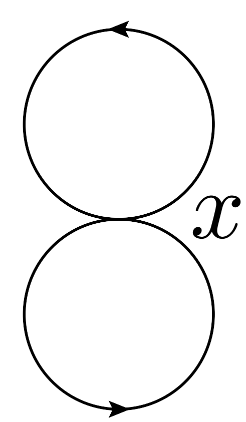
leading term

$$\frac{\lambda}{4!}\langle\phi^4\rangle = \frac{\lambda}{8}\langle\phi^2\rangle^2$$

$$\langle\phi(x)^2\rangle^2_{ren} \sim \alpha^2\ 4(\log(a(\tau)))^2$$

$$(T_\rho{}^\rho)_{\rm pot} \sim -\frac{2\lambda}{8} \times \alpha^2 4(\log(a(\tau)))^2$$

$$\therefore \Lambda_{eff} \sim \Lambda + \lambda \times \alpha^2(\log(-\frac{1}{H\tau}))^2$$



Effective cosmo const has time dependence.

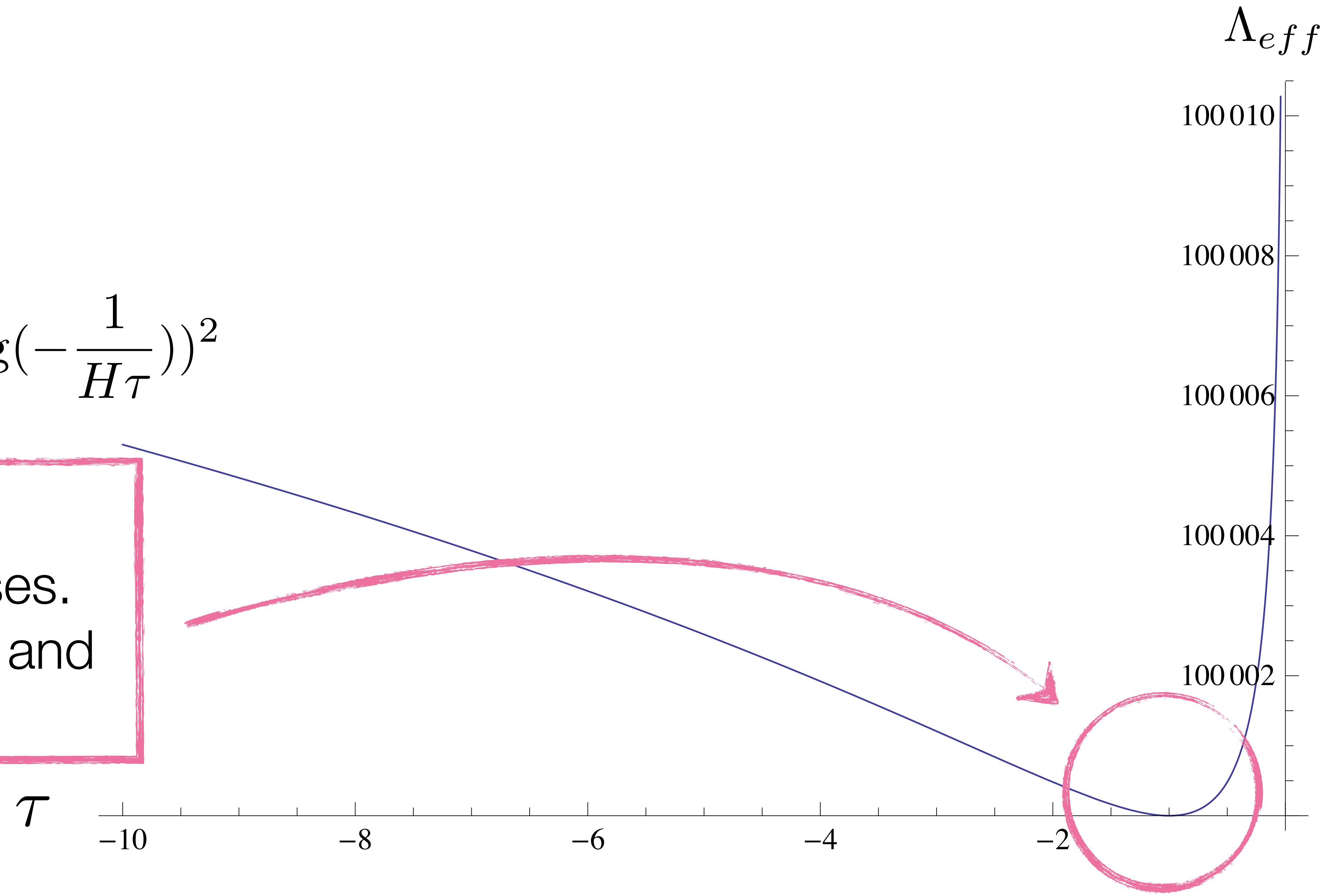
5 Field theory in lower-dim dS space

Graph

ϕ^4 theory (2D)

$$\therefore \Lambda_{eff}$$

$$\sim \Lambda + \lambda \times \alpha^2 (\log(-\frac{1}{H\tau}))^2$$



In this region effective cosmo const decreases. But lowest value is Λ and it is very large.

In this model we do not see IR screening effect.

5 Field theory in lower-dim dS space

Liouville theory

Effective potential term

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - e^{\lambda\phi}$$

leading term

$$\langle T_{\mu\nu} \rangle_{\text{pot}} = -g_{\mu\nu} \left\langle e^{\lambda\phi} \right\rangle$$

$$\left\langle e^{\lambda\phi} \right\rangle = e^{\left(\frac{1}{2}\lambda^2 \langle \phi^2 \rangle\right)}$$

$$\left\langle \phi^2 \right\rangle \sim \left\langle \phi^2(x) \right\rangle_{\text{rem}} \sim \frac{1}{2\pi} \log a(\tau)$$

$$e^{\left(\frac{1}{2}\lambda^2 \langle \phi^2 \rangle\right)} \sim e^{\left(\frac{1}{2}\lambda^2 \frac{1}{2\pi} \log a(\tau)\right)}$$

$e^{\lambda\phi}$ obtained in a closed form.

$$= \left(-\frac{1}{H\tau}\right)^{\frac{\lambda^2}{4\pi}}$$

$$\therefore \Lambda_{eff} = \Lambda + \kappa\left(-\frac{1}{H\tau}\right)^{\frac{\lambda^2}{4\pi}}$$

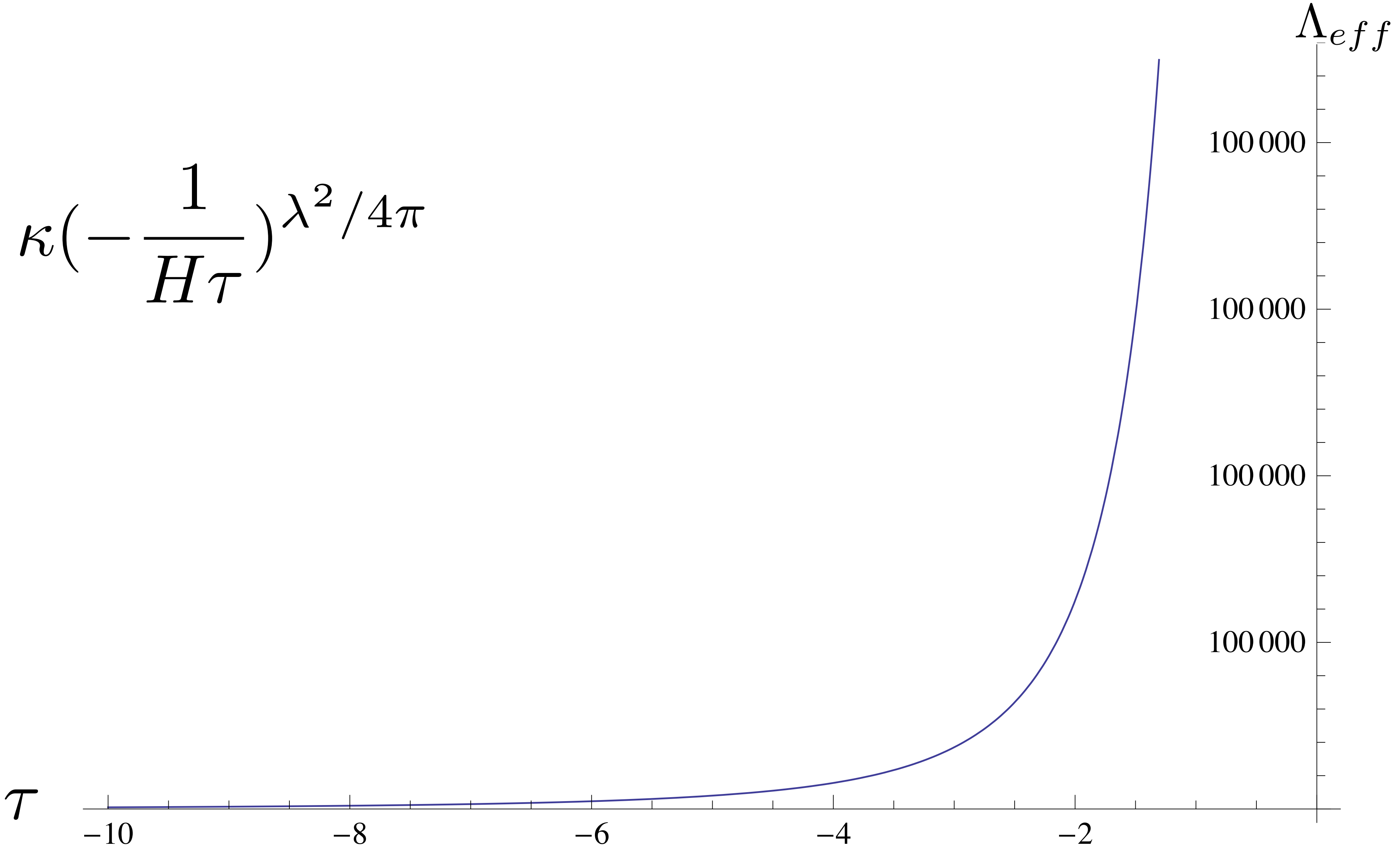
the time dependence is different from perturbative theory.

ϕ^4 theory (2D) $\Lambda_{\text{eff}} \sim \Lambda + \lambda \times \alpha^2 (\log(-\frac{1}{H\tau}))^2$

5 Field theory in lower-dim dS space

Liouville theory

$$\therefore \Lambda_{eff} = \Lambda + \kappa(-\frac{1}{H\tau})^{\lambda^2/4\pi}$$



Quantum effect has a power dependence in τ.

↓

However the effective cosmo const increases as τ approaches 0.

6 Summary of Part1

Contribution of kinetic term (free theory)

No time dependence appears

in 2D $T^{\mu}_{\mu} \neq 0$ Conformal anomaly
in 3D constant effect
in 4D constant effect (KK) (11)

By differential, kinetic term's contribution is constant.

Contribution of potential term

There appears time dependence.

perturbative theory

$$\log a(\tau)$$

non-perturbative theory

$$a(\tau)^{\frac{\lambda^2}{4\pi}}$$

But, it does not have IR screening effect on cosmo const $\Lambda_{\mathbf{eff}}$.

Part II Liouville theory and 2D gravity

—— 2D toy model for 4D gravity + matter

1 Liouville theory

Liouville theory

The theory has a few remarkable properties.

1 It is solvable at quantum level in 2 D Minkowski space.

Liouville theory is solvable in flat space.

↓

There are an infinite number of conserved quantities.

2 It has conformal invariance.

3 Liouville theory may be regarded as 2D quantum gravity.

$$S_{\mathrm{L}} = \frac{1}{4\pi}\int d^2x\sqrt{\hat{g}}\left(\frac{1}{2}\hat{g}^{ab}\partial_a\phi\partial_b\phi + \Lambda e^{\gamma\phi} + \cdots\right)$$

2 Solvable models

Another class of solvable models (non-relativistic)

KdV eq., MKdV eq.

It has infinitely many conserved quantities.

MKdV eq.

$$u_t = u_{\sigma\sigma\sigma} - 6u^2 u_\sigma$$

Conservation laws are easy to construct for MKdV eq.

A first few of them

$$I_1 = \int_0^{2\pi} d\sigma \frac{1}{2} u^2 \quad , \quad I_2 = \int_0^{2\pi} d\sigma \frac{1}{2}(u^4 + (u_\sigma)^2) \; , \quad \dots$$

By use of EOM

Sasaki Yamanaka

$$\dot{I}_1 = \int_0^{2\pi} d\sigma u \dot{u} = \int_0^{2\pi} d\sigma [-\frac{1}{2}(u_\sigma^2)_\sigma - 2(u)_\sigma^3] = 0$$

$$\dot{I}_2 = \cdots = 0$$

3 Conformal invariance

Liouville theory has conformal inv in 2D flat space.

$$\mathcal{L} = -\frac{1}{2} g^{ab}\partial_a\phi\partial_b\phi - e^{\lambda\phi} \qquad T^c{}_c|_{g_{ab}=\eta_{ab}} = 0$$

In de Sitter space, an additional term is necessary to achieve conformal inv.

$$S = \int d^2x\sqrt{\hat{g}}[-\frac{1}{2}\hat{g}^{ab}\partial_a\phi\partial_b\phi + Q\hat{R}\phi + \mu e^{2b\phi}]$$

$$= \int d^2x\sqrt{\hat{g}}\{\frac{1}{b^2}[-\frac{1}{2}\hat{g}^{ab}\partial_a\varphi\partial_b\varphi + Q'\hat{R}\varphi + \mu e^{2\varphi}]\}$$ (rescaling $\varphi = b\phi$)

Proof can be made by noting EOM

$$D^a D_a\varphi - \hat{R} = \mu e^{2\varphi}$$

and conformal transformation

$$\varphi \to \varphi + \delta\sigma(x) \qquad \hat{g}_{ab} \to e^{2\delta\sigma(x)}\hat{g}_{ab}$$

$$\sqrt{\hat{g}_{ab}}\hat{R}[\hat{g}] \to \sqrt{\hat{g}_{ab}}(\hat{R}[\hat{g}] + D^c D_c\delta\sigma(x))$$

4 2dimensional gravity

- 2D Quantum gravity (QG)   (Kawai et al. 87)

  2D gravity with cosmo const μ (but without R)

$$Z = \int DX^\mu Dg_{ab}(x) e^{iS[g_{ab},X^\mu]} \qquad S_{2D}[g_{ab}, X^\mu] = \int dx^2 \sqrt{g}(g^{ab}\partial_a X^\mu \partial_b X_\mu + \mu)$$

After integrating over X^μ, Liouville theory arises.

$$Z \sim \int D\phi\, e^{iS_L(\phi)}$$

Polyakov 87

$$S_{\mathrm{L}} = \frac{1}{4\pi}\int d^2x \sqrt{\hat{g}}(\frac{1}{2}\hat{g}^{ab}\partial_a\phi\partial_b\phi + Q\phi R(\hat{g}) + \frac{\mu}{2\gamma^2}e^{\gamma\phi})$$

In comparison, Einstein gravity

$$S[g_{ab}(x)] = \int d^2x \sqrt{g}[\frac{1}{\kappa^2}R(g_{ab}) + \Lambda]$$

R is invariant under change of metric g_{ab}, containing no dynamics.

Liouville theory has conformal anomaly.

$$T^c{}_c|_{\hat{g}_{ab}} = c\hat{R} \qquad c = 1 + 3(2Q)^2$$

The $c\hat{R}$ term appears in 2D gravity.

4 2dimensional gravity

Gravitational loop + matter loop

Using Liouville theory, we can evaluate the effects of gravitational loop and matter loop.

$$S_{\mathrm{matter}}[\phi, \chi] = \int d^2x \sqrt{g}[g^{ab}\partial_a\phi\partial_b\phi + QR\phi + \Lambda + g^{ab}\partial_a\chi\partial_b\chi - V(\chi)]$$

↓ Weyl transformation

$$S_{\mathrm{L+matter}}[\phi, \chi] = \int d^2x \sqrt{\hat{g}}[\hat{g}^{ab}\partial_a\phi\partial_b\phi + Q\hat{R}\phi + \Lambda e^{\phi} + \hat{g}^{ab}\partial_a\chi\partial_b\chi - e^{\phi}V(\chi)]$$

The first three terms are the 2D gravity action.

Last term describes the coupling of gravity and matter.

By setting ϕ to ϕ_0, this action is same as the matter action of Part I.

2 D toy model for 4 D gravity + matter

Summary of Liouville theory

We are currently woking on the farrowing problems.

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1 Solvability of the Liouville theory in dS space

How can we rescue the time dependence of Hamiltonian?

We want to extend QFT to dS space and calculate correlation functions.
However, it is not trivial that hamiltonian has time independence.

Solvable theories have infinitely number of conserved quantities.
How can we extend the conservation laws of Liouville theory from Minkowski space to dS space?

2 2D gravity and Liouville theory

We want to calculate the effects of 2D gravity and matter and interaction of gravity and matter appears.

To summarize, we want to calculate the screening effect from 2D gravity and matter.

Summary and outlook

- The potential terms pick up time dependence due to loop effects. However, IR screening effects don't appear. (Part I)

  We have constructed a good 2 D toy model (Liouville theory) for 4 D QG + matter loops (Part II).

  Liouville and matter loops for cosmo const $\Lambda_{\rm eff}$ are being computed.

- We extend Liouville theory solvability from flat space to dS space. Question : Can we construct "conservation" laws in dS space?

  If it is solvable, we may calculate quantum corrections to cosmo const $\Lambda_{\rm eff}$ in closed form.

4 2 dimensional gravity

Gravitational loop + matter loop

Using Liouville theory, we can evaluate the effects of gravitational loop and matter loop.

$$S_{\mathrm{L+matter}}[\phi,\chi]=\int d^2x\sqrt{\hat{g}}[\hat{g}^{ab}\partial_a\phi\partial_b\phi+Q\hat{R}\phi+\Lambda e^{\phi}+\hat{g}^{ab}\partial_a\chi\partial_b\chi-e^{\phi}V(\chi)]$$

Example)

$$\mathcal{L}=(\text{Liouville theory})-\frac{1}{2}\hat{\mathrm{g}}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-\frac{\lambda}{4!}\mathrm{e}^{2\mathrm{b}\phi}\varphi^4$$

This term describes the coupling term of gravitation and matter.

gravity loop and matter loop

$$e^{2b\phi}\varphi^4=\{1+(2b\phi)+\frac{1}{2}(2b\phi)^2+\cdots\}\varphi^4$$

Summary of Liouville theory

We are currently woking.

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Summary and outlook

- The potential terms pick up time dependence due to loop effects.
  However, IR screening effects don't appear. (Part I)

  How about Liouville theory (2D QG)? (Part II)

  When matter field couples gravity field,
  how does it effect cosmo const Λ.

  2D QG plus matter is good toy model for 4D QG plus matter.

- We expend solvability from flat space to dS space to check
  solvable model in flat space is solvable too in dS space.

  If yes, we can calculate them in non-perturbative ways.