Entanglement of Local Operators in large N CFTs

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with M.Nozaki and T.Takayanagi
arXiv:1405.5946[hep-th]

Outline

- Motivation
- Renyi entropies for excited states
- Large N and large c
- Holographic analysis
- Summary/Future directions

AdS/CFT ("early")

[Maldacena]
[Witten,GKP….]

Physics of local operators in CFT ⟷ Physics of Strings on AdS

$$\langle \mathcal{O}(x_1)\cdots\mathcal{O}(x_n)\rangle = \frac{\delta}{\delta\phi_0(x_1)}\cdots\frac{\delta}{\delta\phi_0(x_1)} Z_{\text{Gravity}}[\phi_0(x)]$$

Lots of developments: 2 & 3-pt functions, Integrability

Well understood and explored…

AdS/CFT ("modern")

Entanglement ⟷ Spacetime Geometry

$$S_A = \frac{\mathrm{Area}(\gamma_A^d)}{4G_N^{d+2}}$$

[Ryu,Takayanagi'06]

Holography, quantum many-body physics, QFT, information theory….

Time dependent phenomena, thermalization,
global and local quench

Universal Laws of entanglement?
Entanglement thermodynamics?

This Talk:

[intro: M.Nozaki's]

Entangled look on early holography

(Can we learn anything new about local operators in CFTs from the perspective of entanglement ?)

Universal physics of excitations created by a local operator acting on the ground state
(milder version of a local quench)

$$|\psi\rangle = O(x)\,|0\rangle$$

Main Tool:

Relative Renyi entropies for states excited by local operators

$$\begin{aligned}\rho(t) &= \mathcal{N}\cdot e^{-iHt}e^{-\epsilon H}O(0,x^i)|0\rangle\langle 0|O^\dagger(0,x^i)e^{-\epsilon H}e^{iHt} \\ &= \mathcal{N}\cdot O(w_2,\bar{w}_2,\mathbf{x})|0\rangle\langle 0|O^\dagger(w_1,\bar{w}_1,\mathbf{x}),\end{aligned}$$

Reduced $\rho_A = \mathrm{Tr}_B(\rho)$

$$\Delta S_A^{(n)} = \frac{1}{1-n}\log\left(\frac{\mathrm{Tr}(\rho_A^n)}{\mathrm{Tr}(\rho_A^{(0)})^n}\right) = \frac{1}{1-n}\log\left[\frac{\langle O(w_1,\bar{w}_1)O^\dagger(w_2,\bar{w}_2)...O(w_{2n},\bar{w}_{2n})\rangle_{\Sigma_n}}{(\langle O(w_1,\bar{w}_1)O^\dagger(w_2,\bar{w}_2)\rangle_{\Sigma_1})^n}\right]$$

Connection with AdS/CFT: Large N or large c

S

[He,Numasawa,Takayanagi,Watanabe'14]

(n=2)

$$\frac{\langle O(w_1,\bar{w}_1)O(w_2,\bar{w}_2)O(w_3,\bar{w}_3)O(w_4,\bar{w}_4)\rangle_{\Sigma_2}}{\left(\langle O(w_1,\bar{w}_1)O(w_2,\bar{w}_2)\rangle_{\Sigma_1}\right)^2} = |z|^{2\Delta_O}|1-z|^{2\Delta_0}G_O(z,\bar{z})$$

At late time $(z,\bar{z}) \to (1,0)$

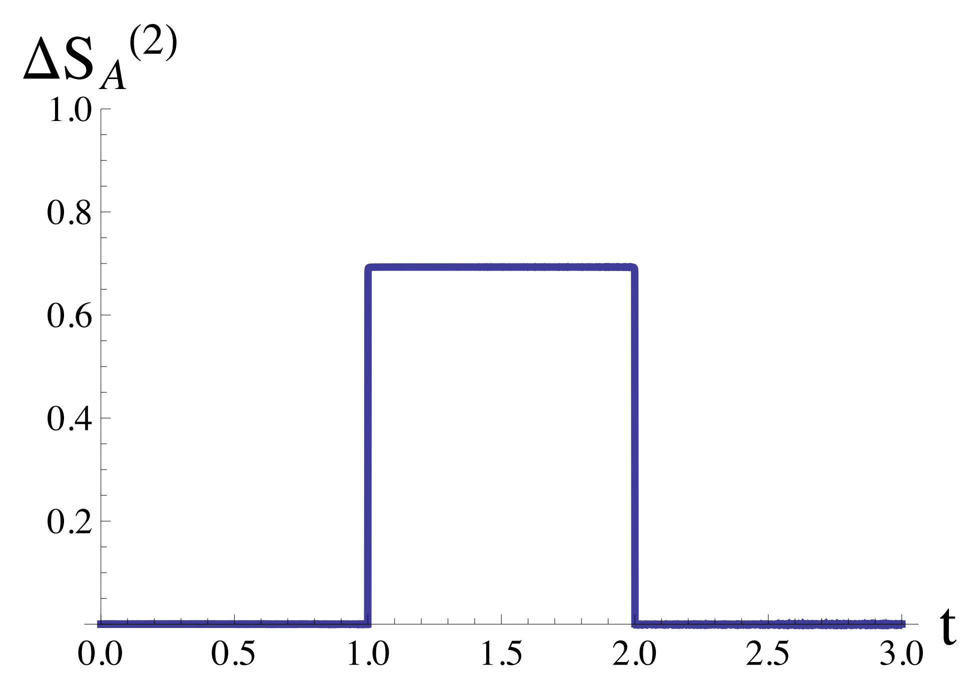
In rational CFTs

$$G(z,\bar{z}) \simeq F_{00}[O]\cdot(1-z)^{-2\Delta_O}\,\bar{z}^{-2\Delta_O}$$

$$\Delta S_A^{(2)} = -\log F_{00}[O] = \log d_O$$

$$d_\Delta = \frac{S_{0\Delta}}{S_{00}}$$ quantum dimension

"EPR pair propagating through the system"



2d CFTs at large c

[PC,M.Nozaki,T.Takayanagi14]

Conformal block expansion

$$G(z,\bar{z}) = \sum_b (C^b_{OO^\dagger})^2 F_O(b|z)\bar{F}_O(b|\bar{z})$$

at large central charge c

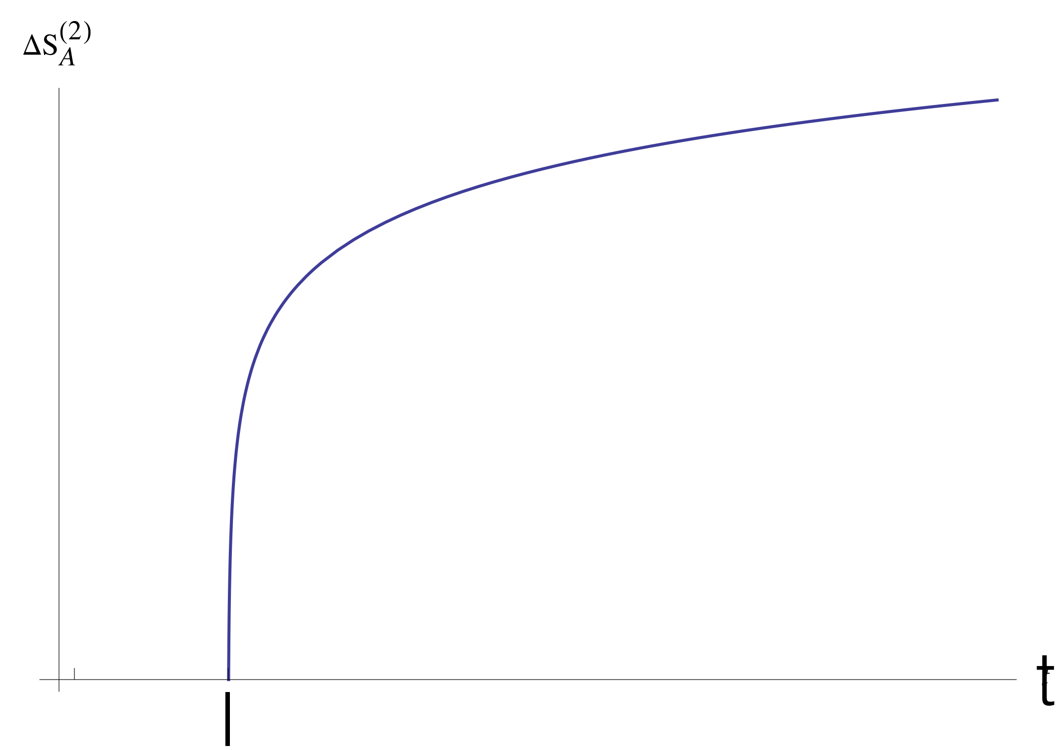
[Fateev,Ribault'11]

$$F_O(b|z) \simeq z^{\Delta_b - 2\Delta_O} \cdot {}_2F_1(\Delta_b, \Delta_b, 2\Delta_b, z)$$

at late time

$$\Delta S_A^{(2)} \simeq 4\Delta_O \cdot \log \frac{2t}{\epsilon}$$

similar to a local quench



CFTs and large N

Operator: $$Tr(\mathcal{Z}^J) = Tr(\phi_1 + i\phi_2)^J$$

e.g. J=2 (Free Field)

$$\Delta S_R^{(n)} = \frac{1}{1-n}\log\left(2^{1-2n} + \frac{1}{2^n N^{2(n-1)}}\right)$$

!!

n $\geq$ 2

$$\Delta S_A^{(n)} = \frac{Jn-1}{n-1}\log 2$$

(n=1) von-Neuman entropy

$$\Delta S_R^{(1)} = \log\left(2\sqrt{2}N\right)$$

N^2 d.o.f

Ground State

Both scale as c~N^2

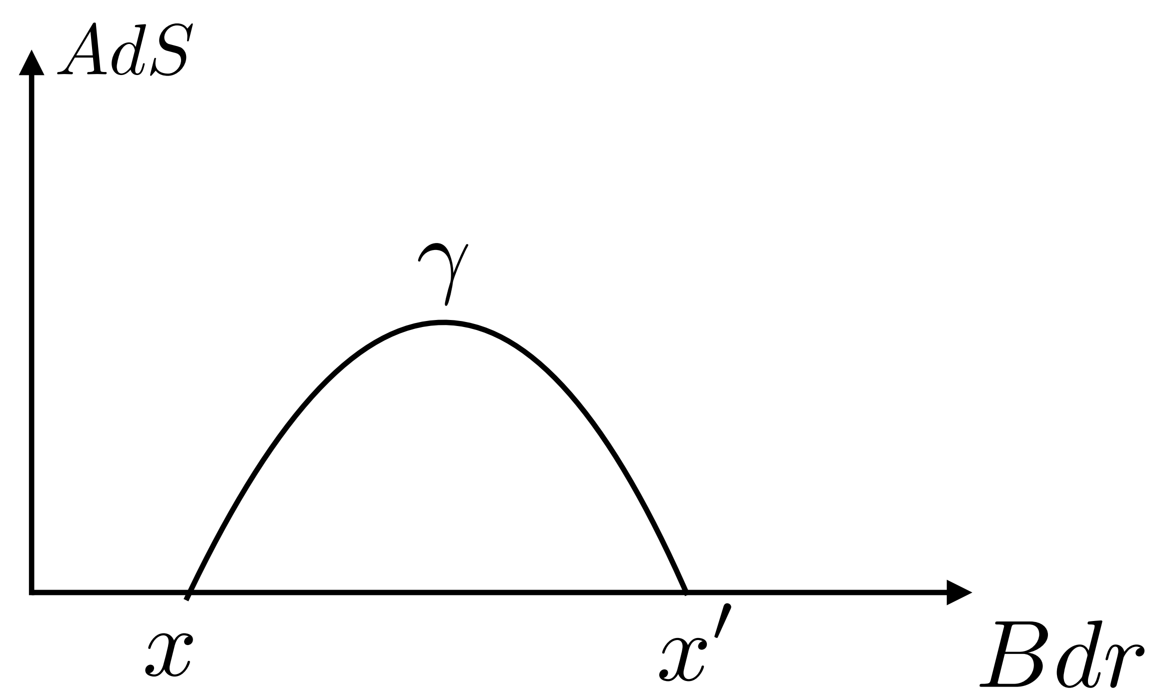
Holographic Checks

Geodesics and propagators

$$\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle = \frac{\delta}{\delta \phi_0(x_1)} \cdots \frac{\delta}{\delta \phi_0(x_1)} Z_{\text{Gravity}}[\phi_0(x)]$$

for operators with "large" conformal dimension (semiclassical)

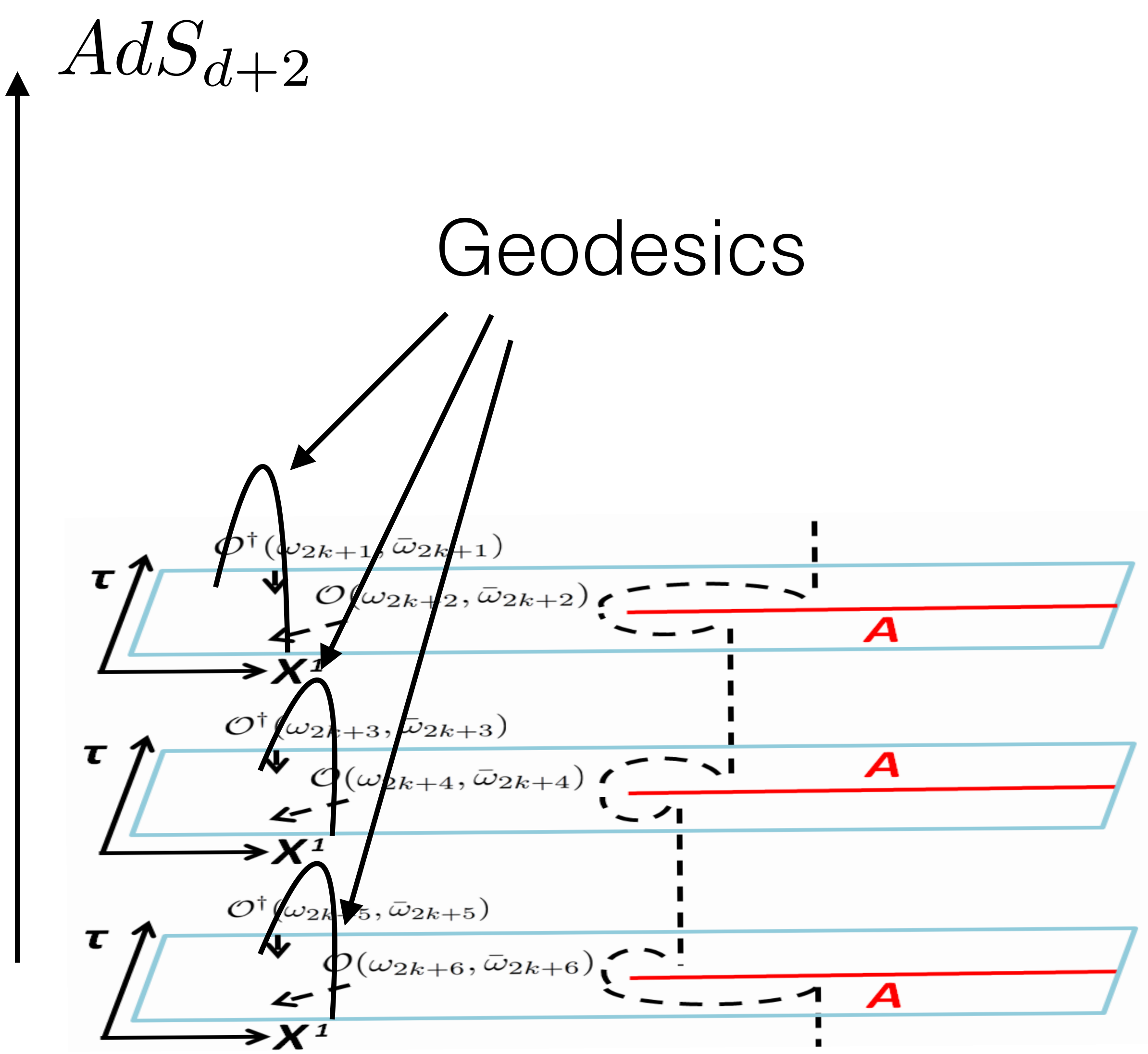
$$\langle \Phi_\Delta(x) \Phi_\Delta(x') \rangle \sim e^{-\frac{\Delta}{R} L(\gamma)}$$



Geodesics in topological Black Holes

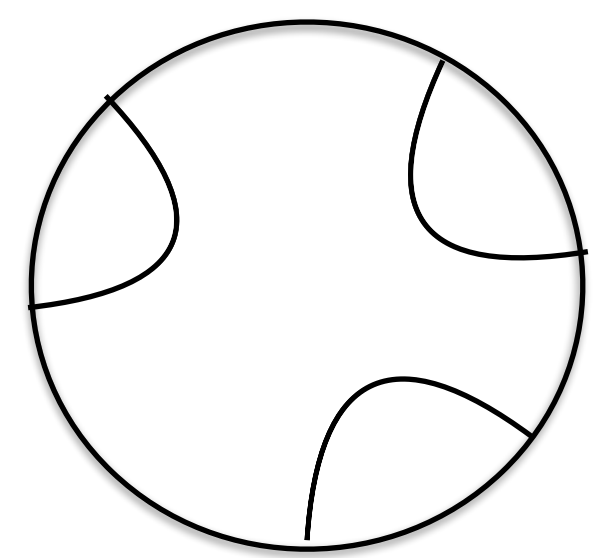
[Emparan'99]

horizon H_d^n



H_d^n

Witten Diagram



Geodesics in topological Black Holes

$$ds^2 = f(r)d\tau^2 + \frac{dr^2}{f(r)} + r^2 d\phi^2 + r^2 e^{-2\phi} dx_i^2$$

$$f(r) = -1 - \frac{\mu}{r^{d-2}} + \frac{r^2}{R^2}$$

Temperature and "n"

$$\beta = 2\pi n R$$

holographic ratio of the propagators (eg d=2)

$$e^{-\frac{2\Delta_O}{R}\left(L^{(n)} - L^{(1)}\right)} = \left(\frac{\cosh\left(\Delta\phi\right) - \cos\left(\frac{\Delta\tau}{R}\right)}{n^2\left(\cosh\left(\frac{\Delta\phi}{n}\right) - \cos\left(\frac{\Delta\tau}{n\,R}\right)\right)}\right)^{2\Delta_O}$$

perfectly matches the 2d CFT result

Results $n \geq 2$

AdS_3

$$\Delta S_A^{(n)} \simeq \frac{2n\Delta_O}{n-1} \log\left(\frac{n \sin\left(\frac{\pi}{n}\right) t}{\epsilon}\right) - \frac{1}{n-1} \log 2$$

higher d at late time

$$\Delta S_A^{(n)} \simeq \frac{4n\Delta}{d(n-1)} \log\left(\frac{F_{(d,n)}\, t}{2\,\epsilon}\right) + C_{(n,d)} - \frac{1}{n-1} \log 2$$

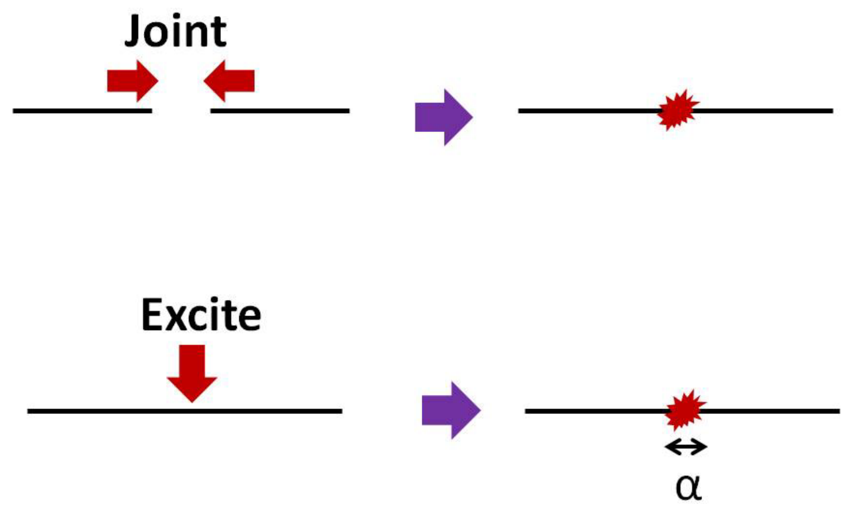
n->1

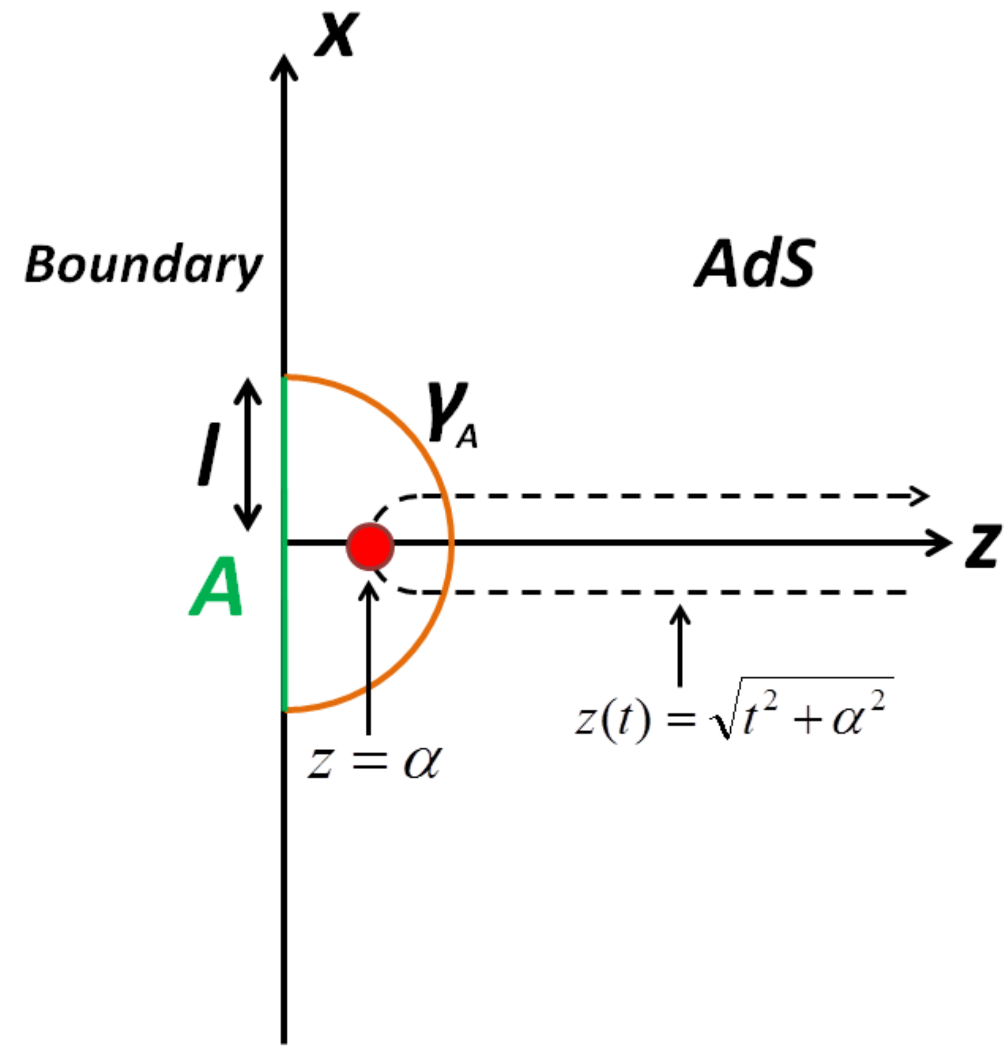
n->1 from holography?

Falling particle in AdS (n=1)

[Nozaki,Numasawa,Takayanagi'13]

CFT (chain)





In our setup: $\alpha \equiv \epsilon$

$$\Delta S_A^{(1)} = 0 \qquad (t < l)$$

$$\Delta S_A^{(1)} \simeq \frac{c}{6} \log \frac{t}{\epsilon} + \frac{c}{6} \log \frac{\Delta}{c} \qquad (t \gg l)$$

General picture (summary)

rational CFT

$$D_n = (d_O)^{n-1}$$

free field (large N)

$$D_n = 2^{Jn-1} + O(N^{-2})$$

$$\Delta S_A^{(n)} = -\frac{1}{n-1}\log\frac{\langle O^\dagger O\cdots O^\dagger O\rangle_{\Sigma_n}}{(\langle O^\dagger O\rangle_{\Sigma_1})^n}$$
$$\simeq -\frac{1}{n-1}\log\left[\frac{1}{D_n} + \mu_n\cdot\left(\frac{\epsilon}{t}\right)^{\nu_n}\right]$$

Large c
(strongly interacting)

$$\Delta S_A^{(n)} \simeq \frac{\nu_n}{n-1}\log\frac{t}{\epsilon}$$

$$\nu_n \simeq \frac{4n\Delta}{d} + O\left(\frac{\Delta^2}{c}\right)$$

$$D_n \sim e^{b_n\cdot N^{a_n}}$$

$$\lim_{n\to 1}\frac{\nu_n}{n-1} \simeq c \quad ?$$

Our Results:

- von-Neuman (n=1) and Renyi entropies behave very differently
- Naive large N limit breaks down for the Entanglement Entropy
- Universal scaling of the Renyi entropy for excited states at large c
- Holographic analysis consistent with large c and large N analysis…

Thank You