Glueball instability and thermalization driven by dark radiation

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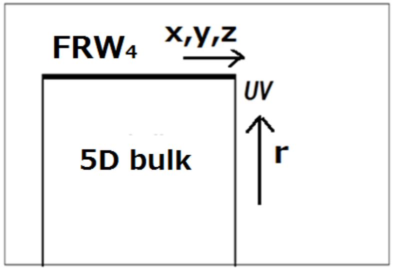
Introduction

We consider 5-dimensional bulk whose 4D boundary at $r \to \infty$ (UV) has **Friedmann-Robertson-Walker ($\boldsymbol{FRW_4}$)** metric with negative cosmological constant ($-\boldsymbol{\lambda}$.)



There also appears a parameter $\boldsymbol{C}$ (or $\boldsymbol{c_0}$) as an integration constant of Einstein Equation for 5D bulk. ***C*** corresponds to the energy density of the 4D boundary theory.

We consider the effect of $\boldsymbol{\lambda}$ and ***C*** to the 4D boundary theory by studying the, glueball spectrum, Entanglement Entropy by using the holography of the 5D bulk.

We find that there is an phase transition of 4D boundary theory at critical ***C.***

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5D bulk with FRW_4 boundary

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Entanglement Entropy

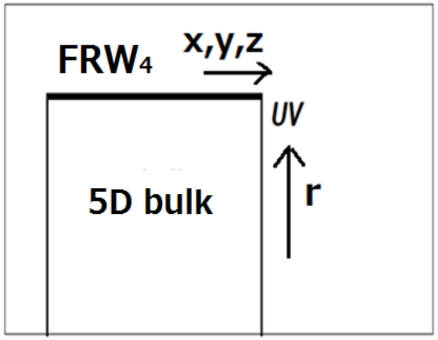
Summary

5D bulk

5D bulk metric is obtained in the following ansatz,

$$ds_5^2 = \frac{r^2}{R^2}\left(-n(r,t)^2 dt^2 + A(r,t)^2\, a_0^2(t)\gamma_{ij}(x)dx^i dx^j\right) + \frac{R^2}{r^2}dr^2$$

R: constant (AdS_5 radius)



We will find $n(r,t)$ and $A(r,t)$ which satisfies

$$n(r,t) \to 1, \quad A(r,t) \to 1 \quad \text{for } r \to \infty,$$

The ***4D*** boundary ($r \to \infty$) metric becomes FRW_4 metric.

$$ds_4^2 = -dt^2 + a_0^2(t)\gamma_{ij}(x)dx^i dx^j$$

$a_0(t)$: scale factor

$$\gamma_{ij}(x) = \delta_{ij}\left(1 + \frac{k}{4}\Sigma_{i=1}^3 \left(x^i\right)^2\right)^{-2} \qquad (k = 0, \ \pm 1)$$

We consider $k = -1$ **(negative curvature) case**

Einstein Equation for 5D bulk

$a_0(t)$, $A(r,t)$ and $n(r,t)$ are determined by 5D Einstein Equation.

$$R_{MN} = -\Lambda g_{MN} \quad (M, N = 0 \cdots 5) \qquad \left(\Lambda = \frac{4}{\mathrm{R}^2}\right)$$

With the following ansatz, $\frac{1}{\dot{a}_0(t)} \frac{d(a_0(t)A(r,t))}{dt} = \frac{r}{R} n(r,t)$ *(P. binetruy, et al. 2000) (D. Langlois' 2003)*

the 5D Einstein Equation becomes

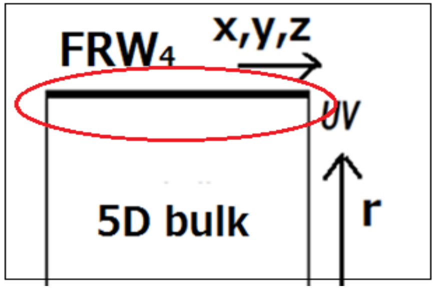
$$\left(\frac{\dot{a}_0(t)}{a_0(t)}\right)^2 - \frac{1}{a_0^2(t)} = -\frac{r^2}{R^4} A(r,t)^2 + \left(\frac{r}{R}\left(\frac{r}{R} A(r,t)\right)'\right)^2 + \frac{CR^2}{a_0^4(t) r^2 A^2(r,t)}$$

$\left(\bullet = \frac{\partial}{\partial t} \text{ and } ' = \frac{\partial}{\partial r} \right)$

C is given as an integral constant and it is called " holographic dark radiation" .

Friedman equation

Furthermore, we impose the Friedman equation for boundary $\boldsymbol{FRW_4}$



$$ds_4^2 = -dt^2 + a_0^2(t)\gamma_{ij}(x)dx^i dx^j$$

$$\left(\frac{\dot{a}_0(t)}{a_0(t)}\right)^2 - \frac{1}{a_0^2(t)} = -\lambda(t) \qquad \lambda(>0) : \text{a cosmological constant}$$

We assume that time evolution of $\lambda(t)$ and $a_0(t)$ is very slow.

$$\dot{\lambda}(t) \sim 0, \qquad \frac{\dot{a}_0(t)}{a_0(t)} \sim 0$$

5D bulk metric

Then, we can get the $\boldsymbol{A(r,t)}$ and $\boldsymbol{n(r,t)}$ as follows

$$\boldsymbol{A} = \left(\left(1+\left(\frac{r_0}{r}\right)^2\right)^2 + c_0\left(\frac{R}{r}\right)^4\right)^{1/2} \qquad \boldsymbol{n} = \frac{\left(\left(1+\left(\frac{r_0}{r}\right)^2\right)^2 - c_0\left(\frac{R}{r}\right)^4\right)}{A}$$

$c_0 \equiv \mathrm{CR}^2/(4a_0^4)$ **: energy density of dual 4D Yang-Mills theory**

$r_0 \equiv \frac{R^2}{2}\sqrt{\lambda}$ **: cosmological constant of boundary 4d space-time.**

(K.Ghoroku and A. Nakamura 2012)

We will use $\boldsymbol{c_0}$ and $\boldsymbol{r_0}$ instead of $\boldsymbol{C}$ and $\boldsymbol{\lambda}$.

Since we assume $\dot{\lambda} \sim 0$ and $\frac{\dot{a}_0(t)}{a_0(t)} \sim 0$

$\boldsymbol{c_0, r_0 \sim}$ **constant**

5D bulk ($c_0 \geq 0$, $r_0 = 0$)

When $\boldsymbol{r_0 = 0}$ (4D flat boundary metric) and $\boldsymbol{c_0 \geq 0}$, 5D bulk metric becomes

$$ds_5^2 = \frac{r^2}{R^2}\left(-\frac{\left(1-c_0\left(\frac{R}{r}\right)^4\right)^2}{1+c_0\left(\frac{R}{r}\right)^4}dt^2 + \left(1 + c_0\left(\frac{R}{r}\right)^4\right)dx^i dx^i\right) + \frac{R^2}{r^2}dr^2$$

By changing coordinates as $\tilde{r} = r\sqrt{1 + \frac{R^4}{r^4}c_0}$

The above metric becomes **5D AdS-Schwarzschild black hole**

$$ds_5^2 = \frac{\tilde{r}^2}{R^2}\left(-f(\tilde{r})dt^2 + \left(dx^i\right)^2\right) + \frac{R^2 d\tilde{r}^2}{\tilde{r}^2 f(\tilde{r})} \qquad f(r) = 1 - \frac{4R^4 c_0}{\tilde{r}^4}$$

Thus c_0 cotributes to the temperature.

5D bulk $(c_0 \geq 0,\ r_0 \geq 0)$

5D bulk solution becomes

$$ds_5^2 = \frac{r^2}{R^2}\left(-n(r)^2 dt^2 + A(r)^2 a_0^2(t)\gamma_{ij}(x)dx^i dx^j\right) + \frac{R^2}{r^2}dr^2$$

$$A(r) = \left(\left(1 + \left(\frac{r_0}{r}\right)^2\right)^2 + c_0\left(\frac{R}{r}\right)^4\right)^{1/2} \qquad n(r) = \frac{\left(\left(1+\left(\frac{r_0}{r}\right)^2\right)^2 - c_0\left(\frac{R}{r}\right)^4\right)}{A}$$

At $r = r_H \equiv \left(\sqrt{c_0}R^2 - r_0^2\right)^{1/2}, \quad g_{tt} \propto n(r_H) = 0.$

When $c_0 > \frac{r_0^4}{R^4}$, there is an **"event horizon"** at $r = r_H$ and the Hawking temperature T_H is given by

$$T_H = \frac{r_H\left(1 + \frac{r_0^2 + \sqrt{c_0}R^2}{r_H^2}\right)}{\pi R^2 A(r_H)}$$

Hawking Temperature

Hawking temperature $\boldsymbol{T_H}$

$$T_H = \frac{r_H\left(1+\frac{r_0^2+\sqrt{c_0}R^2}{r_H^2}\right)}{\pi R^2 A(r_H)} \qquad r_H \equiv \left(\sqrt{c_0}R^2 - r_0^2\right)^{1/2}$$

As c_0 (energy density) becomes large, T_H increases.

As r_0 (cosmological constant) becomes large, T_H decreases.

At $c_0 = r_0^4/R^4, \quad T_H = 0$.

Thus at $c_0 = \frac{r_0}{R^4}$, there is a phase transition between confinement phase and deconfinement phase.

5D bulk $(c_0 \geq 0, \mathrm{r}_0 \geq 0)$

When $\mathbf{0} \leq c_0 \leq \frac{r_0^4}{R^4}$, $r_H \equiv \left(\sqrt{c_0}R^2 - r_0^2\right)^{1/2}$ is not a real number and there is no "event horizon"

$\leftrightarrow$ Dual 4D field theory is in the **"confinement phase".**

Stable Glueball spectrum by the 5D bulk metric fluctuation

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Glueball mass spectrum

Glueball spectrum can be obtained by the fluctuation $\boldsymbol{h_{ij}(t,\ x^i, r)}$ of the 5D bulk ($\boldsymbol{g_{MN}}$). *(R.C. Brower, S.D.Mathur and C.I. Tan. 2003)*

$$\frac{1}{\sqrt{-g}}\partial_M\left(\sqrt{-g}\,g^{MN}\partial_N h_{ij}\right) = 0$$

By decomposing $\quad \boldsymbol{h_{ij}(x^\mu, r) = p_{ij}\chi(x^\mu)\phi(r)}$

The equation of 4D part $\boldsymbol{\chi(x^\mu)}$ is given by

$$\frac{1}{g_4}\partial_\mu\sqrt{g_4}\,g^{\mu\nu}\partial_\nu\chi(x^\mu) = m^2\chi(x^\mu)$$

$\boldsymbol{m}$: Glueball mass

Glueball mass spectrum

Equation for $\boldsymbol{\phi}(\boldsymbol{r})$ becomes

$$\partial_r^2\phi + g_2(r)\partial_r\phi + \left(\frac{R}{r}\right)^4 \frac{m^2}{n(r)^2}\phi(r) = 0$$

$$\bar{g}_2(r) = \partial_r\left(log\left[\left(\frac{r}{R}\right)^5 n(r)A(r)^3\right]\right)$$

$\boldsymbol{m}$: **glueball mass**

Glueball mass spectrum ($r_0 \geq 0, \quad c_0 = 0$)

First we consider the $c_0 = 0$ case (analytic calculation).

By defining $x \equiv \frac{r}{r_0}$, equation for $\phi(r)$ is given by

$$\partial_x^2 \phi + g_2(x) \partial_x \phi + \frac{R^4 m^2}{r_0^2 x^4 A^2(x)} \phi = 0$$

Where $g_2(x) = \frac{1}{x}\left(5 - \frac{8}{x^2 A(x)}\right) \qquad A(x) = 1 + \frac{1}{x^2}$

ϕ becomes normalizable by choosing m as

$$m^2 = -\lambda(N+1)(N+4) \quad \lambda = \frac{4 r_0^2}{R^4} \qquad N = 0, 1, 2 \cdots$$

The lowest glueball mass (***N=0***) is finite as

$$m_g = 2\sqrt{\lambda}$$

This was obtained by the field theory by ***C.Fronsdal, 1979***

Glueball mass spectrum $(r_0 \geq 0, \quad c_0 \geq 0)$

Next we consider the $\boldsymbol{r_0 \geq 0}$, and $\boldsymbol{c_0 \geq 0}$ case and will show that the lowest glueball mass $\boldsymbol{m_g}$ decreases as $\boldsymbol{c_0}$ becomes large.

equation for $\boldsymbol{\phi(r)}$

$$\boldsymbol{\partial_r^2 \phi + g_2(r) \partial_r \phi + \left(\frac{R}{r}\right)^4 \frac{m^2}{n(r)^2} \phi(r) = 0}$$

$$\boldsymbol{\bar{g}_2(r) = \partial_r \left(log \left[\left(\frac{r}{R}\right)^5 n(r) A(r)^3 \right] \right)}$$

Glueball mass spectrum ($c_0 > 0$)

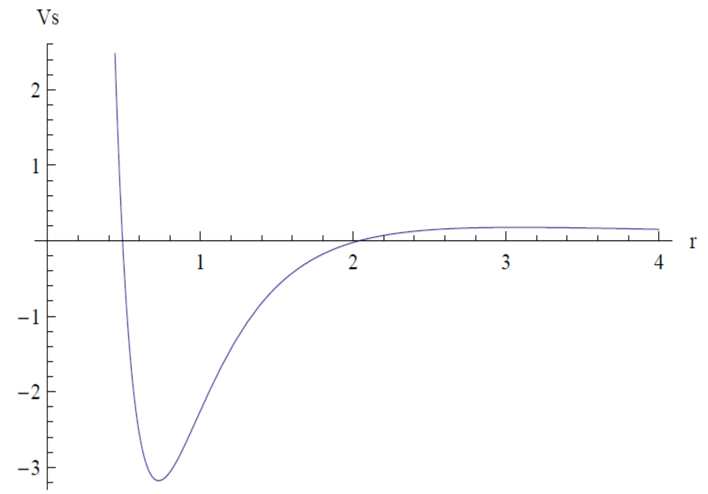
By factorizing **ϕ as** $$\phi = e^{-\frac{1}{2}\int dr\bar{g}_2(r)} f(r)$$

The equation for $f(r)$ becomes the Schrodinger equation

$$-\partial_r^2 f + V(r) f = 0$$

with the potential ***V(r)***

$$V = \frac{1}{4}\bar{g}_2^2 + \frac{1}{2}\partial_r\bar{g}_2 - \frac{m^2}{n^2}\left(\frac{R}{r}\right)^4$$

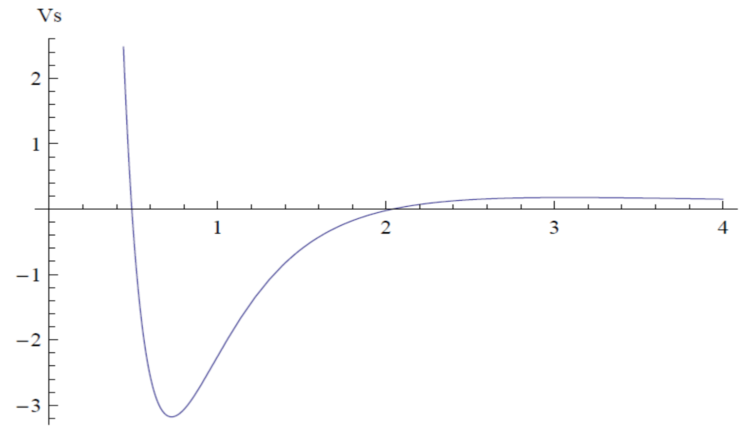


Glueball mass spectrum ($c_0 > 0$)

WKB approximation gives

$$\int \sqrt{-V}\,dr = \left(N+\frac{1}{2}\right)\pi \quad N = 0,1,2\cdots$$

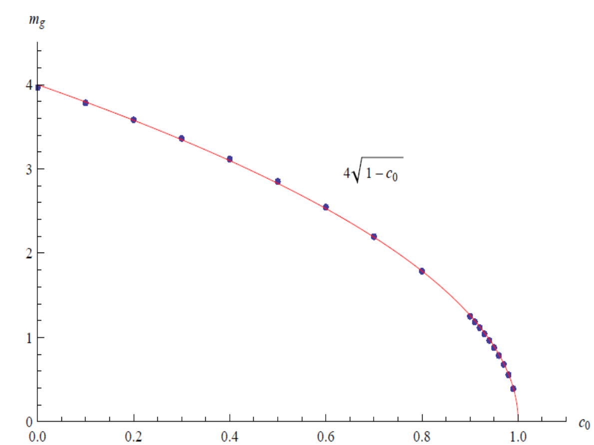
$$V = \frac{1}{4}\bar{g}_2^2 + \frac{1}{2}\partial_r \bar{g}_2 - \frac{m^2}{\bar{n}^2}\left(\frac{R}{r}\right)^4$$



The lowest glueball mass m_g is given when ***N=0*** in the above formula. The relation between $\boldsymbol{m_g}$ and $\boldsymbol{c_0}$ is calculated numerically.

For critical $c_0 = \dfrac{r_0^4}{R^4}$,

Lowest glueball mass $\boldsymbol{m_g}$ becomes zero.



Glueball as an rotating closed string

Glueball with large quantum number: a rotating string in the bulk

Bulk metric: $$ds_5^2 = \frac{r^2}{R^2}\left(-n(r)^2 dt^2 + A(r)^2 ds_3^2\right) + \frac{R^2}{r^2} dr^2$$

where $ds_3^2 = \mathrm{a}_0^2(\mathrm{t})\left(\frac{\mathrm{dp}^2}{1+\mathrm{p}^2} + \mathrm{p}^2\left(\mathrm{d}\theta^2 + \sin^2\ \theta\, d\phi^2\right)\right).$

p radial coordinate in 3d space in FRW_4

The string world sheet coordinates are taken as $(\tau, \sigma) = (t, r)$.

The Ansatz of the closed string rotating around S^2 are $\theta = \theta(r)$ and $\phi = \omega t$.

Spin and Energy

Lagrangian of a closed string

$$L = -\frac{1}{2\pi\alpha'}\int dr\,\frac{r^2}{R^2}A^2\sqrt{\left(\frac{n^2}{A^2} - \omega^2 p^2 \sin^2\theta\, a_0^2(t)\right)\left(\theta^2 p^2 a_0(t) + \frac{1}{A^2}\left(\frac{R}{r}\right)^4\right)}$$

Then, we can obtain the spin and Energy as follows.

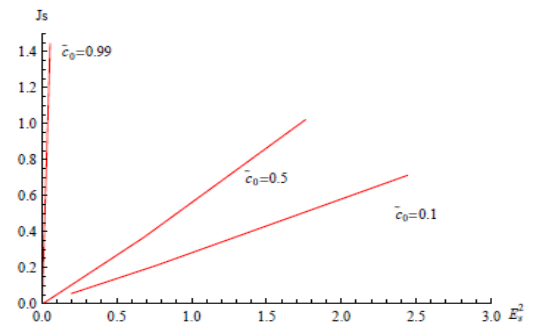
Spin $$J_s = \frac{\partial L}{\partial \omega} = \frac{1}{2\pi\alpha'}\int dr\,\frac{a_0^2 r^2}{R^2}A^2\omega p^2\, sin^2\,\theta\,\sqrt{\frac{\theta^2 p^2 a_0(t) + \frac{1}{A^2}\left(\frac{R}{r}\right)^4}{\frac{n^2}{A^2} - \omega^2 p^2\, sin^2\,\theta a_0^2(t)}}$$

Energy $$E_s = \frac{\omega\partial L}{\partial\omega} - L = \frac{1}{2\pi\alpha'}\int dr\,\frac{r^2}{R^2}n^2\sqrt{\frac{\theta'^2 p^2 a_0(t) + \frac{1}{A^2}\left(\frac{R}{r}\right)^4}{\frac{n^2}{A^2} - \omega^2 p^2\, sin^2\,\theta a_0^2(t)}}$$

Regge Behavior

By solving the equation of motion numerically, we can get a Regge behavior

$$J_s = \alpha_{gluball} E_s^2 \ .$$

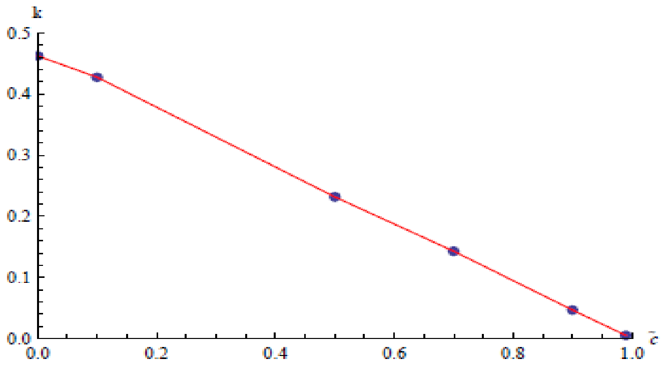


$J_s - E_s$ relation for $r_0 = 1, \ R = 1$

We calculate the relation between String tension $\left(k = \dfrac{1}{8\alpha_{glueball}}\right)$ and c_0 .

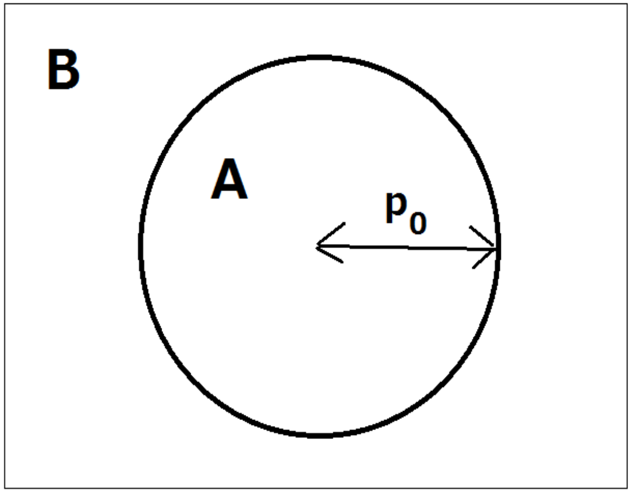
For the critical $c_0 = \dfrac{r_0^4}{R^4}$,

string tension becomes zero.



Entanglement Entropy

We consider the entanglement entropy on the 4D FRW boundary whose entangling region is the 3D sphere with the radius of $\boldsymbol{p_0}$



Entanglement Entropy $\boldsymbol{S_{EE}}$:.

$$\boldsymbol{S_{EE} = -Tr(\rho_A ln\, \rho_A)}$$

$\boldsymbol{\rho_A = Tr_B \rho_{tot}}$: *reduced density matrix*

$\boldsymbol{\rho_{tot}} = |\boldsymbol{\psi}\rangle\langle\boldsymbol{\psi}|$ *:density matrix of the total system*

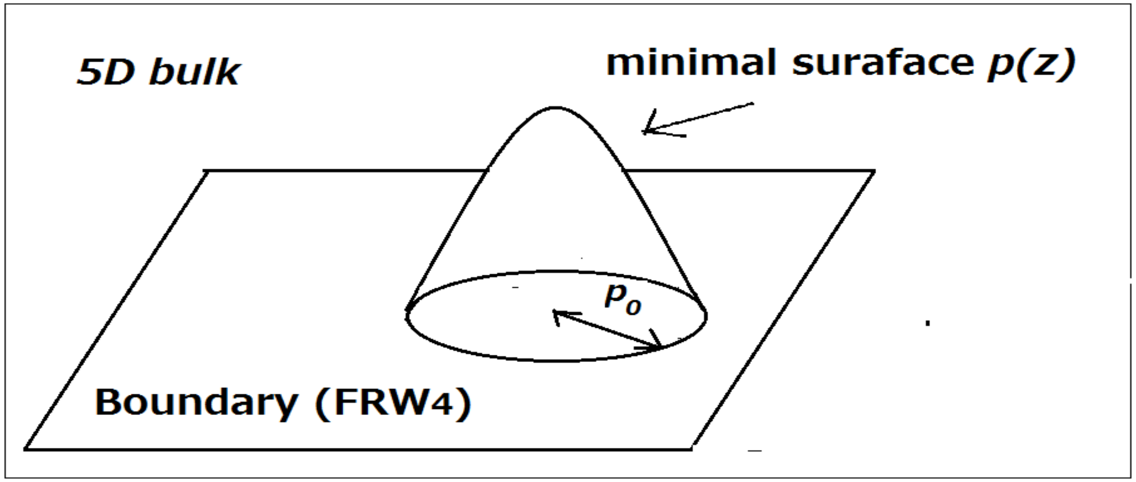
Entanglement Entropy by AdS/CFT

Entanglement Entropy by AdS/CFT *(S. Ryu and T. Takayanagi '06)*

$$S_{EE} = \frac{S_{AREA}}{4G_N^{(5)}}$$

S_{Area} : Area of the minimal surface in ***5D bulk*** space

$G_N^{(5)}$: Newton constant in ***5D bulk***



Entanglement Entropy

By changing coordinates as $z = r_c^2/r$ $\left(r_c \equiv \left(c_0 R^4 + r_0^4\right)^{\frac{1}{4}}\right)$, The spatial part of the bulk metric becomes

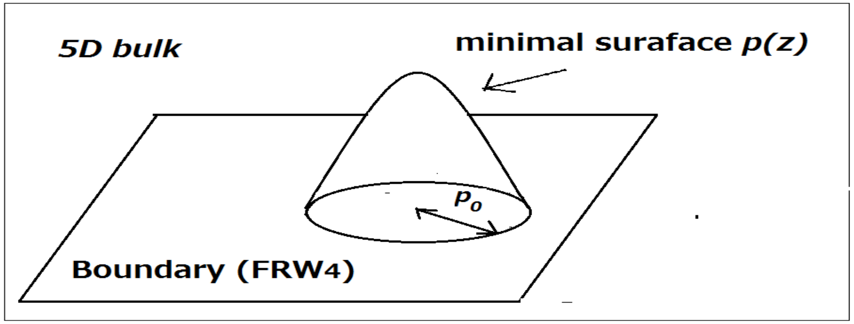
$$ds_{space}^2 = \frac{1}{R^2}\left(z^2 + 2r_0^2 + \frac{r_c^2}{z^2}\right)^2 a_0^2(t)\gamma(p)^2\left(dp^2 + p^2 d\Omega_2^2\right) + \frac{R^2}{z^2}dz^2$$

$$\gamma(p) \equiv \frac{1}{1-\frac{p^2}{4}}$$

$p \equiv \left(\Sigma_{i=1}^3 \left(x^i\right)^2\right)^{\frac{1}{2}}$: the radial coordinate on the 4D boundary (FRW_4)

From the Ryu-Takayanagi conjecture , we consider the minimal surface ***p(z)*** in the bulk.

S. Ryu and T. Takayanagi '06

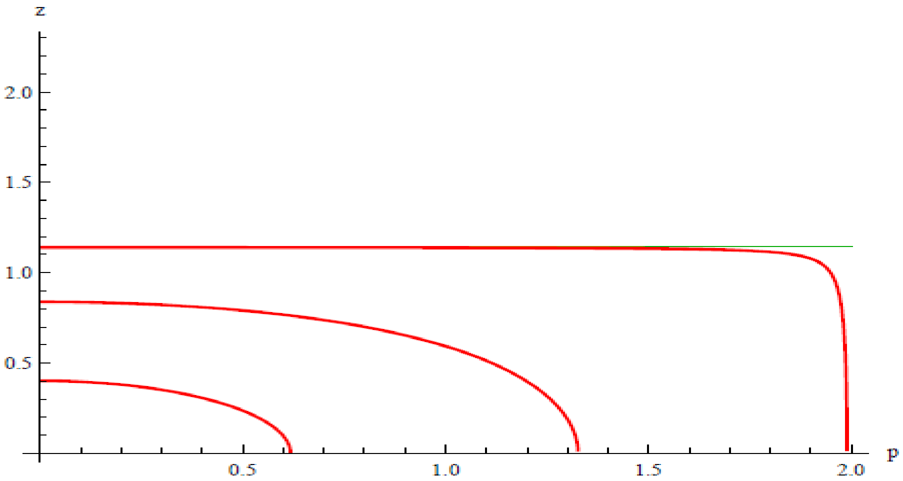


Entanglement enteropy

$$\frac{S_{AREA}}{4\pi} = \int_0^{z(p=0)} p(z)^2 B \sqrt{Bp'(z)^2 + \frac{R^2}{z^2}}$$

$$B \equiv \frac{a_0^2\gamma^2}{R^2}\left(z^2 + \frac{r_c^4}{z^2} + 2r_0^2\right)$$

The minimal surface solution ***p(z)*** is obtained numerically as follows



Entanglement Entropy

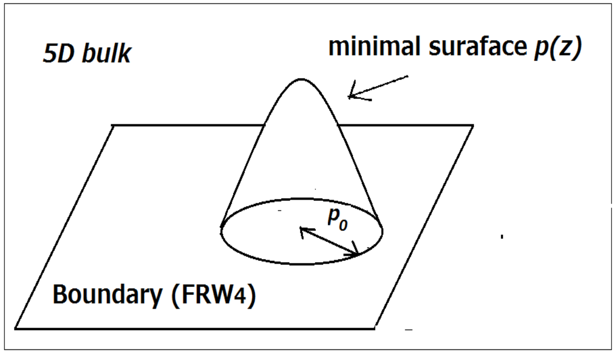
The minimal surface solution ***p(z)*** can be expanded in terms of ***z.***

$$p(z) = p_0 + p_2 z^2 + p_4 z^4 + p_{4L} z^4 \log z + \cdots$$

where $p_2 = -\frac{\left(1-(p_0^2/4)^2\right)R^4}{2a_0^2 p_0 r_c^2}$, $\qquad p_{4L} = -\frac{\left(1-\left(\frac{p_0^2}{4}\right)^2\right)R^8}{4a_0^2 p_0 r_c^8}\left(\frac{\dot{a}_0}{a_0}\right)^2$

p_0: **size of the entangling surface**

p_4 **: arbitrary constants**



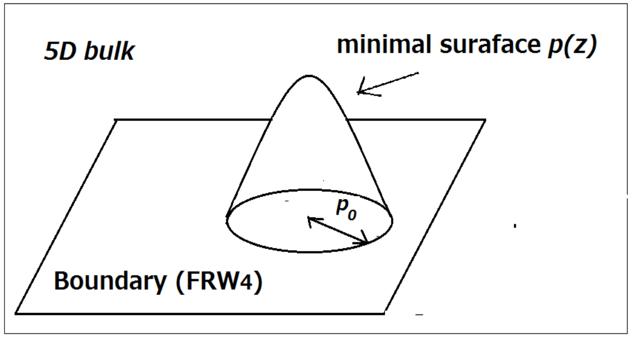
For $\frac{\dot{a}_0(t)}{a_0(t)} \sim 0$, the coefficient of ***log*** term $p_{4L} \sim 0$

Entanglement Entropy

Then, we can get the Entanglement Entropy S_{EE} as

$$S_{EE} = \gamma_1 \frac{Area_A}{4\pi\epsilon^2} + \gamma_2 log(\frac{p_0}{\epsilon}) + S_{finite}$$

$\epsilon \ll 1$: UV cutoff



$$\gamma_1 = \frac{2N^2 r_c^4}{R^4}$$

$$\gamma_2 = N^2 \left(1 + Area_A \left(\frac{\dot{a}_0}{a_0}\right)^2\right)$$:independent of UV cutoff ϵ

$$Area_A \equiv 4\pi(a_0(t)\gamma(p_0))^2 p_0^2$$: proper area of 4D FRW space

$$ds^2_{FRW_4} = -dt^2 + a_0^2(t)\gamma(p)^2(dp^2 + p^2 d\Omega_2^2) \qquad \gamma(p) \equiv \frac{1}{1 - \frac{p^2}{4}}$$

Entanglement Entropy

the Entanglement Entropy $\boldsymbol{S_{EE}}$

$$\boldsymbol{S_{EE}} = \boldsymbol{\gamma_1 \frac{Area_A}{4\pi\epsilon^2} + \gamma_2 log(\frac{p_0}{\epsilon}) + S_{finite}} \qquad \boldsymbol{\epsilon \ll 1}\textbf{: UV cutoff}$$

$$\boldsymbol{\gamma_2 = N^2 \left(1 + Area_A \left(\frac{\dot{a}_0}{a_0}\right)^2\right)}$$

The second term of $\boldsymbol{\gamma_2}$ is the effect of the curvature of FRW_4

(J. Maldacena, G.L. Pimentel 2013)

For $\boldsymbol{\frac{\dot{a}_0(t)}{a_0(t)} \ll 1}$, $\boldsymbol{\gamma_2 \sim N^2}$: degree of the freedoms in the dual field theory.

Entanglement entropy

Next we consider the UV finite term $\boldsymbol{S_{finite}}$.

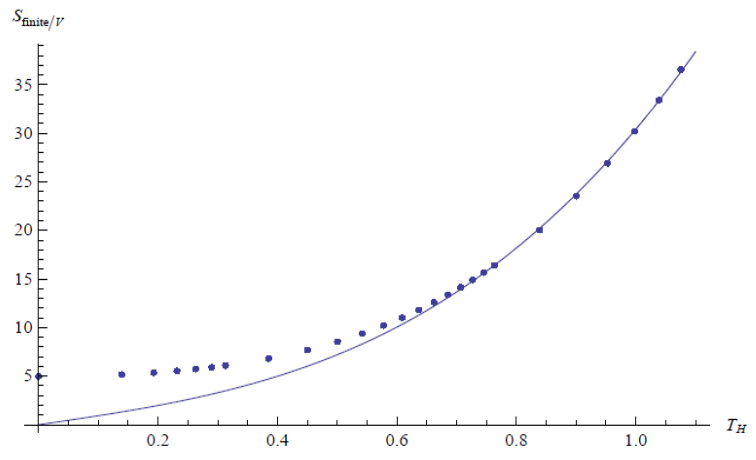
$$S_{EE} = \gamma_1 \frac{Area_A}{4\pi\epsilon^2} + \gamma_2 log(\frac{p_0}{\epsilon}) + S_{finite}$$

The Hawking temperature $\boldsymbol{T_H}$ is given by $T_H = \frac{r_H\left(1+\frac{r_0^2+\sqrt{c_0}R^2}{r_H^2}\right)}{\pi R^2 A(r_H)}$

By numerical calculations with fixed large size p_0, we find that

$\boldsymbol{S_{finite} \propto T_H^3}$ at large $\boldsymbol{T_H}$.

This is the same behavior as
the thermal entropy in 4D theory.



Summary

・We consider 5-dimensional bulk whose 4D boundary has FRW_4 metric with negative cosmological constant $-\boldsymbol{\lambda}$.

・There is also a parameter c_0 corresponding to the energy density of 4D boundary field theory.

・When cosmological constant is zero $\left(\boldsymbol{r_0 \equiv \frac{R^2}{2}\sqrt{\lambda} = 0}\right)$, 5D bulk becomes AdS5-Schwarzschild Black hole.

・When $\boldsymbol{0 \leq c_0 < \frac{r_0^4}{R^4}}$, dual UV field theory is in the "confinement phase".

The lowest glueball mass becomes massive.

Regge behavior by closed string calculation.

・When $\frac{\mathrm{r}_0^4}{\mathrm{R}^4} \leq \boldsymbol{c_0}$, an "event horizon" appears and dual field theory is in the deconfinement phase.

The lowest glueball mass becomes zero,

Holographic Entanglement Entropy becomes Thermal Entropy