Massive fermion in (4+1) dimensional Anti de-Sitter black hole background

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Introduction

Gauge/gravity duality

- The gauge/gravity duality is the theory that a strongly coupled gauge theory in d dimensions corresponds to a classical gravity theory in (d-2) dimensions.
- For example, the dual theory of superfluidity in (3+1) dimensions is Anti de-Sitter (AdS) black hole with Abelian Higgs hair in (4+1) dimensions.

Background

- Black hole with Abelian Higgs hair

\[ S = \int d^4x \sqrt{-\text{det}(g)} \left( \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{2} m^2 A^2 - \frac{1}{4} \lambda A^4 \right) + \frac{1}{2} R - \frac{1}{4} \alpha' R^2 \]

- (4+1) dimensional black hole metric of polar coordinates

\[ ds^2 = -e^{2\rho} dt^2 + e^{-2\rho} dr^2 + r^2 d\Omega_4^2 \]

- The ansatz and boundary condition

\[ A = \sum a_n e^{i\pi x^4} e^{i\pi y^4} \]

The numerical solutions

- The behavior near horizon

\[ r \rightarrow 0 \Rightarrow A \sim e^{-\pi r} \]

- The behavior on the AdS boundary \( r \rightarrow \infty \)

\[ A \sim \frac{1}{r^{\alpha}} \]

The numerical method

- We solve the Dirac equation as the eigenproblem.
- Diagonalization method

- Thus, we need to find the complete basis \( \{ \phi_n \} \).

The scheme

The action of the fermion

- The action of the fermion:

\[ \mathcal{L} = \bar{\psi} i\gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi + \frac{1}{4} \lambda (\bar{\psi} \psi)^2 \]

The gamma matrices and the vielbein

- The gamma matrices in tangent space

\[ \gamma^\mu = \gamma^\mu_{\text{tan}} \]

- The vielbein defined as

\[ e^\mu_{\text{vec}} = g^{\mu \nu} \frac{\partial}{\partial x^\nu} \]

We numerically solve the Dirac equation for the fixed background.

The basis set

1. The symmetry

- The symmetry of the metric in (4+1) dimensions.

SO(4) \xrightarrow{\alpha} SU(2) \times SU(2)

- The metric written by 1-form

\[ ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_4^2 \]

- The Killing vectors

\[ \xi^\mu = (\xi^t, \xi^r, 0, 0, 0) \]

- We can define two angular momenta

\[ L_z, L_z \]

- The commutation relations

\[ [L_z, L_z] = i\hbar \{L_z^2, L_z^2\} \]

2. Wigner D function

- The special case

\[ \mathbf{D}^{(\pm)}_{\lambda}(x,y) \]

- Eigenvalues of \( \mathbf{D}^{(\pm)}_{\lambda}(x,y) \) are the same

- The eigenfunction of \( \xi^z, \xi^r, \xi^z \): Wigner D function

- The eigenstate

\[ | \alpha, \beta \rangle \]

3. The form of Dirac spinor

- The plane-wave basis

\[ | \alpha, \beta \rangle \]

- The Killing vectors

\[ \xi^\mu = (\xi^t, \xi^r, 0, 0, 0) \]

4. The angular basis

- SU(2) Clebsch-Gordan coefficient

\[ c_{\lambda\mu}(\lambda_1, \lambda_2, \lambda_3) \]

- The angular basis

\[ | \alpha, \beta \rangle = \sum c_{\lambda\mu}(\lambda_1, \lambda_2, \lambda_3) | \lambda, \mu \rangle \]

Results

Numerical result 1

- Additional boundary condition for numerical calculation

\[ \partial_\mu (A_\mu x) = 0 \]

- The density \( \rho \)

\[ \rho = 0.25 \]

Numerical result 2

Conclusions and future outlooks

- Our Yukawa coupling model

\[ \mathcal{L}_Y = \bar{\psi} (\partial_\mu - m_j) \psi \frac{1}{2} \square \phi + g \bar{\psi} \psi \phi \]

- can identify as a semi bosonized Gross-Neveu model

- In term of a bosonization \( \xi \rightarrow \xi + \phi \) and letting \( \phi \rightarrow \phi + \phi \)

- From this, we maybe discuss a mechanism of the condensation.

- We apply the basis to the rotating black hole in (4+1) dimensions.