Chern-Simons vector models and duality in 3 dimensions

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String theory and quantum field theory

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Chern-Simons theory

① (Condensed matter physics)

Quantum hole effect

② (Mathematics)

Knot theory, Jones polynomial, A polynomial **[Witten '89]**

③ (string theory)

Cubic string field theory, Open topological string theory **[Witten '85]**

④ (M_theory)

Effective field theories of membranes. **[BLG '07, ABJM '08]**

⑤ (3d CFT)

Infinitely many interacting CFT (conformal zoo). **[Witten '89]** **[Moore_Seiberg '89]**

⑥ (AdS/CFT correspondence)

Dual CFT3 of (HS) gravity on AdS4

Pure (HS) gravity on AdS3

[Gaberdiel_Gopakumar '11]

(Pure) Chern-Simons theory

Action

$$iS_{cs} \quad = \quad \frac{ik}{4\pi}\int \mathrm{tr}(\widetilde{A}d\widetilde{A}+\frac{2}{3}\widetilde{A}^3)$$

Feature

① CS coupling constant (k) is protected as an integer.

② Independent of metric. (Topological).

③ Exact “CFT” parametrized by (k,N) or λ=N/k. N: rank of gauge group

④ Exactly soluble. (Wilson loop ⇔ Knot) . **[Witten ’89]**

Vector (Sigma) models

① (Phenomenology)

Effective field theory of pion, Low energy theorem

Landau-Ginzburg model

② (Large N field theory)

Soluble in 1/N expansion

Dynamical symmetry breaking (or restoration) **[Nambu-Jona-Lasinio '60]**

③ (RG flow) **[Wilson_Kogut '74]**

Nontrivial fixed point **[Wilson_Fisher '72] [Gross-Neveu '74]**

④ (Probe of geometry)

(quantum) description of geometry cf. Polyakov action

⑤ (AdS/CFT correspondence)

Dual CFT3 of HS gravity on AdS4 **[Klebanov_Polyakov '02]**

CS Vector models

preserve conformal symmetry and higher spin symmetry
in the ‘t Hooft limit.

- spectra of singlets are not renormalized in the ‘t Hooft limit.
  (Anomalous dimension is suppressed by 1/N).
- couple to higher spin gravity (Vasiliev) theory
  surviving in the low energy limit. (AdS/CFT)
- soluble in the ‘t Hooft limit and (euclidean) light-cone gauge.
- enjoy novel duality (bosonization) in 3 dimensions
  and novel thermal phase structure.

CS Vector models

Scale invariant Action

① Regular boson theory

$$S_{cs} + \int d^3x \left(D_\mu\bar{\phi}D^\mu\phi + \lambda_6(\bar{\phi}\phi)^3\right)$$

② Critical boson theory

[Wilson_Fisher '72]

$$S_{cs} + \int d^3x \left(D_\mu\bar{\phi}D^\mu\phi + \sigma\bar{\phi}\phi\right)$$

③ Regular fermion theory

$$S_{cs} + \int d^3x \left(\bar{\psi}\gamma^\mu D_\mu\psi\right)$$

④ Critical fermion theory

[Gross-Neveu '74]

$$S_{cs} + \int d^3x \left(\bar{\psi}\gamma^\mu D_\mu\psi + \sigma_f\bar{\psi}\psi + \lambda_6^f\sigma_f^3\right)$$

⑤ "Mixed" sigma model

$$S_{cs} + \int d^3x \Big[D_\mu\bar{\phi}D^\mu\phi + \bar{\psi}\gamma^\mu D_\mu\psi + \lambda_4(\bar{\psi}\psi)(\bar{\phi}\phi) + \lambda_4'(\bar{\psi}\phi)(\bar{\phi}\psi)$$
$$+\lambda_4''\left((\bar{\psi}\phi)(\bar{\psi}\phi) + (\bar{\phi}\psi)(\bar{\phi}\psi)\right) + \lambda_6(\bar{\phi}\phi)^3\Big].$$

Exact correlation functions

are almost determined by almost-conserved conformal symmetry and higher spin symmetry via bootstrap method.

[Maldacena–Zhiboedov '12]

under the normalization

$$\langle J_s J_{s'} \rangle = \tilde{N} \langle J_s J_{s'} \rangle_{bos,fer} \qquad \langle J_0 J_0 \rangle = \frac{\tilde{N}}{1+\tilde{\lambda}^2} \langle J_0 J_0 \rangle_{bos,fer}$$

3pt function

$$\langle J_s J_{s'} J_{s''} \rangle = \tilde{N} \left[\frac{\tilde{\lambda}^2}{1+\tilde{\lambda}^2} \langle J_s J_{s'} J_{s''} \rangle_{bos} + \frac{1}{1+\tilde{\lambda}^2} \langle J_s J_{s'} J_{s''} \rangle_{fer} + \frac{\tilde{\lambda}}{1+\tilde{\lambda}^2} \langle J_s J_{s'} J_{s''} \rangle_{int} \right]$$

↓

Critical boson ↔ Regular fermion

3d bosonization

and so on...

Exact correlation functions

are almost determined by almost-conserved conformal symmetry and higher spin symmetry via bootstrap method.

Explicit computation

Critical boson: $\tilde{N} = 2N_B \frac{\sin \pi\lambda_B}{\pi\lambda_B}$ $\tilde{\lambda} = -\cot \frac{\pi\lambda_B}{2}$

Regular fermion: $\tilde{N} = 2N_F \frac{\sin \pi\lambda_F}{\pi\lambda_F}$ $\tilde{\lambda} = \tan \frac{\pi\lambda_F}{2}$

[Aharony_Gur-Ari_Yacoby, Gur-Ari_Yacoby '12]

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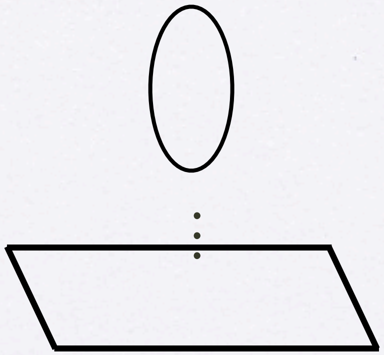
[Aharony_Gur-Ari_Yacoby, Gur-Ari_Yacoby '12]

Duality!

$$k_F \to -k_B, \quad N_F \to |k_B| - N_B, \quad \lambda_F = \frac{N_F}{k_F} \to \mathrm{sign}\lambda_B - \lambda_B$$

→ level-rank duality!!

Thermal free energy



gauge: A_- =0

Procedure

① Integrate out gauge field with gauge: A_- =0.

② Introduce auxiliary singlet fields Σ to kill all interaction.

(Hubbard-Stratonovich transformation)

③ Integrate out φ, ψ.

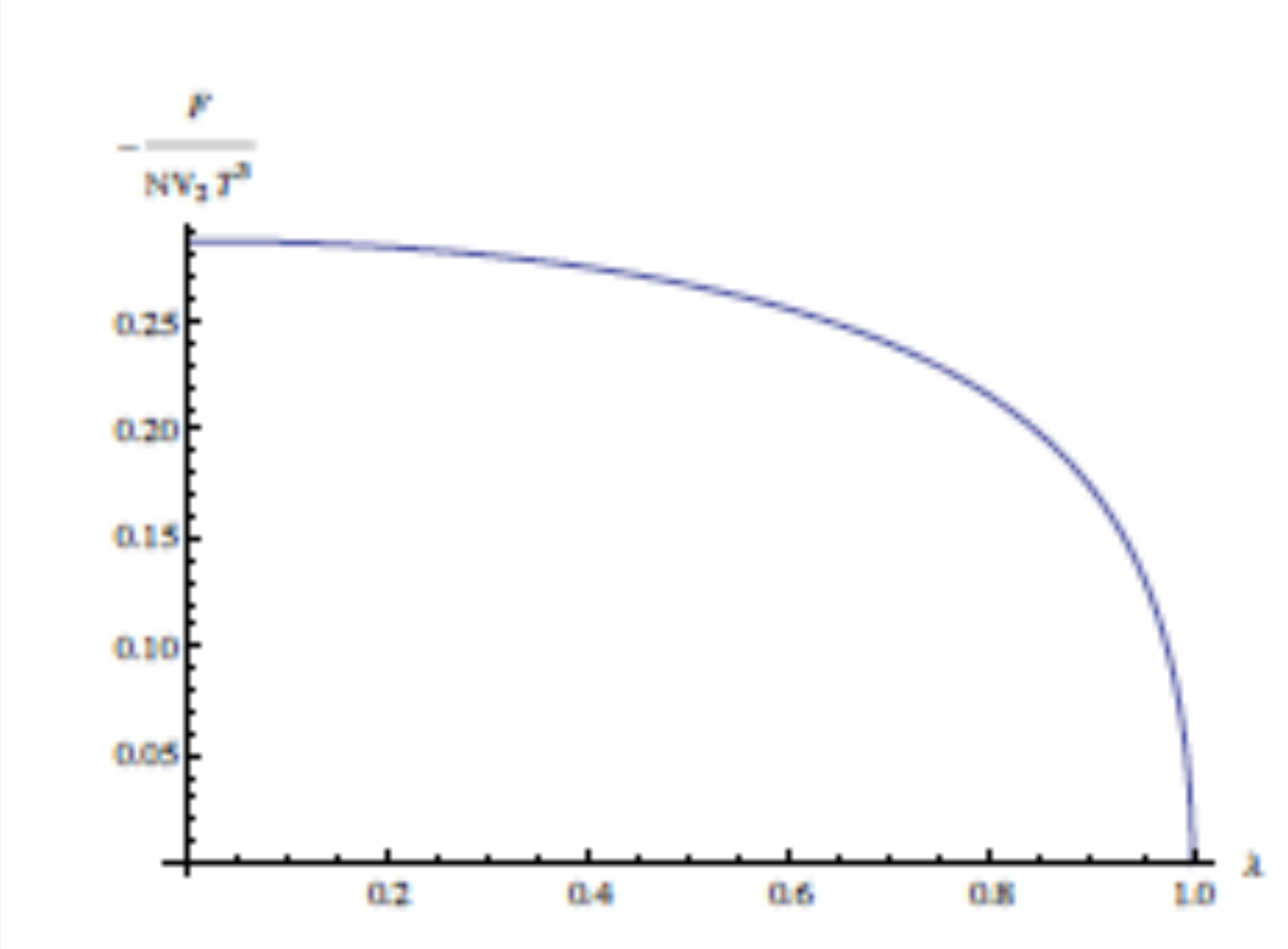
④ Evaluate it by saddle point approx. under translationally inv. config.

S.Giombi_S.Minwalla_S.Prakash_S.Trivedi_S.Wadia_X.Yin Eur.Phys.J.C72(2012)

Thermal free energy

CS Fermion vector model

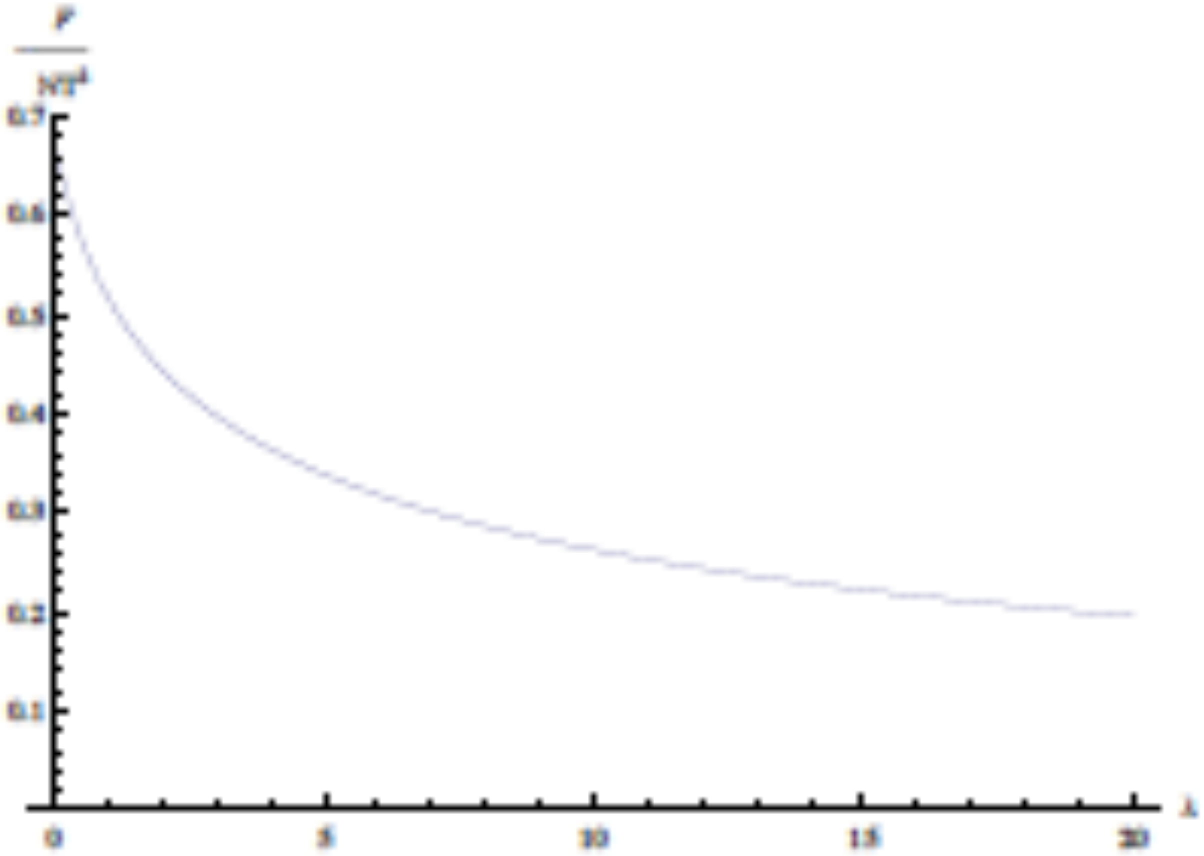
S.Giombi_S.Minwalla_S.Prakash_S.Trivedi_S.Wadia_X.Yin Eur.Phys.J.C72(2012)



Thermal free energy

N=2 SUSY CS vector model (1 chiral multplet)

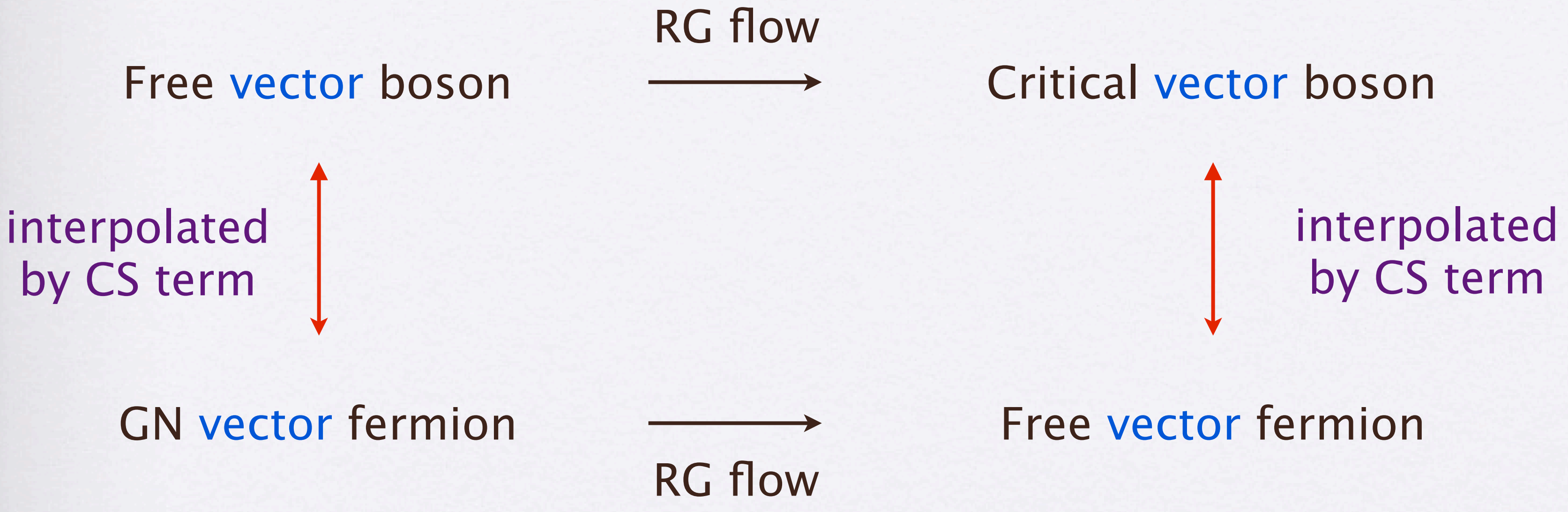
S.Jain_S.P.Trivedi_S.R.Wadia_SY JHEP10(2012)194



Puzzle against 3d duality

(1) “3d bosonization” [Aharony_Guri-Ari_Yacoby ’12], [Maldacena_Ziboedov ’12]

[Guri-Ari_Yacoby ’12]



Free vector boson —RG flow→ Critical vector boson

interpolated by CS term

interpolated by CS term

GN vector fermion —RG flow→ Free vector fermion

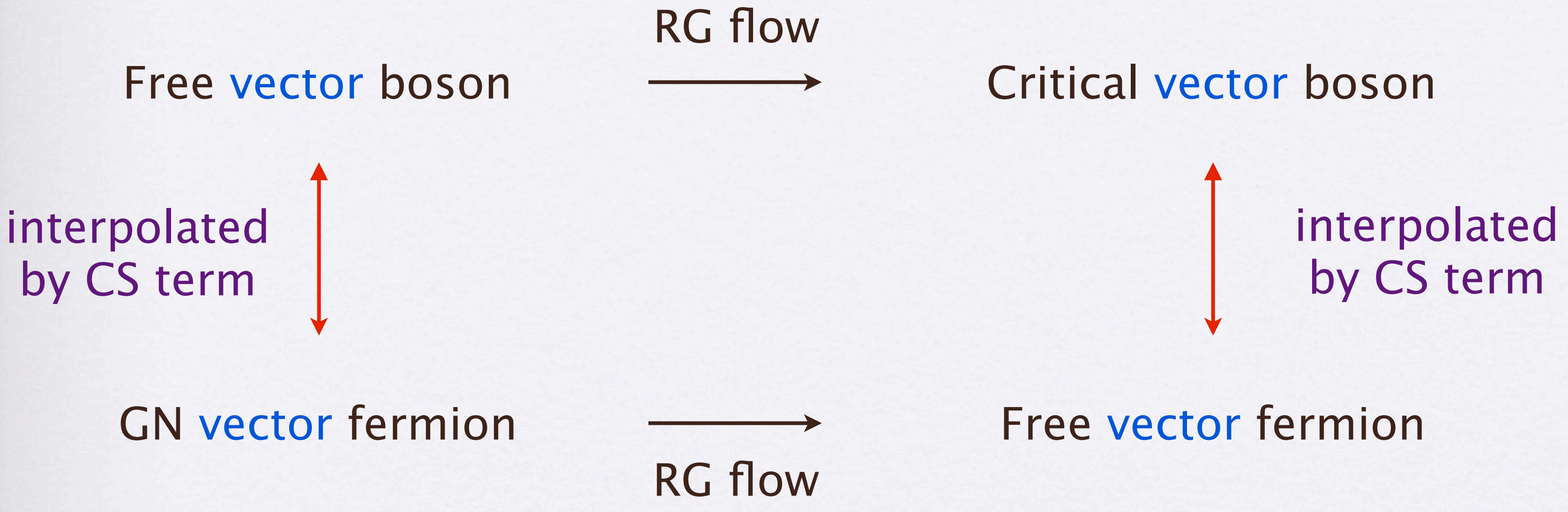
(2) Seiberg-like duality [Giveon_Kutasov ’08] [Benini_Closset_Cremonsi ’11]

N=2 case $N \to |k| - N$ $k \to -k$ $|\lambda| \to 1 - |\lambda|$ with $N/|\lambda|$ fixed.

Puzzle against 3d duality

(1) "3d bosonization" **[Aharony_Guri-Ari_Yacoby '12], [Maldacena_Ziboedov '12]**

[Guri-Ari_Yacoby '12]



RG flow

Free vector boson → Critical vector boson

interpolated by CS term ↕ interpolated by CS term

GN vector fermion → Free vector fermion

RG flow

(2) Seiberg-like duality **[Giveon_Kutasov '08]** **[Benini_Closset_Cremonsi '11]**

N=2 case $N \to |k| - N \quad k \to -k \qquad |\lambda| \to 1 - |\lambda| \text{ with } N/|\lambda| \text{ fixed}$

Consider "fermionic" holonomy distribution!!

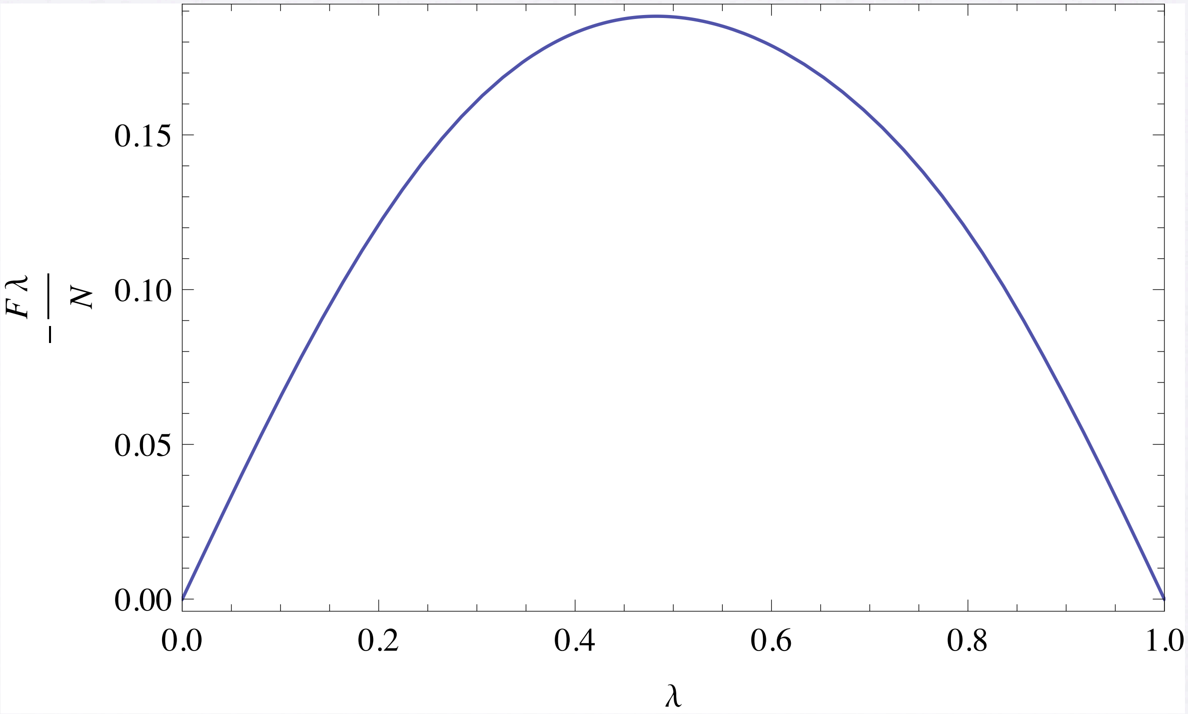
O.Aharony_S.Giombi_G.Gur-Ari_J.Maldacena_R.Yacoby. (arXiv:1210.4109)

Thermal free energy

high-temperature and fermionic holonomy

N=2 SUSY CS vector model (1 chiral multplet)

O.Aharony_S.Giombi_G.Gur-Ari_J.Maldacena_R.Yacoby. (arXiv:1210.4109)



$$|\lambda| \to 1 - |\lambda| \quad !!$$

Thermal partition function

CS vector model on S² x S¹ in high temperature

S.Jain_S.Minwalla_T.Sharma_T.Takimi_S.Wadia_SY arXiv:1301.6169

$$Z_{CS} = \left(\prod_{m=1}^{N} \sum_{n_m \in \mathbf{Z}}\right)\left(\prod_{l \neq m} 2\sin\frac{\alpha_l(\vec{n}) - \alpha_m(\vec{n})}{2}\right) e^{-T^2 V_2 f(U)}\Big|_{\alpha_m = \frac{2\pi n_m}{k}}$$

$V(U) = T^2 V_2 f(U)$ f(U) = free energy density on the flat space

cf. **YM on S² x S¹**

$$Z_{\mathrm{YM}} = \int DU \exp[-V_{\mathrm{YM}}(U)] = \prod_{m=1}^{N} \int_{-\infty}^{\infty} d\alpha_m \left[\prod_{l \neq m} 2\sin\left(\frac{\alpha_l - \alpha_m}{2}\right) e^{-V_{YM}(U)}\right]$$

Thermal partition function

CS vector model on $S^2 \times S^1$ in high temperature

S.Jain_S.Minwalla_T.Sharma_T.Takimi_S.Wadia_SY arXiv:1301.6169

$$Z_{CS} = \left(\prod_{m=1}^{N} \sum_{n_m \in \mathbf{Z}} \right) \left(\prod_{l \neq m} 2 \sin \frac{\alpha_l(\vec{n}) - \alpha_m(\vec{n})}{2} \right) e^{-T^2 V_2 f(U)} \Big|_{\alpha_m = \frac{2\pi n_m}{k}}$$

$$V(U) = T^2 V_2 f(U)$$ f(U) = free energy density on the flat space

In the large N, holonomy distributes on [-π,π] densely.

$$\rho(\alpha) = \frac{1}{N} \sum_{m=1}^{N} \delta(\alpha - \alpha_m) \qquad \rho(\alpha) \leq \frac{1}{2\pi\lambda}$$

cf. **YM on $S^2 \times S^1$**

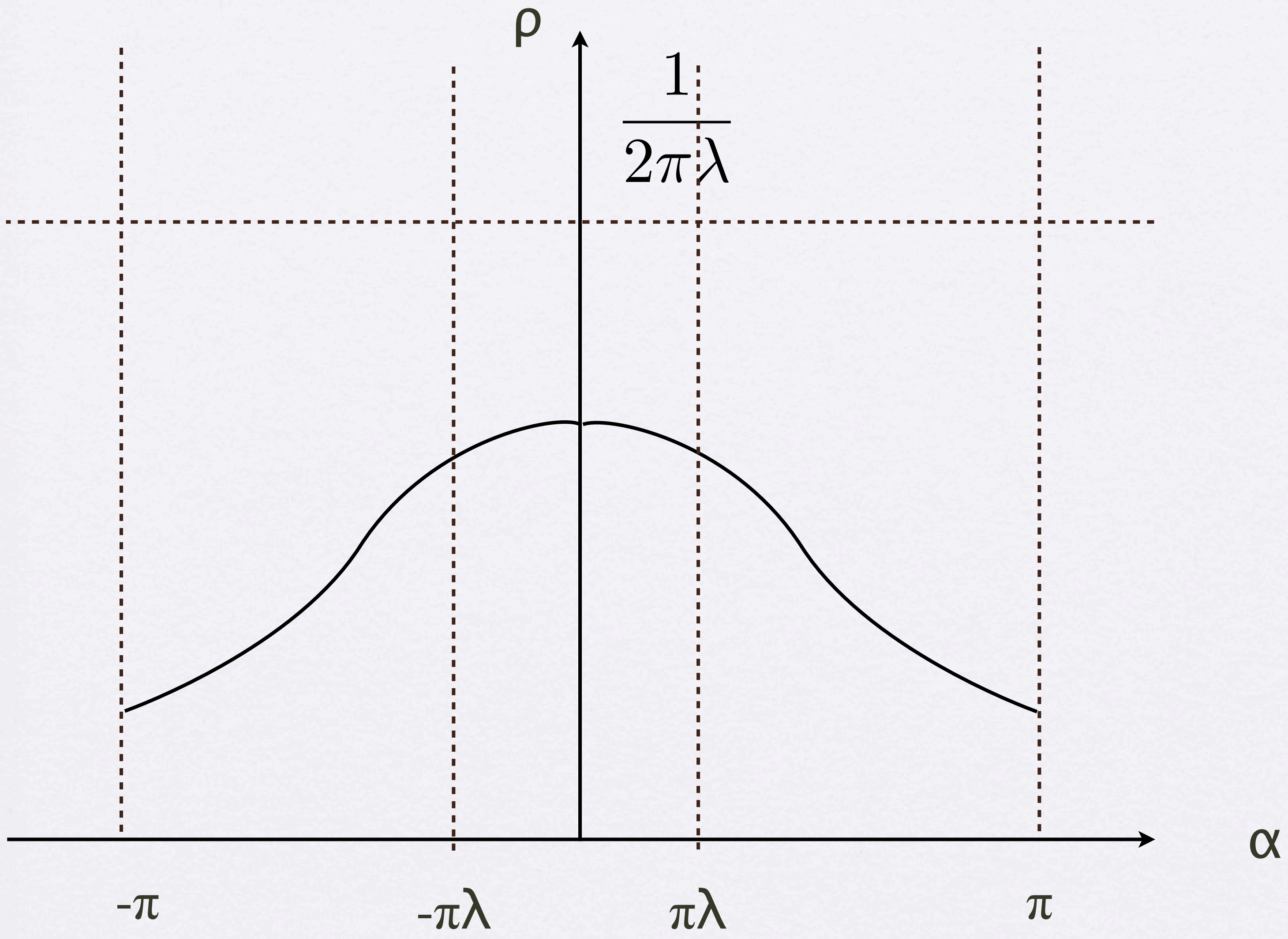
$$Z_{\rm YM} = \int DU \exp[-V_{\rm YM}(U)] = \prod_{m=1}^{N} \int_{-\infty}^{\infty} d\alpha_m \left[\prod_{l \neq m} 2 \sin \left(\frac{\alpha_l - \alpha_m}{2} \right) e^{-V_{YM}(U)} \right]$$

Phases of CS vector model

$V_2 T^2 = \zeta N$

$\zeta << 1$

No gap phase

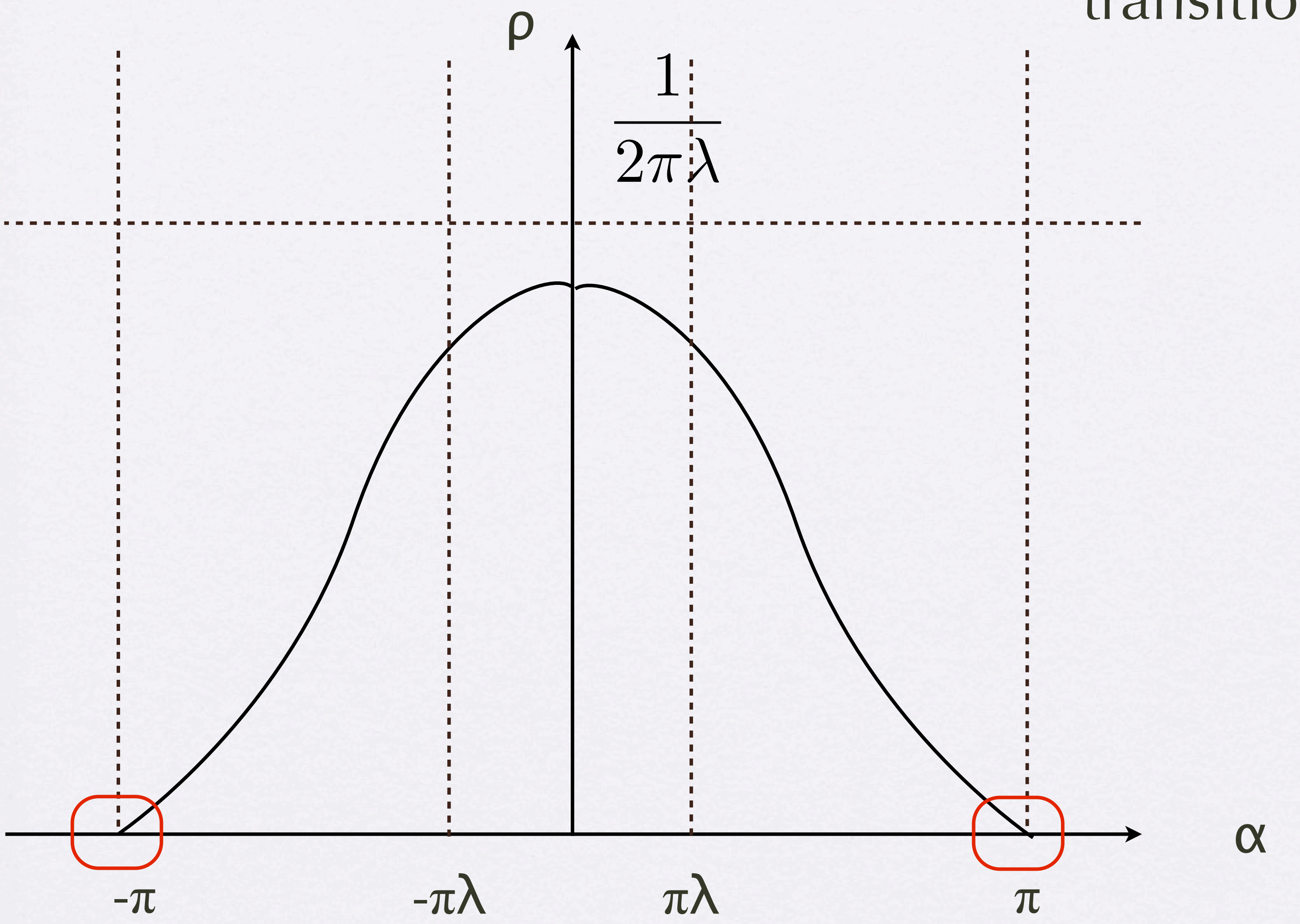


Phases of CS vector model

$V_2 T^2 = \zeta N$

$\zeta = \zeta_{GWW}(\lambda)$

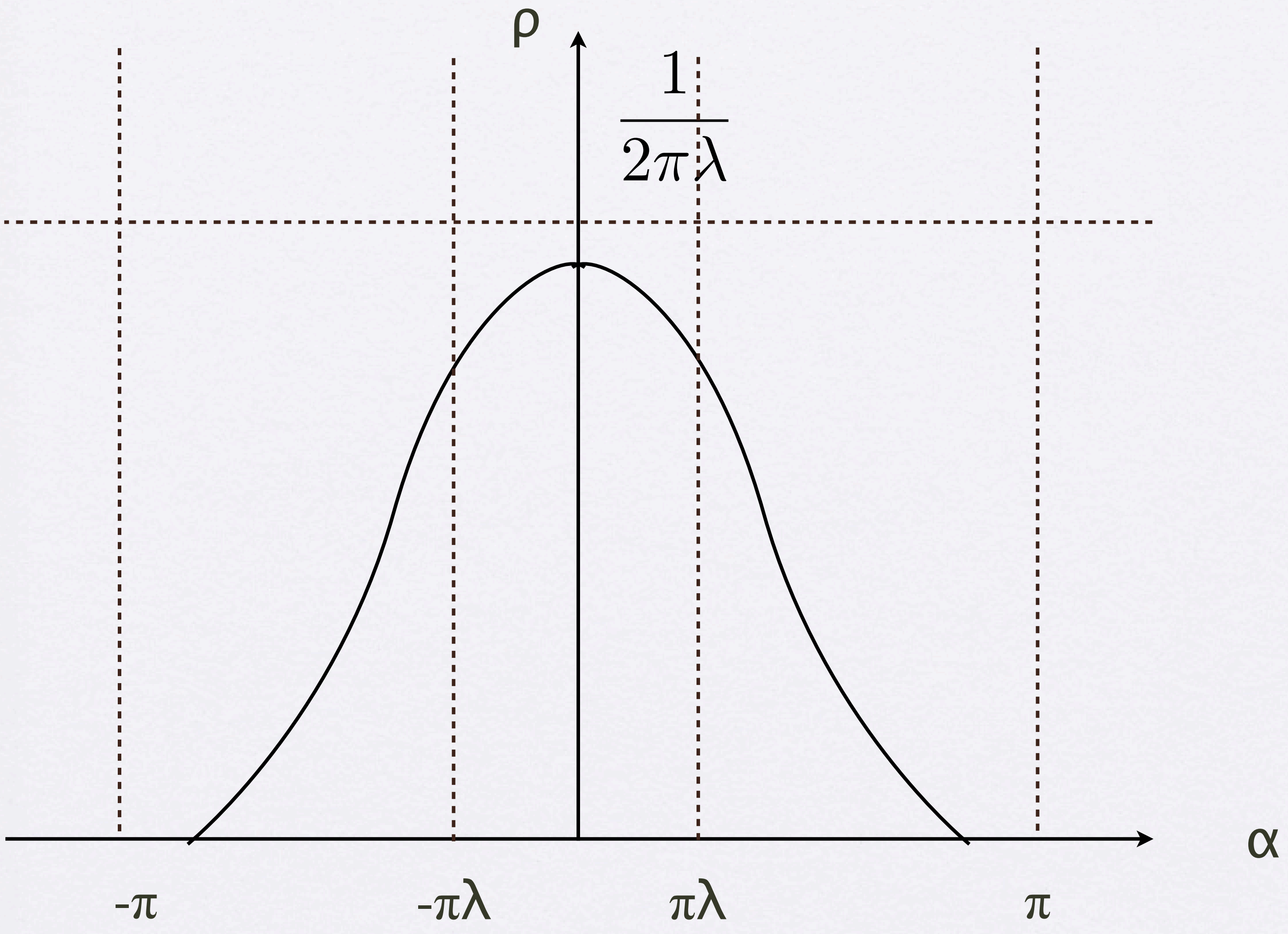
GWW phase transition!



Phases of CS vector model

$V_2T^2 = \zeta N$ $\zeta_{GWW}(\lambda)<\zeta<\zeta_{UGWW}(\lambda)$

Lower gap phase

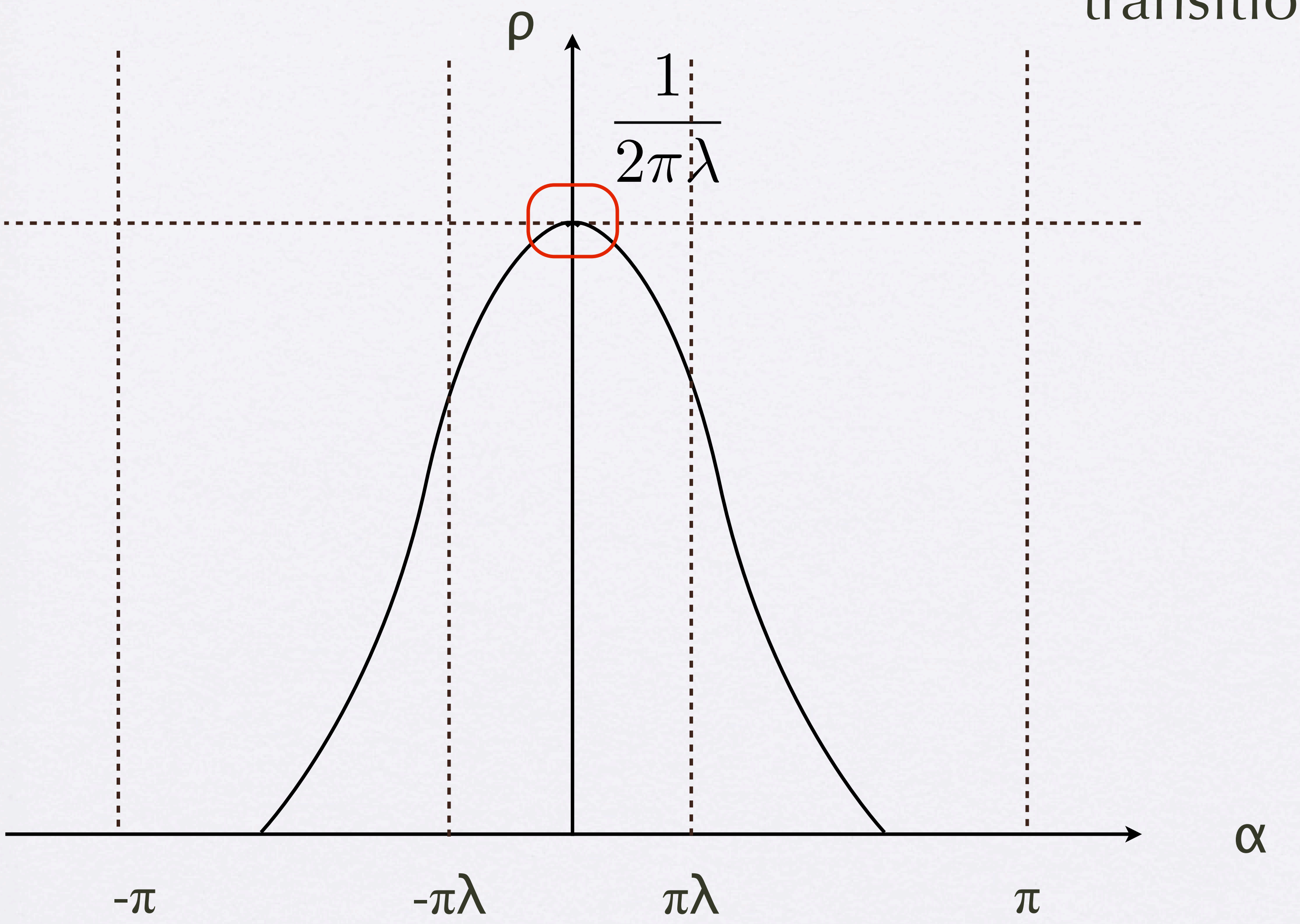


Phases of CS vector model

$V_2 T^2 = \zeta N$

$\zeta = \zeta_{\mathrm{UGWW}}(\lambda)$

Upper GWW transition!

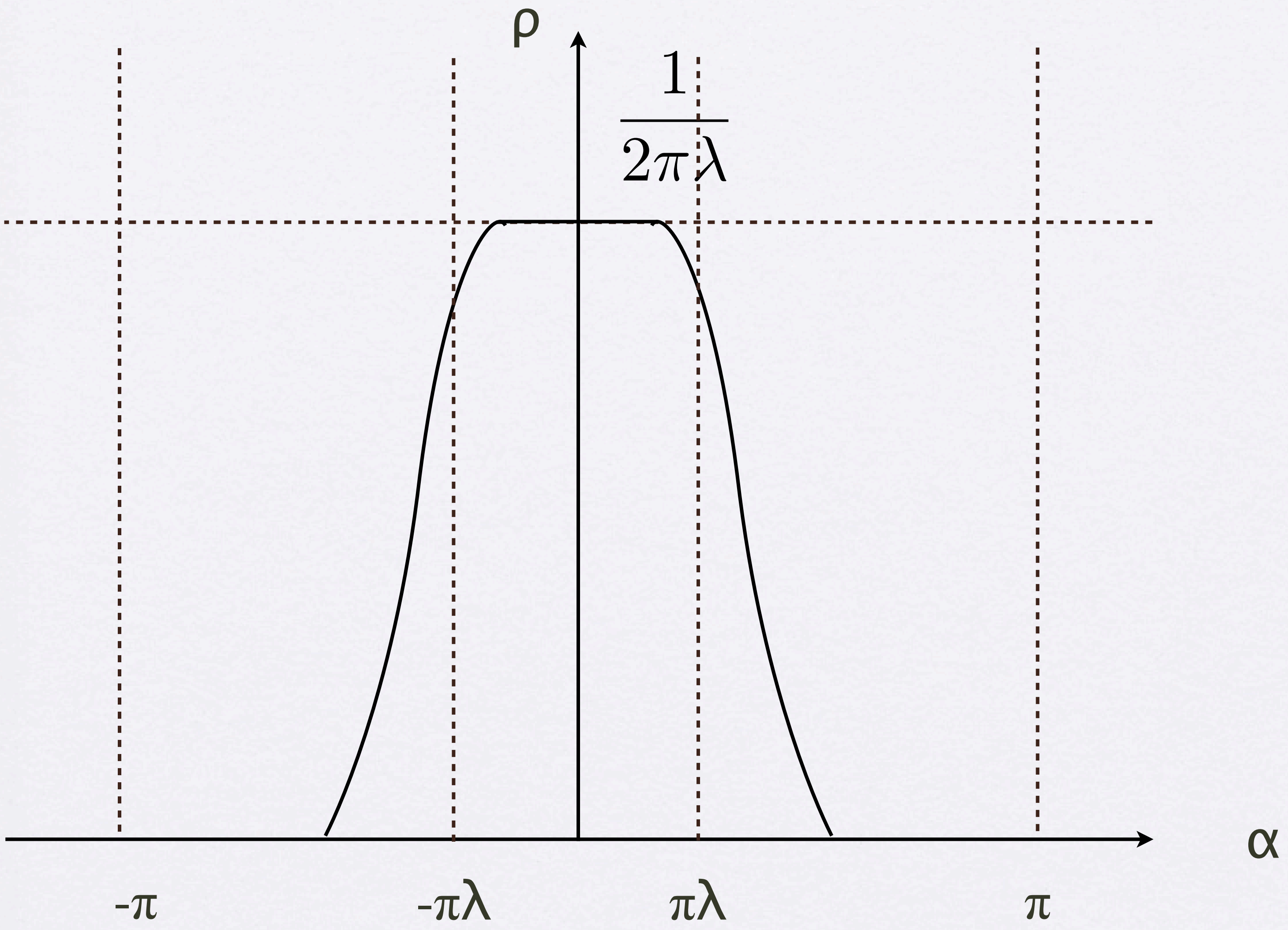


Phases of CS vector model

$V_2 T^2 = \zeta N$

$\zeta_{\mathrm{UGWW}}(\lambda) < \zeta$

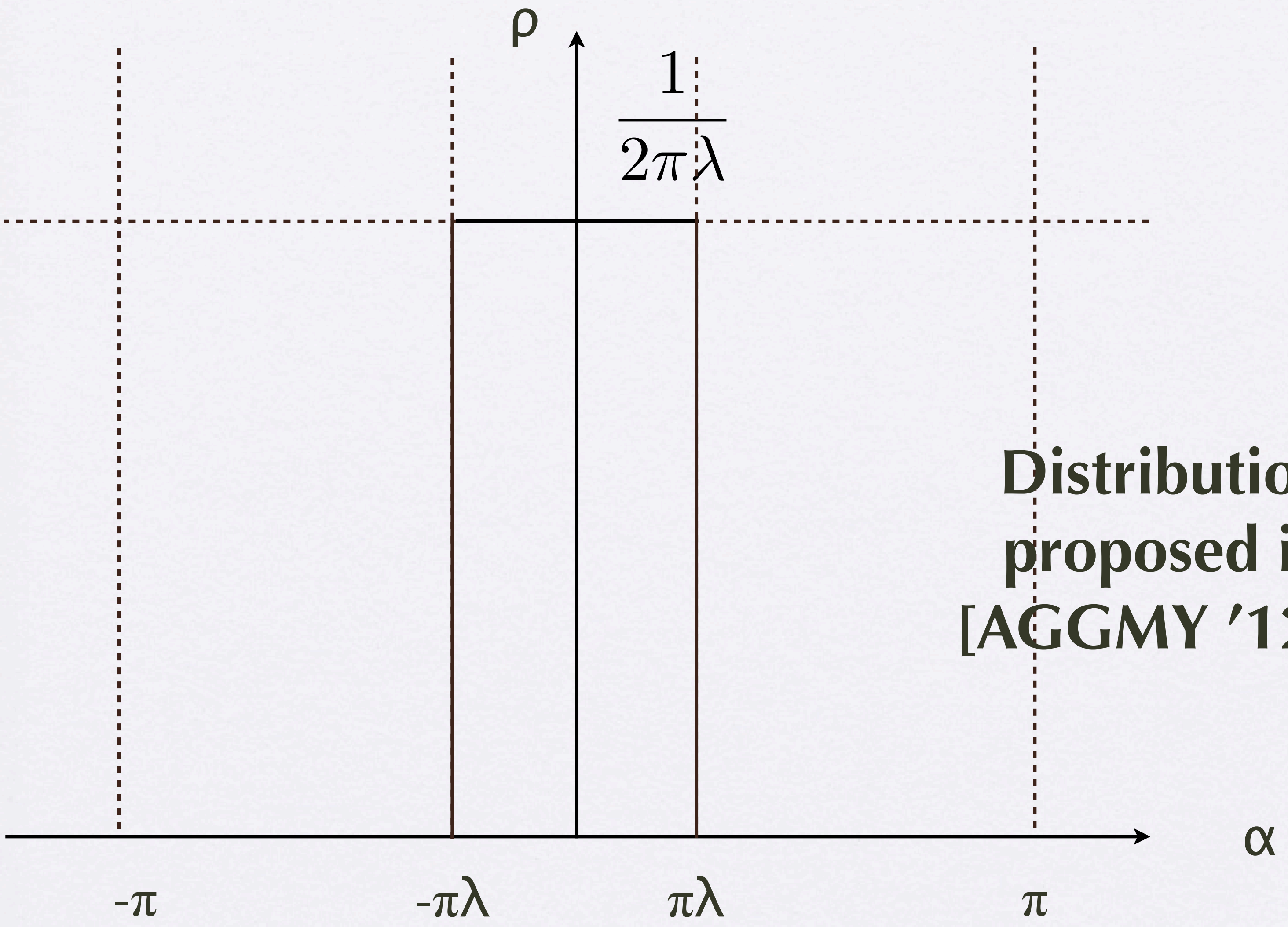
2 gap phase



Phases of CS vector model

$V_2 T^2 = \zeta N$ $\zeta = \infty$ (flat limit)

2 gap phase



Distribution proposed in [AGGMY '12]!!

3d duality & deformation

O.Aharony_S.Giombi_G.Gur-Ari_J.Maldacena_R.Yacoby. (arXiv:1210.4109)

(i) N=2 CS vector model ↻ GK duality

(ii) CS boson-fermion vector model ↻ self duality

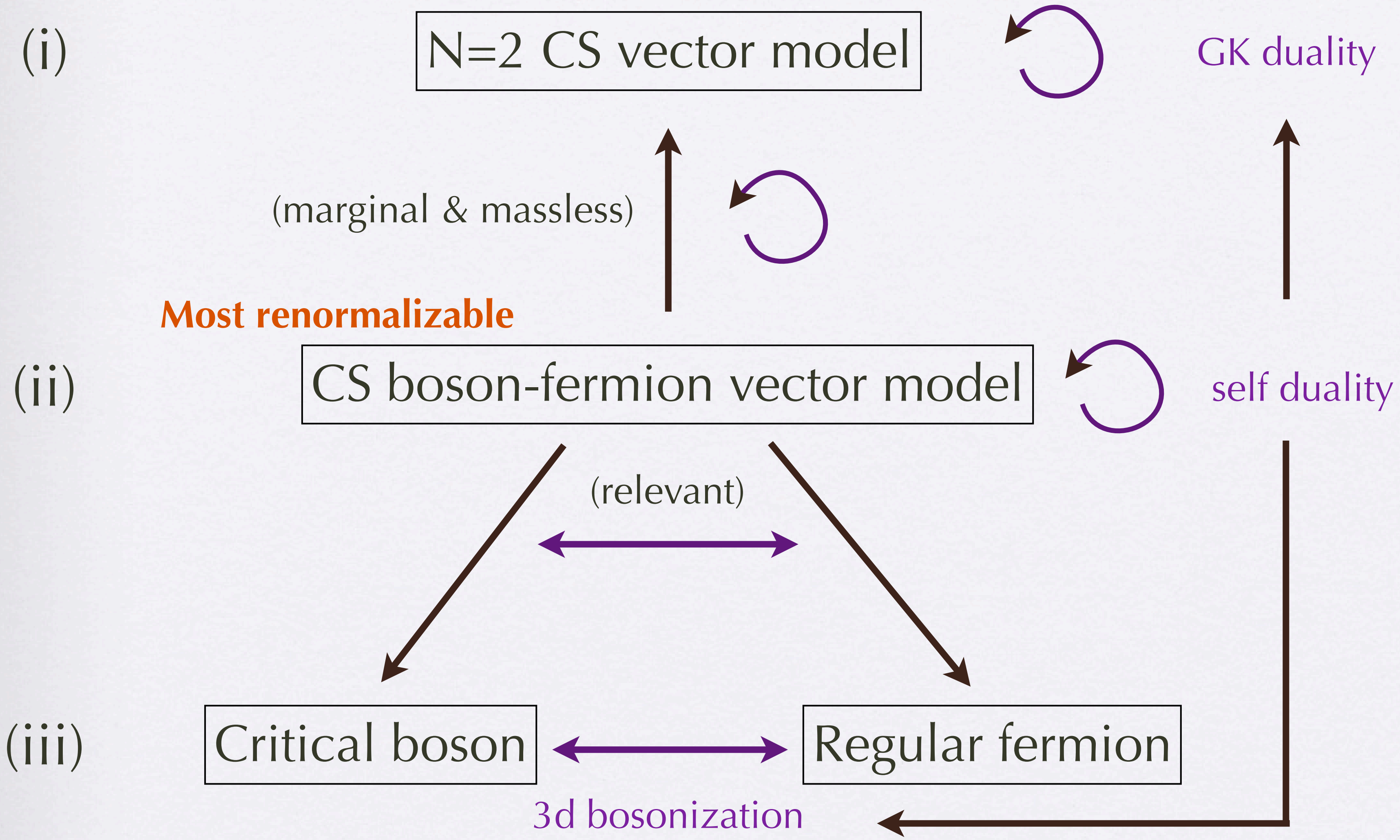
(iii) Critical boson ⟷ Regular fermion

3d bosonization

3d duality & deformation

O.Aharony_S.Giombi_G.Gur-Ari_J.Maldacena_R.Yacoby. (arXiv:1210.4109)

S.Jain_S.Minwalla_SY arXiv:1305.7235



(i) N=2 CS vector model

GK duality

(marginal & massless)

Most renormalizable

(ii) CS boson-fermion vector model

self duality

(relevant)

(iii) Critical boson

Regular fermion

3d bosonization

Summary

- CS vector models are solvable in the 't Hooft limit with euclidean light-cone gauge.

- CS vector models have SUSY (Giveon-Kutasov) and non-SUSY (3d bosonization) duality.

- Strong evidence for these dualities has been provided by thermal free energy by taking account of fermionic holonomy distribution.

- New phase appeared due to fermionic holonomy distribution.

- SUSY and non-SUSY duality have been connected by RG-flow.